1. Chapter 1
    1. chapter 1: Magnetic Circuits and Magnetic Materials
    2. PROBLEMS - chapter 1
2. Chapter 2
    1. Chapter 2: Transformers
    2. PROBLEMS - chapter 2
3. Chapter 3
    1. Chapter 3:Electromechanical-Energy-Conversion
    2. PROBLEMS - chapter 3
4. Chapter 4
    1. Chapter 4: Introduction to Rotating Machines
    2. PROBLEMS - chapter 4
5. Chapter 5
    1. Chapter 5: Synchronous Machines
    2. PROBLEMS - chapter 5
6. Chapter 6
    1. Chapter 6: Polyphase Induction Machines
    2. PROBLEMS - chapter 6
7. Chapter 7
    1. Chapter 7: DC Machines
    2. PROBLEMS - chapter 7
8. Chapter 8
    1. Chapter 8: Variable - Reluctance Machines and Stepping Motors
    2. PROBLEMS - chapter 8
9. Chapter 9
    1. Chapter 9: Single- and Two-Phase Motors
    2. PROBLEMS - chapter 9
10. Chapter 10
    1. Speed and Torque Control

chapter 1: Magnetic Circuits and Magnetic Materials

Chapter 1: Magnetic Circuits and Magnetic Materials

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- The objective of this course is to study the devices used in the interconversion of electric and mechanical energy, with emphasis placed on electromagnetic rotating machinery.
- The transformer, although not an electromechanical-energy-conversion device, is an important component of the overall energy-conversion process.
- Practically all transformers and electric machinery use ferro-magnetic material for shaping and directing the magnetic fields that acts as the medium for transferring and converting energy. Permanent-magnet materials are also widely used.
- The ability to analyze and describe systems containing magnetic materials is essential for designing and understanding electromechanical-energy-conversion devices.
- The techniques of magnetic-circuit analysis, which represent algebraic approximations to exact field-theory solutions, are widely used in the study of electromechanical-energy-conversion devices.

§1.1 Introduction to Magnetic Circuits

Assume the frequencies and sizes involved are such that the displacement-current term in Maxwell’s equations, which accounts for magnetic fields being produced in space by time-varying electric fields and is associated with electromagnetic radiations, can be neglected.

- H : magnetic field intensity, amperes/m, A/m, A-turn/m, A-t/m
- B : magnetic flux density, webers/m2, Wb/m2, tesla (T)
- 1 Wb = 10^8 lines (maxwells); 1 T = 10^4 gauss
- (1.1)From (1.1), we see that the source of H is the current density J .The line integral of the tangential component of the magnetic field

intensity H around a closed contour C is equal to the total current passing through any surface S linking that contour.

$$\oint_c H.\mathrm{dl} = \int_s J.\mathrm{da}$$

- (1.2)Equation (1.2) states that the magnetic flux density B is conserved. No net flux enters or leaves a closed surface.There exists no monopole charge sources of magnetic fields.

$$\oint_s B.\mathrm{da} = 0$$

- A magnetic circuit consists of a structure composed for the most part of high-permeability magnetic material. The presence of high-permeability material tends to cause magnetic flux to be confined to the paths defined by the structure.

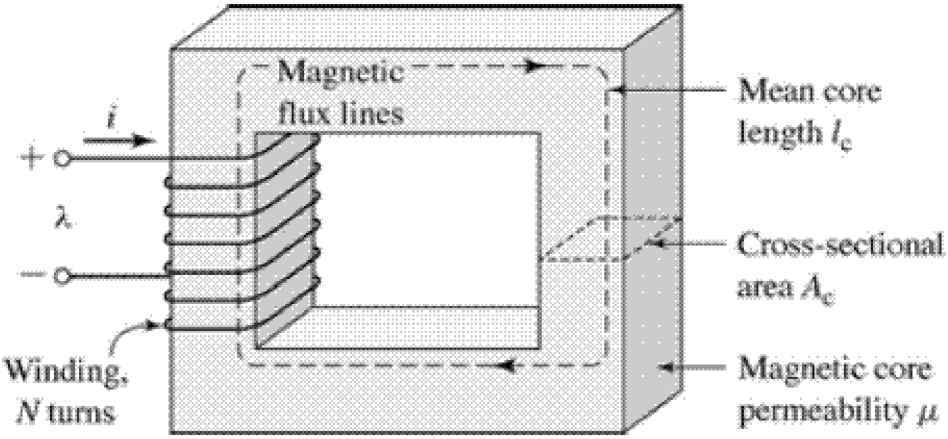


Figure 1.1Simple magnetic circuit.

- In Fig. 1.1, the source of the magnetic field in the core is the ampere-turn product N i , the magnetomotive force (mmf) F acting on the magnetic circuit.
- The magnetic flux φ (in weber, Wb) crossing a surface S is the surface integral of the normal component B :

$$\varphi = \oint_s B.\mathrm{da} \text{ (1.3)}$$

- φ_c : flux in core, B_c : flux density in core

$\varphi_c = B_c A_c$ (1.4)

- H_c : average magnitude H in the core. The direction of $H_c$can be found from the RHR.

$$F = \text{Ni} = \oint \text{Hdl} \qquad (1.5)$$
$$F = \text{Ni} = H_c l_c$$

- The relationship between the magnetic field intensity H and the magnetic flux density B:

$B = \mu H$ (1.6)

- Linear relationship?
- $\mu = \mu_r \mu_o$, μ: magnetic permeability, Wb/A-t-m = H/m
- $\mu_o = 4\pi.10^{-7}$: the permeability of free space
- μ_r: relative permeability, typical values: 2000-80,000

A magnetic circuit with an air gap is shown in Fig.1.2. Air gaps are present for moving elements. The air gap length is sufficiently small. φ : the flux in the magnetic circuit.

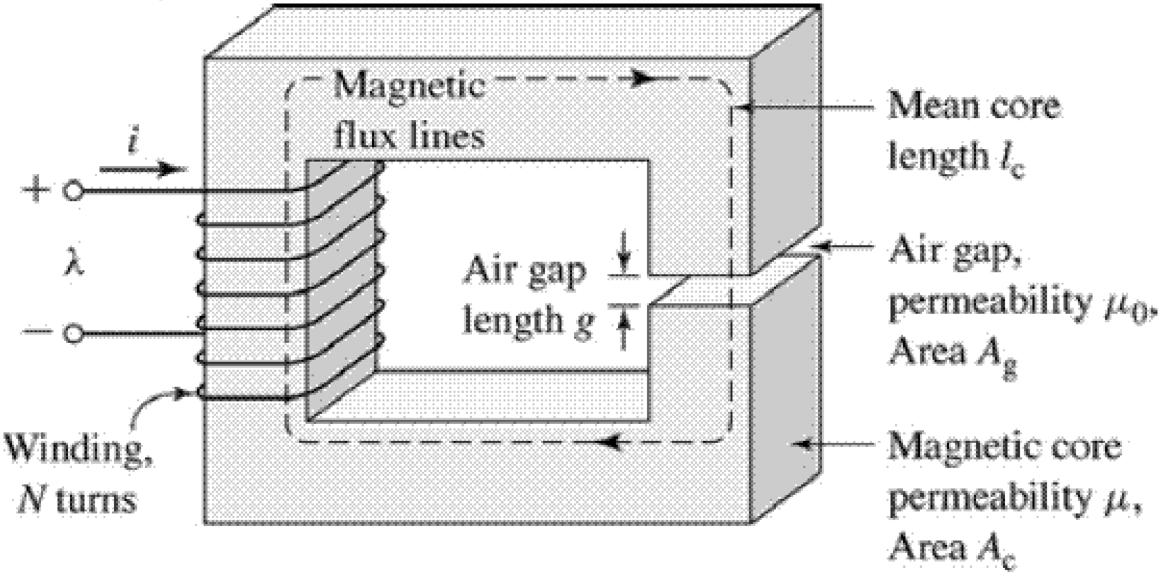

Magnetic flux lines
Mean core length l_c
i
+
λ
−
Air gap length g
Air gap, permeability μ_0, Area A_g
Winding, N turns
Magnetic core permeability μ, Area A_c

Figure 1.2Magnetic circuit with air gap.

$B_c = \frac{\varphi}{A_c}$ (1.7)

$B_g = \frac{\varphi}{A_g}$ (1.8)

$F = H_c l_c + H_g l_g$ (1.9)

$F = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g$ (1.10)

$F = \varphi(\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g})$ (1.11)

- R_c , R_g : the reluctance of the core and the air gap, respectively,

$R_c = \frac{l_c}{\mu A_c}$ (1.12)

$R_g = \frac{g}{\mu_0 A_g}$ (1.13)

$F = \varphi(R_c + R_g)$ (1.14)

$\varphi = \frac{F}{R_c + R_g}$ (1.15)

$\varphi = \frac{F}{\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}}$ (1.16)

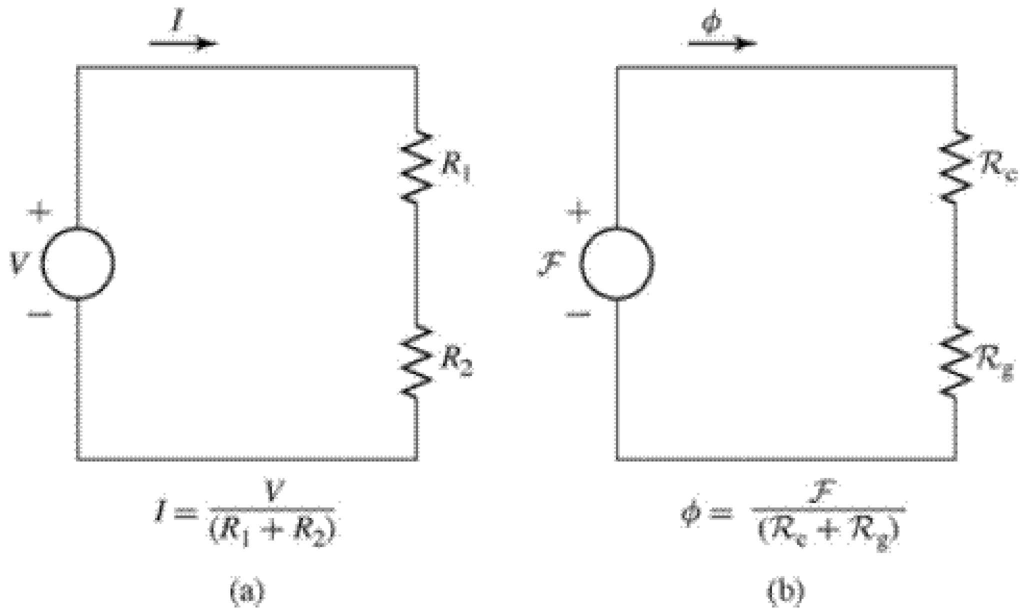
- In general, for any magnetic circuit of total reluctance R_{tot}, the flux can be found as

$\varphi = \frac{F}{R_{\text{tot}}}$ (1.17)

The permeance P is the inverse of the reluctance

$P_{\text{tot}} = \frac{1}{R_{\text{tot}}}$ (1.18)

- Fig. 1.3: Analogy between electric and magnetic circuits:



I
R_1
V
R_2
$I = \frac{V}{(R_1 + R_2)}$
(a)
ϕ
$\mathcal{R}_c$
$\mathcal{F}$
$\mathcal{R}_g$
$\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$
(b)

Figure 1.3:Analogy between electric and magnetic circuits: (a) electric ckt, (b) magnetic ckt.

- Note that with high material permeability: $R_c << R_g$ and thus $R_{\text{tot}} << R_g$

$$\varphi \approx \frac{F}{R_g} = \frac{F\mu_0 A_g}{g} = \mathrm{Ni}\frac{\mu_0 A_g}{g} \quad (1.19)$$

- Fig. 1.4: Fringing effect, effective A_g increased.

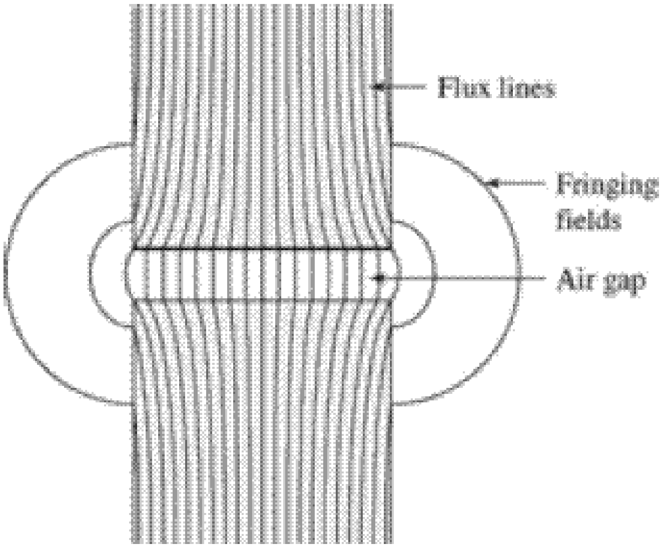


Figure 1.4 Air-gap fringing fields.

- In general, magnetic circuits can consist of multiple elements in series and parallel.

$F = \oint \mathrm{Hdl} = \sum_k F_k = \sum_k H_k l_k$ (1.20)

$F = \int_s J.\mathrm{da}$ (1.21)

$V = \sum_k R_k i_k$ (1.22)

$\sum_n i_n = 0$ (1.23)

$\sum_n \varphi_n = 0$ (1.24)

§1.2 Flux Linkage, Inductance, and Energy

- Faraday's Law:

$\oint_c E.\mathrm{ds} = -\frac{d}{\mathrm{dt}} \int_s B.\mathrm{da}$ (1.25)

- λ: the flux linkage of the winding, ϕ : the instantaneous value of a time-varying flux,
- e : the induced voltage at the winding terminals

$$\begin{gathered} e = N\frac{d\phi}{\mathrm{dt}} = \frac{d\lambda}{\mathrm{dt}} \\ \lambda = N\phi \end{gathered} \quad (1.26)$$

- L : the inductance (with material of constant permeability), H = Wb-t/A

$L = \frac{\lambda}{i}$ (1.27)

$L = \frac{N^2}{R_{\mathrm{tot}}}$ (1.28)

- The inductance of the winding in Fig. 1.2:

$L = \frac{N^2}{(g/\mu_0 A_g)} = \frac{N^2 \mu_0 A_g}{g}$ (1.29)

- Magnetic circuit with more than one windings, Fig. 1.5:

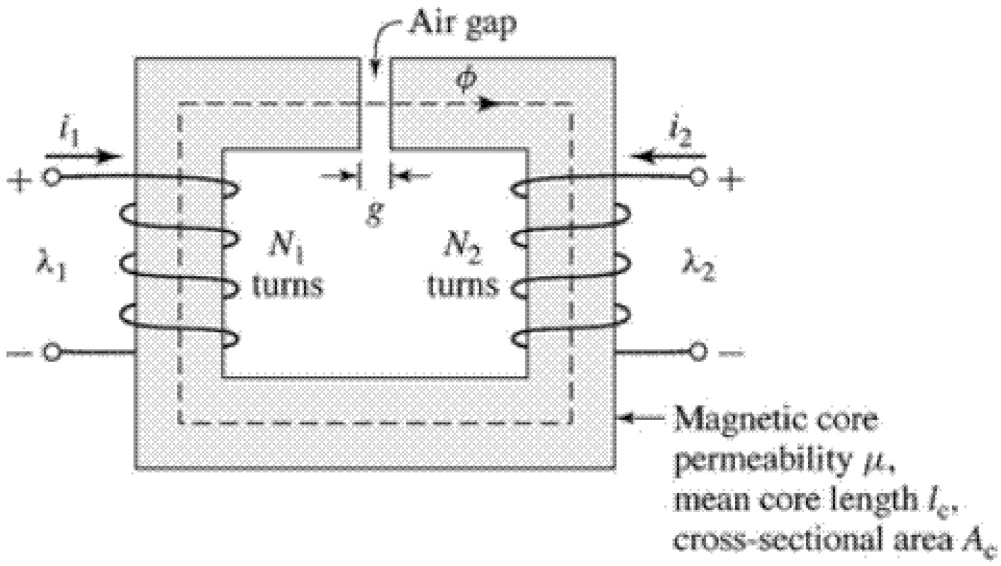

Air gap
ϕ
i_1
i_2
+
+
λ_1
λ_2
g
N_1 turns
N_2 turns
−
−
Magnetic core permeability μ, mean core length l_c, cross-sectional area A_c

Figure 1.5Magnetic circuit with two windings.

$F = N_1 i_1 + N_2 i_2$ (1.30)

$\varphi = (N_1 i_1 + N_2 i_2) \frac{\mu_0 A_c}{g}$ (1.31)

$\lambda_1 = N_1 \varphi = N_1^2 (\frac{\mu_0 A_c}{g}) i_1 + N_1 N_2 (\frac{\mu_0 A_c}{g}) i_2$ (1.32)

$\lambda_1 = L_{11} i_1 + L_{12} i_2$ (1.33)

$L_{11} = N_1^2 \frac{\mu_0 A_c}{g}$ (1.34)

$L_{12} = N_1 N_2 \frac{\mu_0 A_c}{g} = L_{21}$ (1.35)

$\lambda_2 = N_2 \varphi = N_1 N_2 (\frac{\mu_0 A_c}{g}) i_1 + N_2^2 (\frac{\mu_0 A_c}{g}) i_2$ (1.36)

$\lambda_2 = L_{21} i_1 + L_{22} i_2$ (1.37)

$L_{22} = N_2^2 \frac{\mu_0 A_c}{g}$ (1.38)

- Induced voltage, power (W = J/s), and stored energy:

$e = \frac{d}{\mathrm{dt}}(\mathrm{Li})$ (1.39)

$e = L\frac{\mathrm{di}}{\mathrm{dt}} + i\frac{\mathrm{dL}}{\mathrm{dt}}$ (1.40)

$p = \mathrm{ie} = i\frac{d\lambda}{\mathrm{dt}}$ (1.41)

$\varDelta W = \int_{t_1}^{t_2} \mathrm{pdt} = \int_{\lambda_1}^{\lambda_2} \mathrm{id}\lambda$ (1.42)

$\varDelta W = \int_{\lambda_1}^{\lambda_2} \mathrm{id}\lambda = \int_{\lambda_1}^{\lambda_2} \frac{\lambda}{L} d\lambda = \frac{1}{\mathrm{2L}}(\lambda_2^2 - \lambda_1^1)$ (1.43)

$W = \frac{1}{\mathrm{2L}}\lambda^2 = \frac{L}{2}i^2$ (1.44)

§1.3 Properties of Magnetic Materials

- The importance of magnetic materials is twofold:

- Magnetic materials are used to obtain large magnetic flux densities with relatively low levels of magnetizing force.
- Magnetic materials can be used to constrain and direct magnetic fields in well defined paths.

- Ferromagnetic materials, typically composed of iron and alloys of iron with cobalt, tungsten, nickel, aluminum, and other metals, are by far the most common magnetic materials.

- They are found to be composed of a large number of domains.
- When unmagnetized, the domain magnetic moments are randomly oriented.
- When an external magnetizing force is applied, the domain magnetic moments tend to align with the applied magnetic field until all the magnetic moments are aligned with the applied field, and the material is said to be fully saturated.

- When the applied field is reduced to zero, the magnetic dipole moments will no longer be totally random in their orientation and will retain a net magnetization component along the applied field direction.

- The relationship between B and H for a ferromagnetic material is both nonlinear and multivalued.

- In general, the characteristics of the material cannot be described analytically but are commonly presented in graphical form.
- The most common used curve is the B H curve.
- Dc or normal magnetization curve:
- Hysteresis loop (Note the remanance):

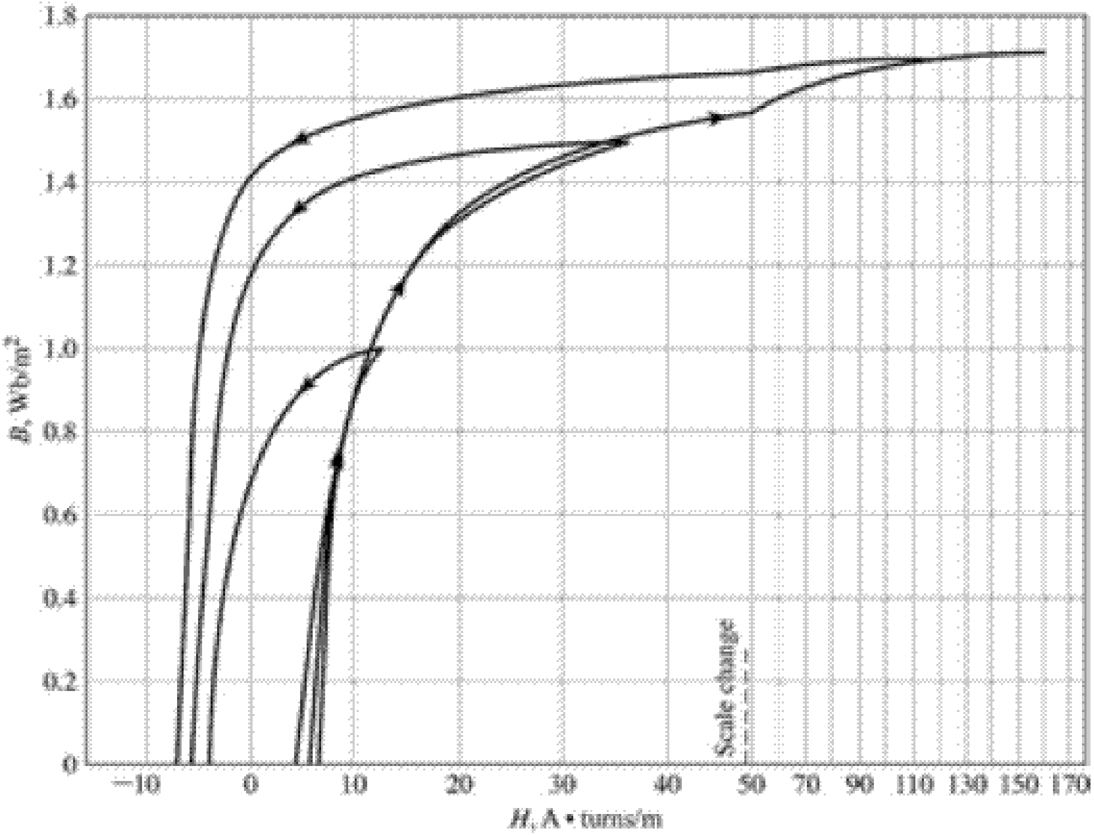


Figure 1.6B-H loops for M-5 grain-oriented electrical steel 0.012 in thick.

Only the top halves of the loops are shown here. (Armco Inc.)

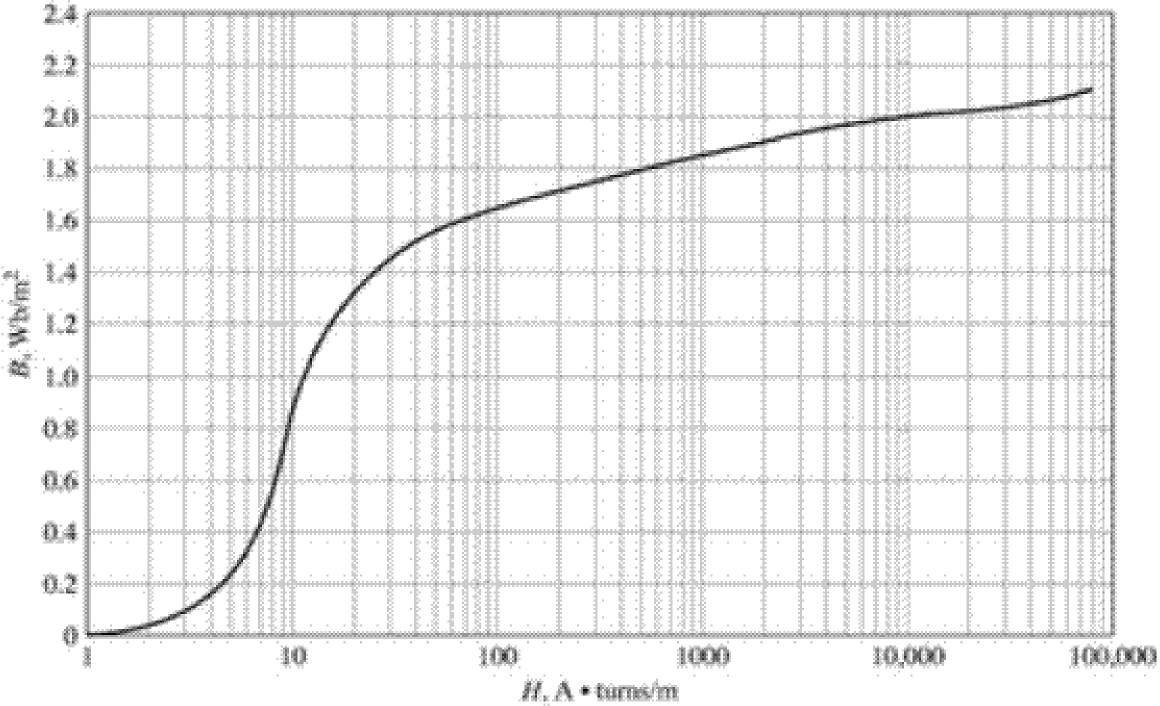


Figure 1.7 Dc magnetization curve for M-5 grain-oriented electrical steel 0.012 in thick.

(Armco Inc.)

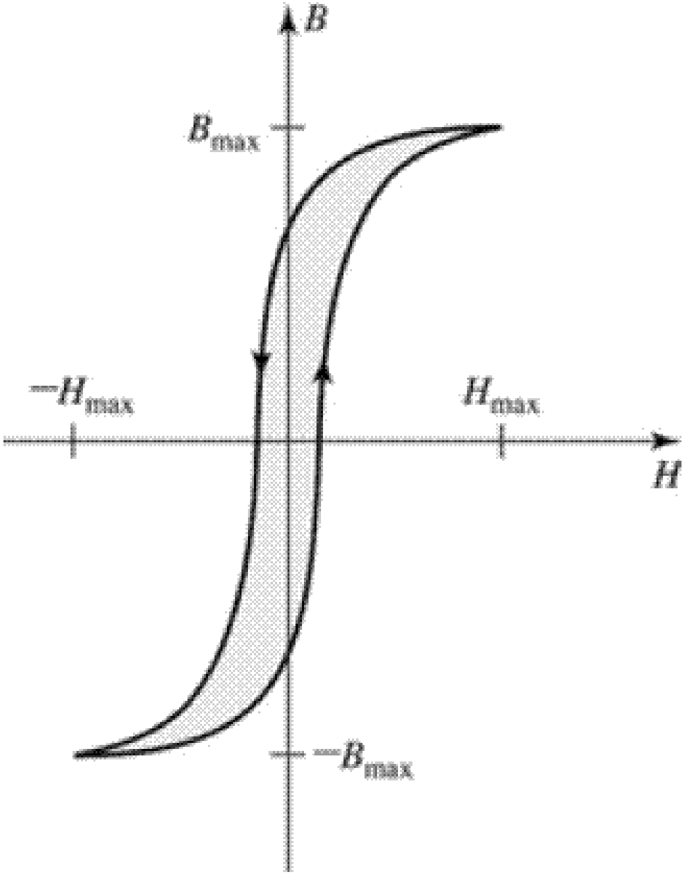


Figure 1.8Hysteresis loop.

§1.4 AC Excitation

- In ac power systems, the waveforms of voltage and flux closely approximate sinusoidal functions of time. We are to study the excitation characteristics and losses associated with magnetic materials under steady-state ac operating conditions.
- Assume a sinusoidal variation of the core flux $\phi(t)$:

$\phi(t) = \varphi_{\max}\text{sin}\omega t = A_c B_{\max}\text{sin}\omega t$ (1.45)

Where $\varphi_{\max}$ =amplitude of core flux in webers

$B_{\max}$ = amplitude of flux density B_c in teslas

ω =angular frequency = 2πf

f = frequency in Hz

- The voltage induced in the N-turn winding is

$e(t) = \omega N\varphi_{\max}\cos(\omega t) = E_{\max}\cos\omega t$ (1.46)

$E_{\max} = \omega N\varphi_{\max} = 2\pi \mathrm{fNA}_c B_{\max}$ (1.47)

- The Root-Mean-Squared (rms) value:

$F_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\mathrm{dt}}$ (1.48)

$E_{\mathrm{rms}} = \frac{2\pi}{\sqrt{2}}\mathrm{fNA}_c B_{\max} = \sqrt{2}\pi \mathrm{fNA}_c B_{\max}$ (1.49)

Note that the rums value of a sinusoidal wave is 12 times its peak value.

- Excitation phenomena, Fig. 1.9:

- ϕ vs $i_\phi \Leftrightarrow B_c$ vs H_c, i_ϕ : exciting current.
- Note that $\phi = B_c A_c$ and that $i_\phi = H_c l_c / N$.

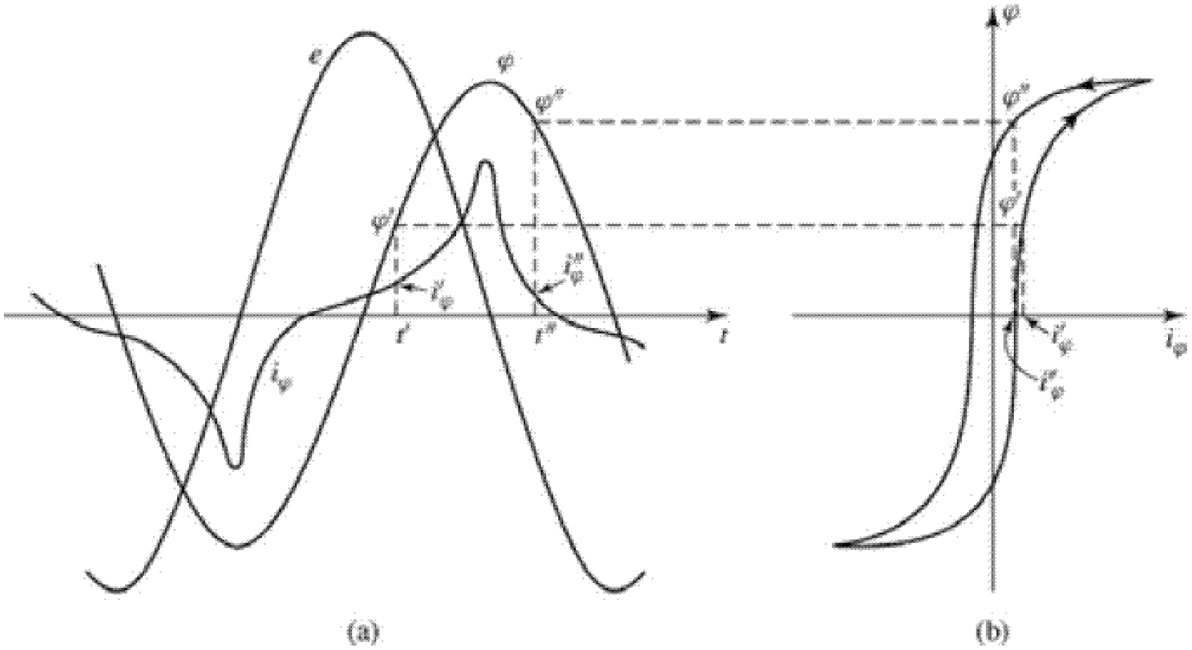


Figure 1.9Excitation phenomena. (a) Voltage, flux, and exciting current;

(b) corresponding hysteresis loop.

$I_{\phi,\text{rms}} = \frac{I_c H_{c,\text{rms}}}{N}$ (1.50)

$E_{\text{rms}} I_{\phi,\text{rms}} = \sqrt{2}\pi \text{fNA}_c B_{\max} \frac{I_c H_{\text{rms}}}{N}$ (1.51)

$= \sqrt{2}\pi \text{fNB}_{\max} H_{\text{rms}} (A_c l_c)$ (1.52)

$P_a = \frac{E_{\text{rms}} I_{\phi,\text{rms}}}{\text{mass}} = \frac{\sqrt{2}\pi f}{\rho_c} B_{\max} H_{\text{rms}}$ (1.53)

P_a: the exciting rms voltamperes per unit mass,

- The rms exciting voltampere can be seen to be a property of the material alone. It depends only on $B_{\max}$because H_{rms} is a unique function of $B_{\max}$.

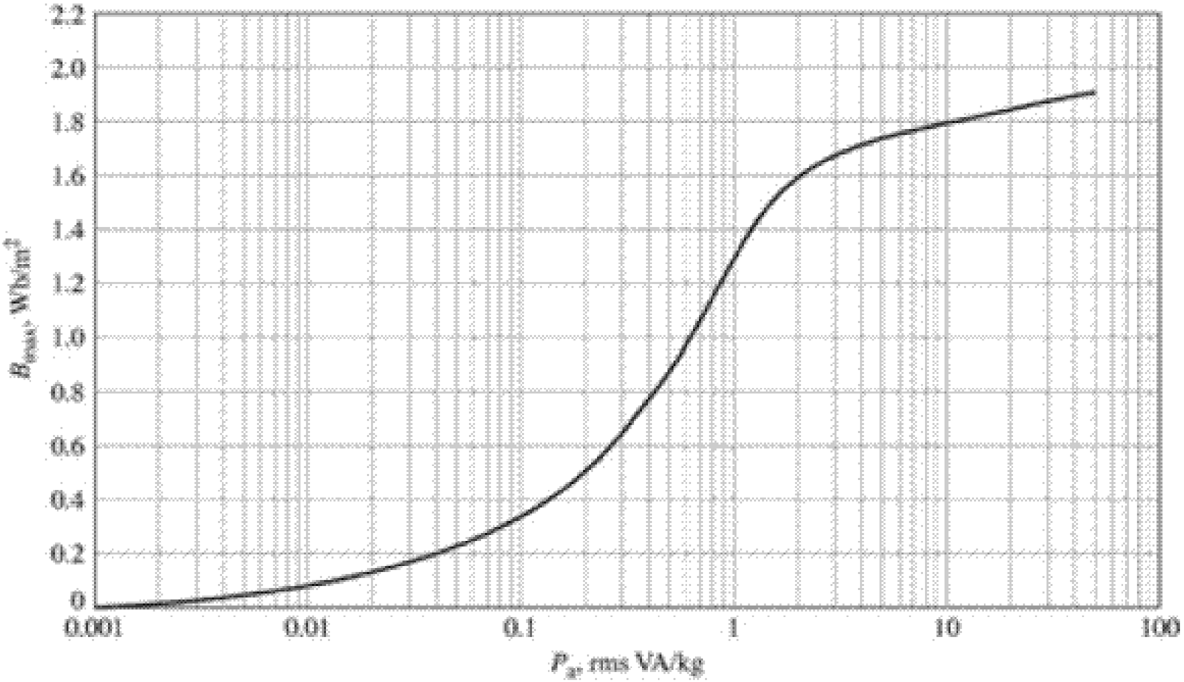


Figure 1.10Exciting rms voltamperes per kilogram at 60 Hz for

M-5 grain-oriented electrical steel 0.012 in thick. (Armco Inc.)

- The exciting current supplies the mmf required to produce the core flux and the power input associated with the energy in the magnetic field in the core.

- Part of this energy is dissipated as losses and results in heating of the core.
- The rest appears as reactive power associated with energy storage in the magnetic field.

This reactive power is not dissipated in the core; it is cyclically supplied and absorbed by the excitation source.

- Two loss mechanisms are associated with time-varying fluxes in magnetic materials.

- The first is ohmic I^2R heating, associated with induced currents in the core material.

  - Eddy currents circulate and oppose changes in flux density in the material.
  - To reduce the effects, magnetic structures are usually built of thin sheets of laminations of the magnetic material.
  - Eddy-current loss $f^2,B_{\max}^2$.

- The second loss mechanic is due to the hysteretic nature of magnetic material.

  - The energy input W to the core as the material undergoes a single cycle

$$W = \oint i_\phi d\lambda = \oint(\frac{H_c l_c}{N})(A_c \mathrm{NdB}_c) = A_c l_c \oint H_c \mathrm{dB}_c \quad (1.54)$$

-
  - For a given flux level, the corresponding hysteresis losses are proportional to the area of the hysteresis loop and to the total volume of material.
  - Hysteresis power loss f.

- Information on core loss is typically presented in graphical form. It is plotted in terms of watts per unit weight as a function of flux density; often a family of curves for different frequencies are given. See Fig.1.12.

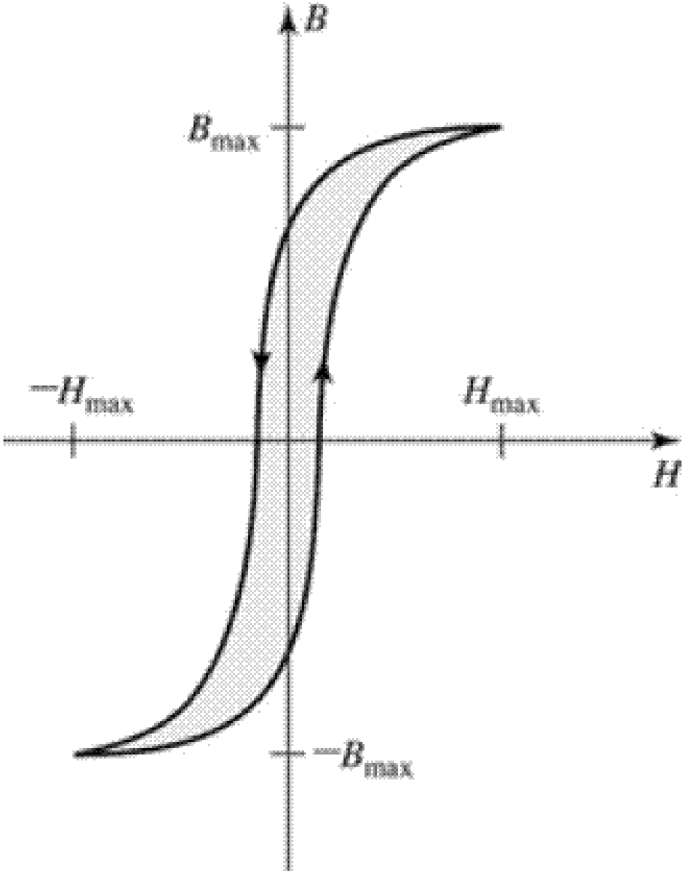


Figure 1.11Hysteresis loop; hysteresis loss is proportional to the loop area (shaded).

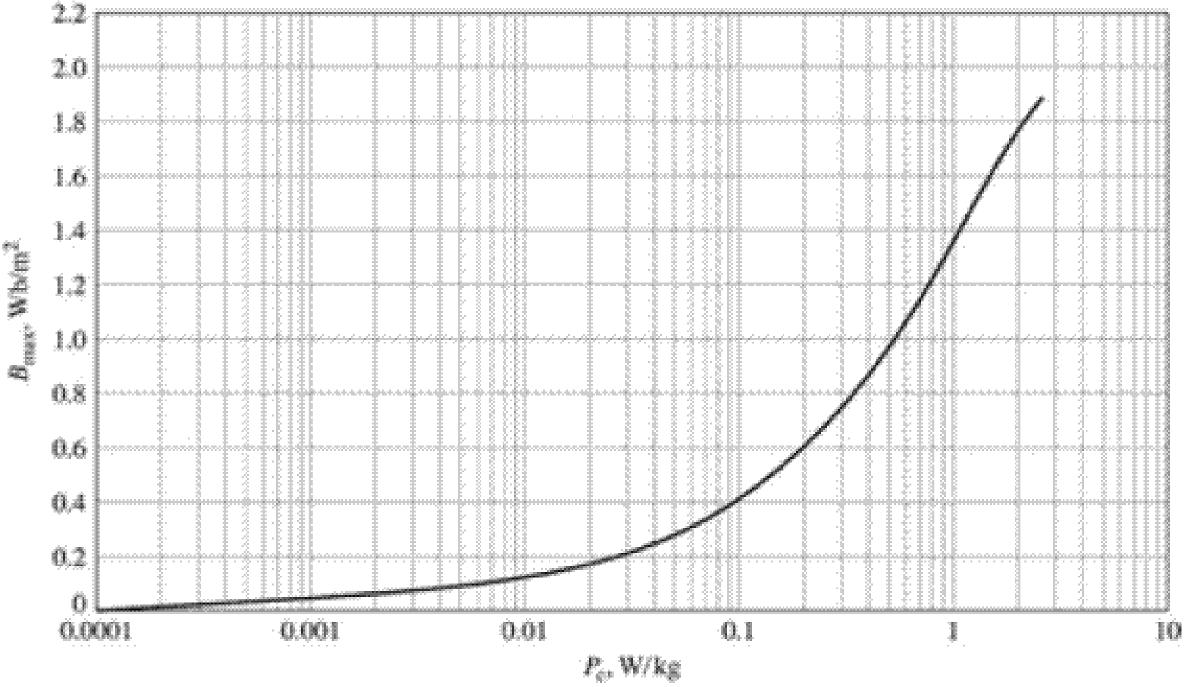


Figure 1.12Core loss at 60 Hz in watts per kilogram for

M-5 grain-oriented electrical steel 0.012 in thick. (Armco Inc).

§1.5 Permanent Magnets

- Certain magnetic materials, commonly known as permanent-magnet materials, are characterized by large values of remanent magnetization and coercivity. These materials produce significant magnetic flux even in magnetic circuits with air gaps.
- The second quadrant of a hysteresis loop (the magnetization curve) is usually employed for analyzing a permanent-magnet material.

- B_r : residual flux density or remanent magnetization,
- H_c: coercivity, (1) a measure of the magnitude of the mmf required to demagnetize the material, and (2) a measure of the capability of the material to produce flux in a magnetic circuit which includes an air gap.
- Large value (> 1 kA/m): hard magnetic material, o.w.: soft magnetic material
- Fig. 1.13(a): Alnico 5, $B_r \simeq 1.22T$, $H_c \simeq -49\mathrm{kA/m}$

- Fig. 1.13(b): M-5 steel, $B_r \simeq 1.4\mathrm{T}$,$H_c \simeq -6\mathrm{kA/m}$
- Both Alnico 5 and M-5 electrical steel would be useful in producing flux in unexcited magnetic circuits since they both have large values of remanent magnetization.

- The significant of remanent magnetization is that it can produce magnetic flux in a magnetic circuit in the absence of external excitation (such as winding currents).

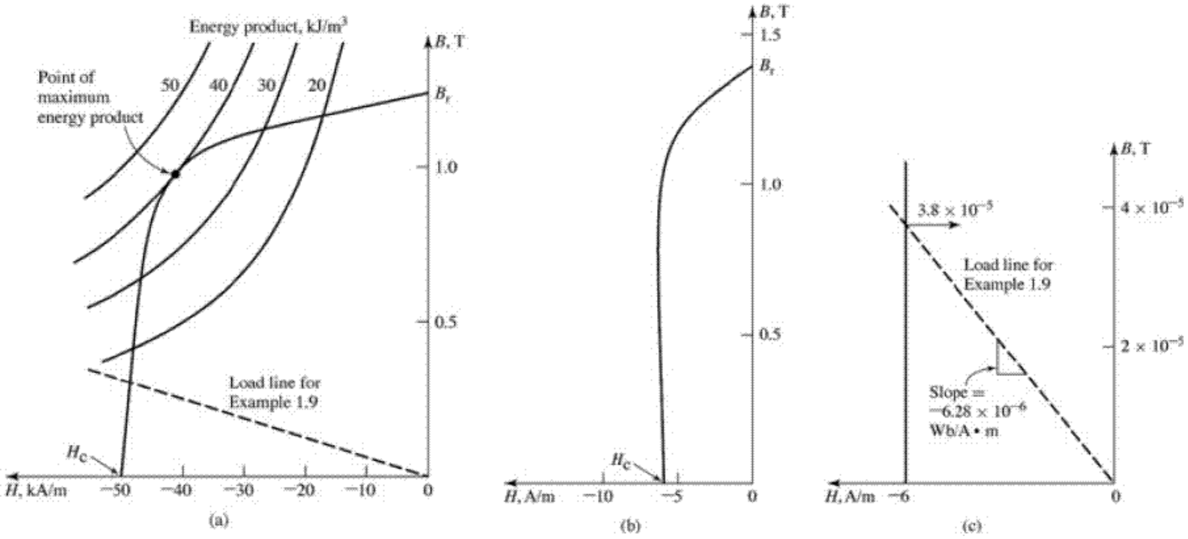


Figure 1.13 (a) Second quadrant of hysteresis loop for Alnico 5; (b) second quadrant of hysteresis loop for M-5 electrical steel; (c) hysteresis loop for M-5 electrical steel expanded for small B. (Armco Inc.)

- Maximum Energy Product: a useful measure of the capability of permanent-magnet material.

- The product of B and H has the dimension of energy density (J/m^3)
- Choosing a material with the largest available maximum energy product can result in the smallest required magnet volume.
- (1.55) can be obtained:

$B_g = \frac{A_m}{A_g} B_m$ (1.55)

$\frac{H_m l_m}{H_g g} = -1$ (1.56)

$$B_g^2 = \mu_0\left(\frac{l_m A_m}{\mathrm{gA}_g}\right)(-H_m B_m) = \mu_0\left(\frac{\mathrm{Vol_{mag}}}{\mathrm{Vol_{airgap}}}\right)(-H_m B_m) \quad (1.57)$$

$$\mathrm{Vol_{mag}} = \frac{\mathrm{Vol_{airgap}} B_g^2}{\mu_0(-H_m B_m)} \quad (1.58)$$

- Equation (1.58) indicates that to achieve a desired flux density in the air gap the required volume of the magnet can be minimized by operating the magnet at the point of maximum energy product.
- A curve of constant B-H product is a hyperbola.
- In Fig.1.13, the maximum energy product for Alnico 5 is 40 kJ/m^3, occurring at the point B =1.0 T and H=40 kA/m .

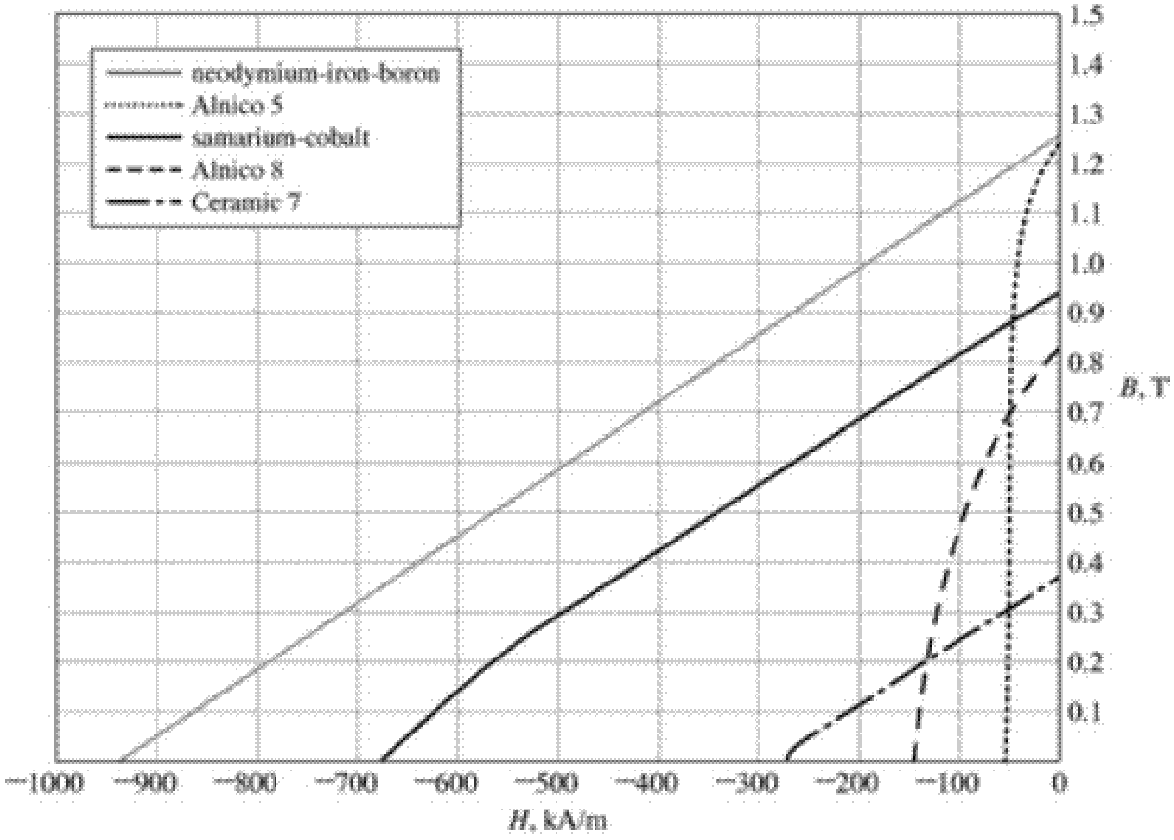


Figure 1.14Magnetization curves for common permanent-magnet materials.

Figure 1.14 shows the magnetization characteristics for a few common permanent magnet materials. Alnico 5 is a widely used alloy of iron, nickel, aluminum, and cobalt, originally discovered in 1931. It has a relatively large residual flux density. Alnico 8 has a lower residual flux density and a higher coercivity than Alnico 5. Hence, it isless subject to demagnetization than Alnico 5. Disadvantages of the Alnico materialsare their relatively low coercivity and their mechanical brittleness.

Ceramic permanent magnet materials (also known as ferrite magnets) are made from iron-oxide and barium- or strontium-carbonate powders and have lower residual flux densities than Alnico materials but significantly higher coercivities. As a result, they are much less prone to demagnetization. One such material, Ceramic 7, is shown in Fig.1.14, where its magnetization characteristic is almost a straight line. Ceramic magnets have good mechanical characteristics and are inexpensive to manufacture; as a result, they are the widely used in many permanent magnet applications.

Samarium-cobalt represents a significant advance in permanent magnet technology which began in the 1960s with the discovery of rare earth permanent magnet materials. From Fig.1.14 it can be seen to have a high residual flux density such as is found with the Alnico materials, while at the same time having a much higher coercivity and maximum energy product. The newest of the rare earth magnetic materials is the neodymium-iron-boron material. It features even larger residual flux density, coercivity, and maximum energy product than does samarium-cobalt.

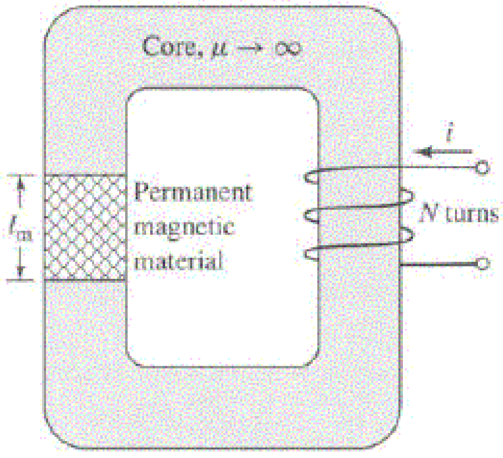


Figure 1.15 Magnetic circuit including both a permanent magnet and an excitation winding.

Consider the magnetic circuit of Fig.1.15. This includes a section of hard magnetic material in a core of highly permeable soft magnetic material as well as an N-turn excitation winding. With reference to Fig.1.16, we assume that the hard magnetic material is initially unmagnetized (corresponding to point a of the figure) and consider what happens as current is applied to the excitation winding. Because the core is assumed to be of infinite permeability, the horizontal axis of Fig.1.16 can be considered to be both a measure of the applied current $i = \mathrm{Hl}_m/N$ as well as a measure of H in the magnetic material.

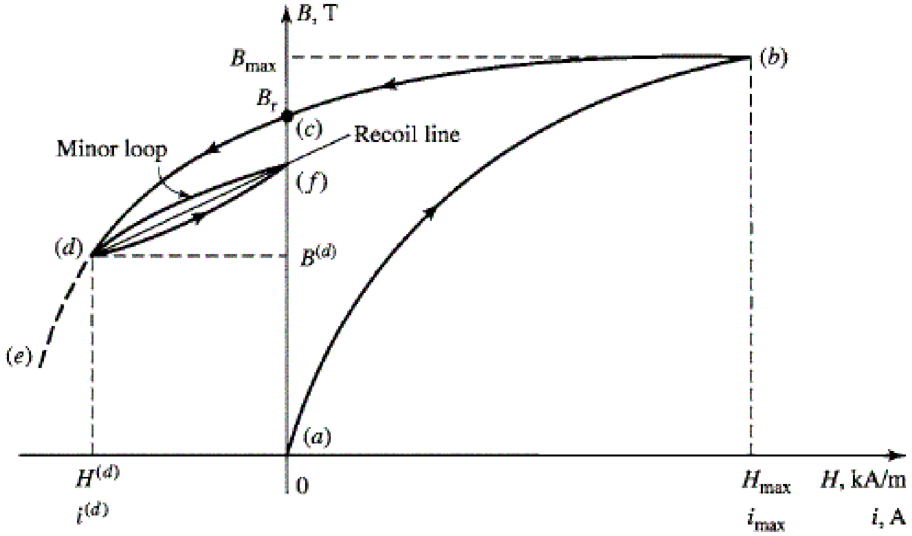


Figure 1.16 Portion of a B-H characteristic showing a minor loop and a recoil line.

As the current i is increased to its maximum value, the B-H trajectory rises from point a in Fig.1.16 toward its maximum value at point b. To fully magnetize the material, we assume that the current has been increased to a value/max sufficiently large that the material has been driven well into saturation at point b. When the current is then decreased to zero, the B-H characteristic will begin to form a hysteresis loop, arriving at point c at zero current. At point c, notice that H in the material is zero but B is at its remanent value Br.

As the current then goes negative, the B-H characteristic continues to trace out a hysteresis loop. In Fig. 1.16, this is seen as the trajectory between points c and d. If the current is then maintained at the value $-i^{(d)}$, the operating point of the magnet will be that of point d. Note that, this same operating point would be reached if the material were to start at point c and, with the excitation held at zero, an air gap of length $g = l_m(A_g/A_m)(-\mu_o H^{(d)}/B^{(d)})$ were then inserted in the core. Should the current then be made more negative, the trajectory would continue tracing out the hysteresis loop toward point e. However, if instead the current is returned to zero, the trajectory does not in general retrace the hysteresis loop toward point c. Rather it begins to trace out a minor hysteresis loop, reaching point f when the current reaches zero. If the current is then varied between zero and $-i^{(d)}$, the B-H characteristic will trace out the minor loop as shown.

As can be seen from Fig. 1.16, the B-H trajectory between points d and f can be represented by a straight line, known as the recoil line. The slope of this line is called the recoil permeability μ_R. We see that once this material has been demagnetized to point d, the effective remanent magnetization of the magnetic material is that of point f which is less than the remanent magnetization B_r which would be expected based on the hysteresis loop. Note that should the demagnetization be increased past point d, for example, to point e of Fig.1.16, a new minor loop will be created, with a new recoil line and recoil permeability.

The demagnetization effects of negative excitation which have just been discussed are equivalent to those of an air gap in the magnetic circuit. For example, clearly the magnetic circuit of Fig.1.15 could be used as a system to magnetize hard magnetic materials. The process would simply require that a large excitation be applied to the winding and then reduced to zero, leaving the material at a remanent magnetization B_r (point c in Fig.1.16).

Following this magnetization process, if the material were removed from the core, this would be equivalent to opening a large air gap in the magnetic circuit, demagnetizing the material. At this point, the magnet has been effectively weakened, since if it were again inserted in the magnetic core, it would follow a recoil line and return to a remanent magnetization somewhat less than B_r. As a result, hard magnetic materials, such as the Alnico materials of Fig.1.14, often do not operate stably in situations with varying mmf and geometry, and there is often the risk that improper operation can significantly demagnetize them. A significant advantage of materials such as Ceramic 7, samarium-cobalt and neodymium-iron-boron is that, because of their "straight-line" characteristic in the second quadrant (with slope close to μ_o), their recoil lines closely match their magnetization characteristic. As a result, demagnetization effects are significantly reduced in these materials and often can be ignored.

At the expense of a reduction in value of the remanent magnetization, hard magnetic materials can be stabilized to operate over a specified region.

PROBLEMS - chapter 1

PROBLEMS

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

1.1 A magnetic circuit with a single air gap is shown in Fig.1.1. The core dimensions are:

Cross-sectional area Ac = $1.8\text{x}10^{-3}m^2$

Mean core length lc = 0.6 m

Gap length g = 2.3 x 10^{-3} m

N = 83 turns

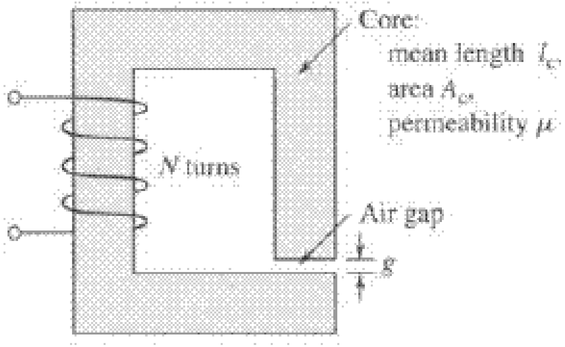


Figure 1.1 Magnetic circuit.

Assume that the core is of infinite permeability ($\mu \to \infty$) and neglect the effects of fringing fields at the air gap and leakage flux. (a) Calculate the reluctance of the core R_C and that of the gap R_g. For a current of i = 1.5 A, calculate (b) the total flux φ, (c) the flux linkages) λ of the coil, and (d) the coil inductance L.

1.2 Repeat Problem 1.1 for a finite core permeability of $\mu = 2500\mu_0$.

1.3 Consider the magnetic circuit of Fig.1.1 with the dimensions of Problem1.1. Assuming infinite core permeability, calculate (a) the number of turns required to achieve an inductance of 12 mH and (b) the inductor current which will result in a core flux density of 1.0 T.

1.4 Repeat Problem 1.3 for a core permeability of $\mu = 1300\mu_0$.

1.5 The magnetic circuit of Problem 1.1 has a nonlinear core material whose permeability as a function of B_m is given by

$$\mu = \mu_o\left[1 + \frac{3499}{\sqrt{1+0.047(B_m)^{7.8}}}\right]$$

where B_m is the material flux density.

a. Using MATLAB, plot a dc magnetization curve for this material (B_m vs. H_m) over the range 0 $B \leq$ 2.2 T.

b. Find the current required to achieve a flux density of 2.2 T in the core.

c. Again, using MATLAB, plot the coil flux linkages as a function of coil current as the current is varied from 0 to the value found in part (b).

1.6 The magnetic circuit of Fig.1.2 consists of a core and a moveable plunger of width l_p, each of permeability . The core has cross-sectional area Ac and mean length . The overlap area of the two air gaps Ag is a function of the plunger position x and can be assumed to vary as

$$A_g = A_c\left[1 - \frac{x}{X_0}\right]$$

You may neglect any fringing fields at the air gap and use approximations consistent with magnetic-circuit analysis.

a. Assuming that $\mu \rightarrow \infty$, derive an expression for the magnetic flux density in the air gap B_g as a function of the winding current I and as the

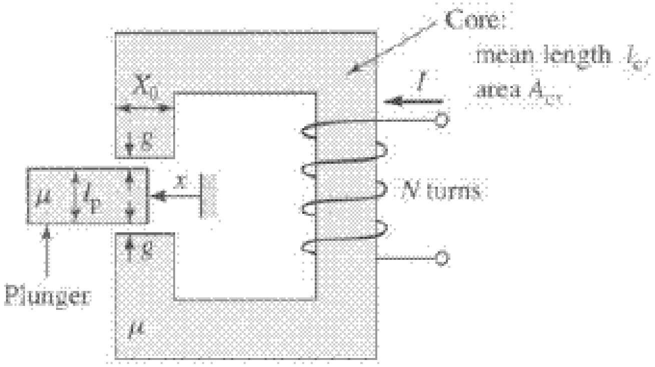


Figure 1.2 Magnetic circuit for Problem 1.6.

plunger position is varied ($0 \leq x \leq 0.8\mathrm{X}_0$). What is the corresponding flux density in the core?

b. Repeat part (a) for a finite permeability μ.

1.7 The magnetic circuit of Fig.1.2 and Problem 1.6 has the following dimensions"

$A_C = 8.2\mathrm{cm}^2$ $l_C = 23\mathrm{cm}$

$l_p = 2.8\mathrm{cm}$ g = 0.8 mm

X_0 = 2.5 cm N = 430 turns

a. Assuming a constant permeability of $\mu = 2800\mu_0$, calculate the current required to achieve a flux density of 1.3T in the air gap when the plunger is fully retracted (x =0).

b. Repeat the calculation of part (a) for the case in which the core and plunger are composed of a nonlinear material whose permeability is given by

$$\mu = \mu_0\left[1 + \frac{1199}{\sqrt{1+0.05B_m^8}}\right]$$

where B_m is the magnetic flux density in the material.

c. For the nonlinear material of part (b), use MATLAB to plot the air-gap flux density as a function of winding current for x = 0 and x = 0.5 X_0.

1.8 An inductor of the form of Fig.1.1 has dimensions:

Cross-sectional area $A_C = 3.6\mathrm{cm}^2$

Mean core length l_C = 15 cm

N - 75 turns

Assuming a core permeability of $\mu = 2100\mu_0$ and neglecting the effects of leakage flux and fringing fields, calculate the air-gap length required to achieve an inductance of 6.0 mH.

1.9 The magnetic circuit of Fig.1.3 consists of rings of magnetic material in a stack of height h. The rings have inner radius Ri and outer radius Ro. Assume that the iron is of infinite permeability ($\mu \to \infty$) and neglect the effects of magnetic leakage and fringing. For:

R_i =3.4 cm

R_0 = 4.0 cm

h =2cm

g = 0.2 cm

calculate:

a. the mean core length l_C and the core cross-sectional area A_C.

b. the reluctance of the core R_C and that of the gap R_g.

For N = 65 turns, calculate:

c. the inductance L.

d. current i required to operate at an air-gap flux density of B_g = 1.35T.

e. the corresponding flux linkages ***SORRY, THIS MEDIA TYPE IS NOT SUPPORTED.*** of the coil.

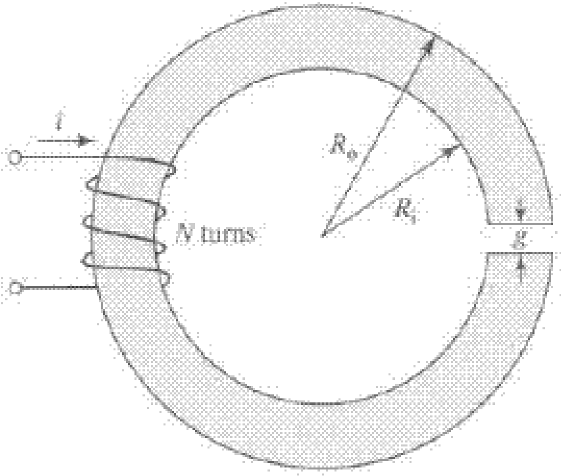


Figure 1.3 Magnetic circuit.

1.10 Repeat Problem 1.9 for a core permeability of ***SORRY, THIS MEDIA TYPE IS NOT SUPPORTED.*** .

1.11 Using MATLAB, plot the inductance of the inductor of Problem 1.9 as a function of relative core permeability as the core permeability varies for ***SORRY, THIS MEDIA TYPE IS NOT SUPPORTED.*** = 100 to μ_x = 10000. (Hint: Plot the inductance versus the log of the relative permeability.) What is the minimum relative core permeability required to insure that the inductance is within 5 percent of the value calculated assuming that the core permeability is infinite?

1.12 The inductor of Fig. 1.4 has a core of uniform circular cross-section of area A_C, mean length lc and relative permeability/Zr and an N-turn winding. Write an expression for the inductance L.

1.13 The inductor of Fig.1.27 has the following dimensions:

A_C = 1.0 cm^2

l_C = 15 cm

g = 0.8 mm

N = 480 turns

Neglecting leakage and fringing and assuming μ_x = 1000, calculate the inductance.

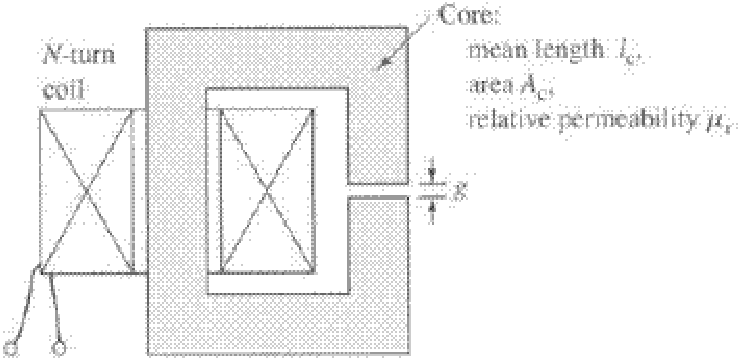


Figure 1.4 Inductor.

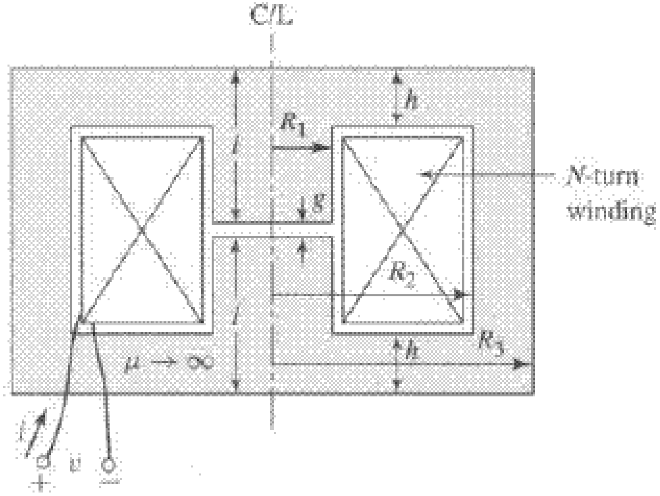


Figure 1.5 Pot-core inductor .

1.14 The inductor of Problem 1.13 is to be operated from a 60-Hz voltage source. (a) Assuming negligible coil resistance, calculate the rms inductor voltage corresponding to a peak core flux density of 1.5 T. (b) Under this operating condition, calculate the rms current and the peak stored energy.

1.15 Consider the magnetic circuit of Fig. 1.5. This structure, known as a pot-core, is typically made in two halves. The N-turn coil is wound on a cylindrical bobbin and can be easily inserted over the central post of the core as the two halves are assembled. Because the air gap is internal to the core, provided the core is not driven excessively into saturation, relatively little magnetic flux will "leak" from the core, making this a particularly attractive configuration for a wide variety of applications, both for inductors such as that of Fig. 1.27 and transformers.

Assume the core permeability to be $\mu = 2500\mu_0$ and N = 200 turns. The following dimensions are specified:

R_1= 1.5cm R_2= 4cm l = 2.5cm

h = 0.75cm g = 0.5mm

a. Find the value of R_3 such that the flux density in the outer wall of the core is equal to that within the central cylinder.

b. Although the flux density in the radial sections of the core (the sections of thickness h) actually decreases with radius, assume that the flux density remains uniform. (i) Write an expression for the coil inductance and (ii) evaluate it for the given dimensions.

c. The core is to be operated at a peak flux density of 0.8 T at a frequency of 60 Hz. Find (i) the corresponding rms value of the voltage induced in the winding, (ii) the rms coil current, and (iii) the peak stored energy.

d. Repeat part (c) for a frequency of 50 Hz.

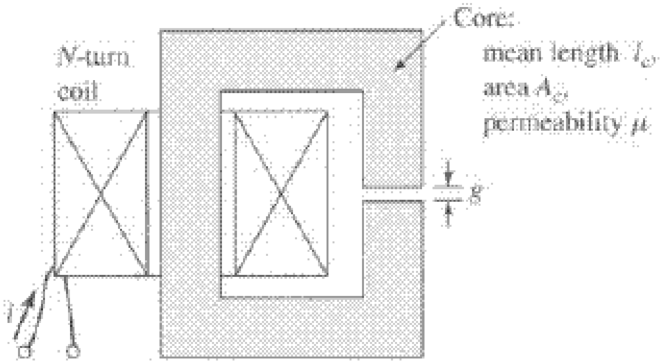


Figure 1.6 Inductor.

1.16 A square voltage wave having a fundamental frequency of 60 Hz and equal positive and negative half cycles of amplitude E is applied to a 1000-

turn winding surrounding a closed iron core of 1.25 x 10-3 m^2 cross section. Neglect both the winding resistance and any effects of leakage flux.

a. Sketch the voltage, the winding flux linkage, and the core flux as a function of time.

b. Find the maximum permissible value of E if the maximum flux density is not to exceed 1.15 T.

1.17 An inductor is to be designed using a magnetic core of the form of that of Fig.1.6. The core is of uniform cross-sectional area $A_C = 5.0\mathrm{cm}^2$ and of mean length I_C=25cm.

a. Calculate the air-gap length g and the number of turns N such that the inductance is 1.4 mH and so that the inductor can operate at peak currents of 6 A without saturating. Assume that saturation occurs when the peak flux density in the core exceeds 1.7 T and that, below saturation, the core has permeability $\mu = 3200\mu_0$.

b. For an inductor current of 6 A, use Eq. $W_{\mathrm{fld}} = \int\limits_V \left[\frac{B^2}{2\mu}\right]$dV to calculate (i) the magnetic stored energy in the air gap and (ii) the magnetic stored energy in the core. Show that the total magnetic stored energy is given by Eq. $T_{\mathrm{fld}} = \frac{i^2}{2}\frac{\mathrm{dL}(\theta)}{d\theta}$.

1.18 Consider the inductor of Problem 1.17. Write a simple design program in the form of a MATLAB script to calculate the number of turns and air-gap length as a function of the desired inductance. The script should be written to request a value of inductance (in mH) from the user, with the output being the air-gap length in mm and the number of turns.

The inductor is to be operated with a sinusoidal current at 60 Hz, and it must be designed such that the peak core flux density will be equal to 1.7 T when the inductor current is equal to 4.5 A rms. Write your script to reject any designs for which the gap length is out of the range of 0.05 mm to 5.0 mm or for which the number of turns drops below 5.

Using your program find (a) the minimum and (b) the maximum inductances (to the nearest mH) which will satisfy the given constraints. For each of these values, find the required air-gap length and the number of turns as well as the rms voltage corresponding to the peak core flux.

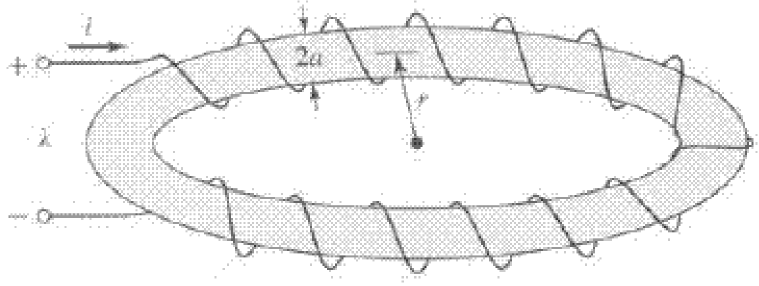


Figure 1.7 Toroidal winding.

1.19 A proposed energy storage mechanism consists of an N-turn coil wound around a large nonmagnetic ($\mu = \mu_0$) toroidal form as shown in Fig. 1.7. As can be seen from the figure, the toroidal form has a circular cross section of radius a and toroidal radius r, measured to the center of the cross section. The geometry of this device is such that the magnetic field can be considered to be zero everywhere outside the toroid. Under the assumption that a r, the H field inside the toroid can be considered to be directed around the toroid and of uniform magnitude

$$H = \frac{\mathrm{Ni}}{2\pi\mathrm{r}}$$

For a coil with N = 1000 turns, r = 10 m, and a = 0.45 m:

a. Calculate the coil inductance L.

b. The coil is to be charged to a magnetic flux density of 1.75 T. Calculate the total stored magnetic energy in the toms when this flux density is achieved.

c. If the coil is to be charged at a uniform rate (i.e., di/dt = constant), calculate the terminal voltage required to achieve the required flux density in 30 sec. Assume the coil resistance to be negligible.

1.20 Figure 1.8 shows an inductor wound on a laminated iron core of rectangular cross section. Assume that the permeability of the iron is infinite. Neglect magnetic leakage and fringing in the two air gaps (total gap length = g). The N-turn winding is insulated copper wire whose resistivity is ρ Ω.m. Assume that the fraction f_w of the winding space is available for copper; the rest of the space is used for insulation.

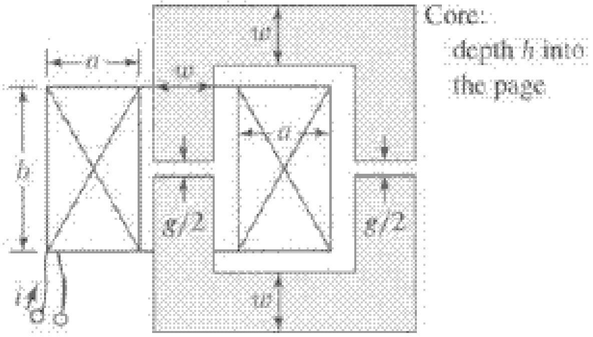


Figure 1.8 Iron-core inductor.

a. Calculate the cross-sectional area and volume of the copper in the winding space.

b. Write an expression for the flux density B in the inductor in terms of the current density J_{cu} in the copper winding.

c. Write an expression for the copper current density J_{cu} in terms of the coil current I, the number of turns N, and the coil geometry.

d. Derive an expression for the electric power dissipation in the coil in terms of the current density J_{cu}.

e. Derive an expression for the magnetic stored energy in the inductor in terms of the applied current density J_{cu}.

f. From parts (d) and (e) derive an expression for the L/R time constant of the inductor. Note that this expression is independent of the number of turns in the coil and does not change as the inductance and coil resistance are changed by varying the number of turns.

1.21 The inductor of Fig. 1.8 has the following dimensions:

a = h = ω = 1.5cm b = 2cm g = 0.2cm

The winding factor (i.e., the fraction of the total winding area occupied by conductor) is f_w = 0.55. The resistivity of copper is $1.73x10^{-8}\Omega.m$. When the coil is operated with a constant dc applied voltage of 35 V, the air-gap flux density is measured to be 1.4 T. Find the power dissipated in the coil, coil current, number of turns, coil resistance, inductance, time constant, and wire size to the nearest standard size. (Hint:Wire size can be found from the expression

$$\mathrm{AWG} = 36 - 4.312\left[\frac{A_{\text{wire}}}{1.267\times10^{-8}}\right]$$

where AWG is the wire size, expressed in terms of the American Wire Gauge, and A_{wire} is the conductor cross-sectional area measured in m^2.)

1.22 The magnetic circuit of Fig.1.9 has two windings and two air gaps. The core can be assumed to be of infinite permeability. The core dimensions are indicated in the figure.

a. Assuming coil 1 to be carrying a current I_1 and the current in coil 2 to be zero, calculate (i) the magnetic flux density in each of the air gaps, (ii) the flux linkage of winding l, and (iii) the flux linkage of winding 2.

b. Repeat part (a), assuming zero current in winding 1 and a current I_2 in winding 2.

c. Repeat part (a), assuming the current in winding 1 to be I_1 and the current in winding 2 to be I_2.

d. Find the self-inductances of windings 1 and 2 and the mutual inductance between the windings.

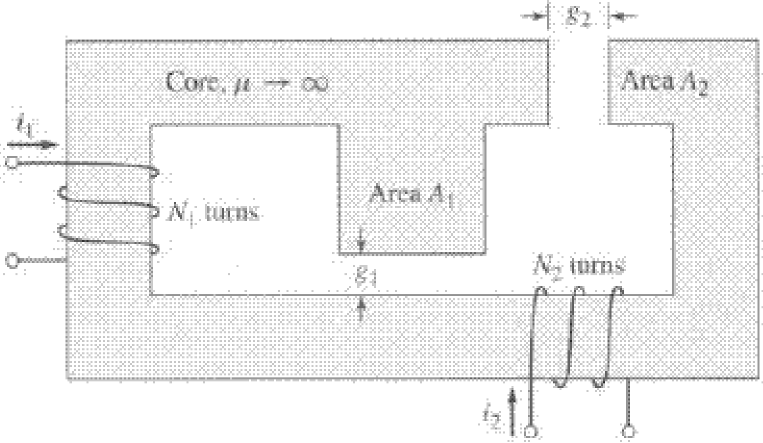


Figure 1.9 Magnetic circuit.

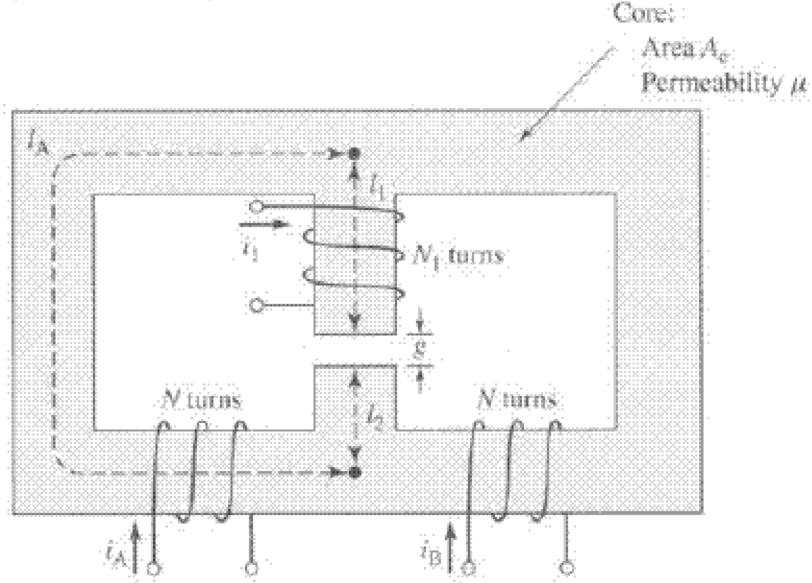


Figure 1.10 Symmetric magnetic circuit.

1.23 The symmetric magnetic circuit of Fig.1.10 has three windings. Windings A and B each have N turns and are wound on the two bottom legs of the core. The core dimensions are indicated in the figure.

a. Find the self-inductances of each of the windings.

b. Find the mutual inductances between the three pairs of windings.

c. Find the voltage induced in winding 1 by time-varying currents i_A(t) and i_B(t) in windings A and B. Show that this voltage can be used to measure the imbalance between two sinusoidal currents of the same frequency.

1.24 The reciprocating generator of Fig.1.11 has a movable plunger (position x) which is supported so that it can slide in and out of the magnetic yoke while maintaining a constant air gap of length g on each side adjacent to the yoke. Both the yoke and the plunger can be considered to be of infinite permeability. The motion of the plunger is constrained such that its position is limited to $0 \leq x \leq \omega$.

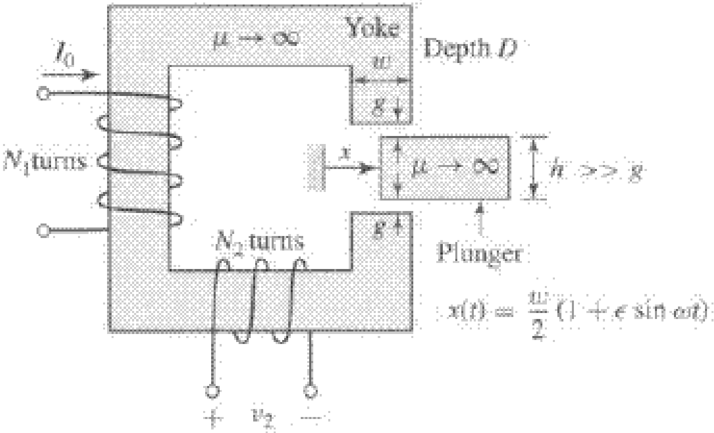


Figure 1.11 Reciprocating generator.

There are two windings on this magnetic circuit. The first has N_1 turns and carries a constant dc current I_0. The second, which has N_2 turns, is open-circuited and can be connected to a load.

a. Neglecting any fringing effects, find the mutual inductance between windings 1 and 2 as a function of the plunger position x.

b. The plunger is driven by an external source so that its motion is given by

$$x(t) = \frac{\omega(1+\varepsilon \sin \omega t)}{2}$$

where $\varepsilon \leq 1$. Find an expression for the sinusoidal voltage which is generated as a result of this motion.

1.25 Figure 1.12 shows a configuration that can be used to measure the magnetic characteristics of electrical steel. The material to be tested is cut or punched into circular laminations which are then stacked (with interspersed insulation to avoid eddy-current formation). Two windings are wound over this stack of laminations: the first, with N_1 turns, is used to excite a magnetic field in the lamination stack; the second, with N_2 turns, is used to sense the resultant magnetic flux.

The accuracy of the results requires that the magnetic flux density be uniform within the laminations. This can be accomplished if the lamination width t = $R_0 - R_1$ is much smaller than the lamination radius and if the excitation winding is wound uniformly around the lamination stack. For the purposes of this analysis, assume there are n laminations, each of thickness ***SORRY, THIS MEDIA TYPE IS NOT SUPPORTED.*** . Also assume that winding 1 is excited by a current $i_1 = I_0$ sinωt.

a. Find the relationship between the magnetic field intensity H in the laminations and current i_1 in winding 1.

b. Find the relationship between the voltage ve and the time rate of change of the flux density B in the laminations.

c. Find the relationship between the voltage $v_0 = G \int v_2$dt and the flux density.

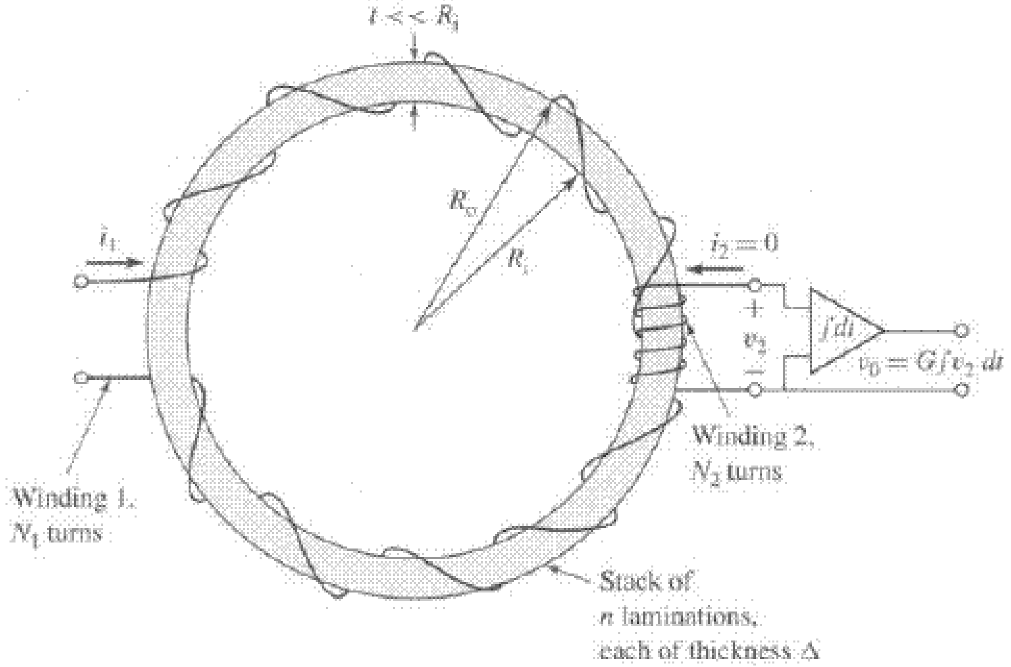


Figure 1.12 Configuration for measurement of magnetic properties of electrical steel.

In this problem, we have shown that the magnetic field intensity H and the magnetic flux density B in the laminations are proportional to the current i_1 and the voltage V_2 by known constants. Thus, B and H in the magnetic steel can be measured directly, and the B-H characteristics as discussed in Sections 1.3 and 1.4 can be determined.

1.26 In order to test the properties of a sample of electrical steel, a set of laminations of the form of Fig.1.12 have been stamped out of a sheet of the electrical steel of thickness 3.0 mm. The radii of the laminations are R_i = 75 mm and R_0 = 82 mm. They have been assembled in a stack of 10 laminations (separated by appropriate insulation to eliminate eddy currents) for the purposes of testing the magnetic properties at a frequency of 100 Hz.

a. The flux in the lamination stack will be excited from a variable-amplitude, 100-Hz voltage source whose peak amplitude is 30 V (peak-to-peak). Calculate the number of turns N_1 for the excitation winding required to insure that the lamination stack can be excited up to a peak flux density of 2.0T.

b. With a secondary winding of N_2 = 20 turns and an integrator gain G = 1000, the output of the integrator is observed to be 7.0 V peak-to-peak. Calculate (i) the corresponding peak flux in the lamination stack and (ii) the corresponding amplitude of the voltage applied to the excitation winding.

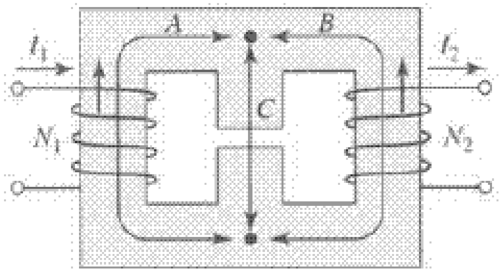


Figure 1.13 Magnetic circuit.

1.27 The coils of the magnetic circuit shown in Fig. 1.13 are connected in series so that the mmf's of paths A and B both tend to set up flux in the center leg C in the same direction. The coils are wound with equal turns, $N_1 = N_2$ = 100.

The dimensions are:

Cross-section area of A and B legs = 7 cm^2

Cross-section area of C legs = 14 cm^2

Length of A path = 17 cm

Length of B path = 17 cm

Length of C path = 5.5 cm

Air gap = 0.4 cm

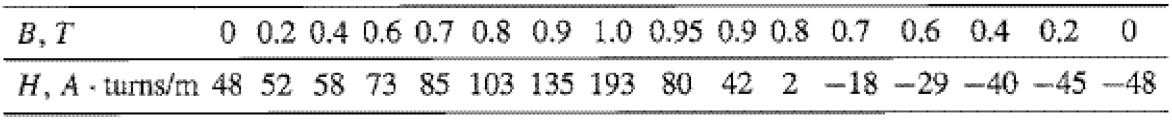
The material is M-5 grade, 0.012-in steel, with a stacking factor of 0.94. Neglect fringing and leakage.

a. How many amperes are required to produce a flux density of 1.2 T in the air gap?

b. Under the condition of part (a), how many joules of energy are stored in the magnetic field in the air gap?

c. Calculate the inductance.

1.28 The following table includes data for the top half of a symmetric 60-Hz hysteresis loop for a specimen of magnetic steel:



B, T	0	0.2	0.4	0.6	0.7	0.8	0.9	1.0	0.95	0.9	0.8	0.7	0.6	0.4	0.2	0
H, $A \cdot$ turns/m	48	52	58	73	85	103	135	193	80	42	2	−18	−29	−40	−45	−48

Using MATLAB, (a) plot this data, (b) calculate the area of the hysteresis loop in joules, and (c) calculate the corresponding 60-Hz core loss in Watts/kg. The density of M-5 steel is 7.65 g/ cm^3.

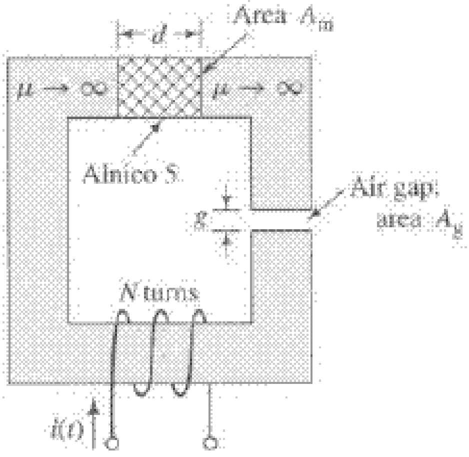


Figure 1.14 Magnetic circuit for.

1.29 It is desired to achieve a time-varying magnetic flux density in the air gap of the magnetic circuit of Fig.1.14 of the form

$$B_g = B_0 + B_1 \text{sin} \omega t$$

where B_0 = 0.5 T and B_1 = 0.25 T. The dc field B0 is to be created by a neodimium-iron-boron magnet, whereas the time-varying field is to be created by a time-varying current.

For $A_g = 6\text{cm}^2$, g = 0.4 cm, and N = 200 turns, find:

a. the magnet length d and the magnet area Am that will achieve the desired dc air-gap flux density and minimize the magnet volume.

b. the minimum and maximum values of the time-varying current required to achieve the desired time-varying air-gap flux density. Will this current vary sinusoidally in time?

Chapter 2: Transformers

Chapter 2: Transformers

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- This chapter is to discuss certain aspects of the theory of magnetically-coupled circuits, with emphasis on transformer action.
- The static transformer is not an energy conversion device, but an indispensable component in many energy conversion systems.

- It is a significant component in ac power systems:

- Electric generation at the most economical generator voltage
- Power transfer at the most economical transmission voltage
- Power utilization at the most voltage for the particular utilization device

- It is widely used in low-power, low-current electronic and control circuits:

-
  - Matching the impedances of a source and its load for maximum power transfer
  - Isolating one circuit from another
  - Isolating direct current while maintaining ac continuity between two circuits

- The transformer is one of the simpler devices comprising two or more electric circuits coupled by a common magnetic circuit.

- Its analysis involves many of the principles essential to the study of electric machinery.

§2.1 Introduction to Transformers

- Essentially, a transformer consists of two or more windings coupled by mutual magnetic flux.

- One of these windings, the primary, is connected to an alternating-voltage.
- An alternating flux will be produced whose magnitude will depend on the primary voltage, the frequency of the applied voltage, and the number of turns.
- The mutual flux will link the other winding, the secondary, and will induce a voltage in it whose value will depend on the number of secondary turns as well as the magnitude of the mutual flux and the frequency.
- By properly proportioning the number of primary and secondary turns, almost any desired voltage ratio, or ratio of transformation, can be obtained.

- The essence of transformer action requires only the existence of time-varying mutual flux linking two windings.

- Iron-core transformer: coupling between the windings can be made much more effectively using a core of iron or other ferromagnetic material.
- The magnetic circuit usually consists of a stack of thin laminations.
- Silicon steel has the desirable properties of low cost, low core loss, and high permeability at high flux densities (1.0 to 1.5 T).

    - Silicon-steel laminations 0.014 in thick are generally used for transformers operating at frequencies below a few hundred hertz.

- Two common types of construction: core type and shell type (Fig. 2.1).

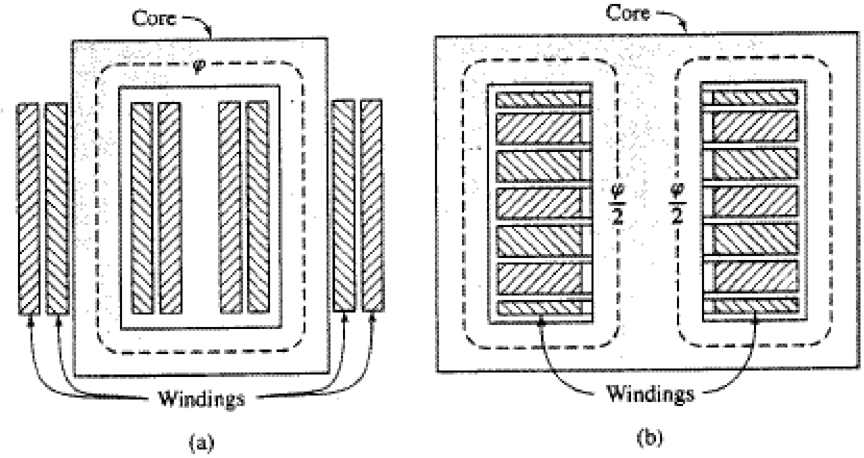


Figure 2.1 Schematic views of (a) core-type and (b) shell-type transformers.

Most of the flux is confined to the core and therefore links both windings.

- Leakage flux links one winding without linking the other.
- Leakage flux is a small fraction of the total flux.
- Leakage flux is reduced by subdividing the windings into sections and by placing them as close together as possible.

§2.2 No-Load Conditions

- Figure 2.4 shows in schematic form a transformer with its secondary circuit open and an alternating voltage v_1 applied to its primary terminals.

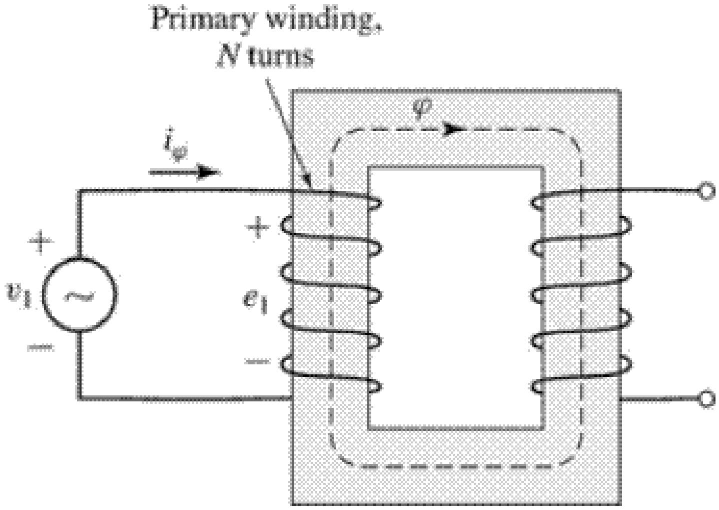


Figure 2.4 Transformer with open secondary.

- The primary and secondary windings are actually interleaved in practice.
- A small steady-state current i_ϕ(the exciting current) flows in the primary and establishes an alternating flux in the magnetic current.
- e_1 = emf induced in the primary (counter emf)

λ_1 = flux linkage of the primary winding

ϕ= flux in the core linking both windings

N_1= number of turns in the primary winding

- The induced emf (counter emf) leads the flux by 90^o

$e_1 = \frac{d\lambda_1}{\mathrm{dt}} = N_1 \frac{d\phi}{\mathrm{dt}}$ (2.1)

$v_1 = R_1 i_\phi + e_1$ (2.2)

- $e_1 \approx v_1$ if the no-load resistance drop is very small and the waveforms of voltage and flux are very nearly sinusoidal.

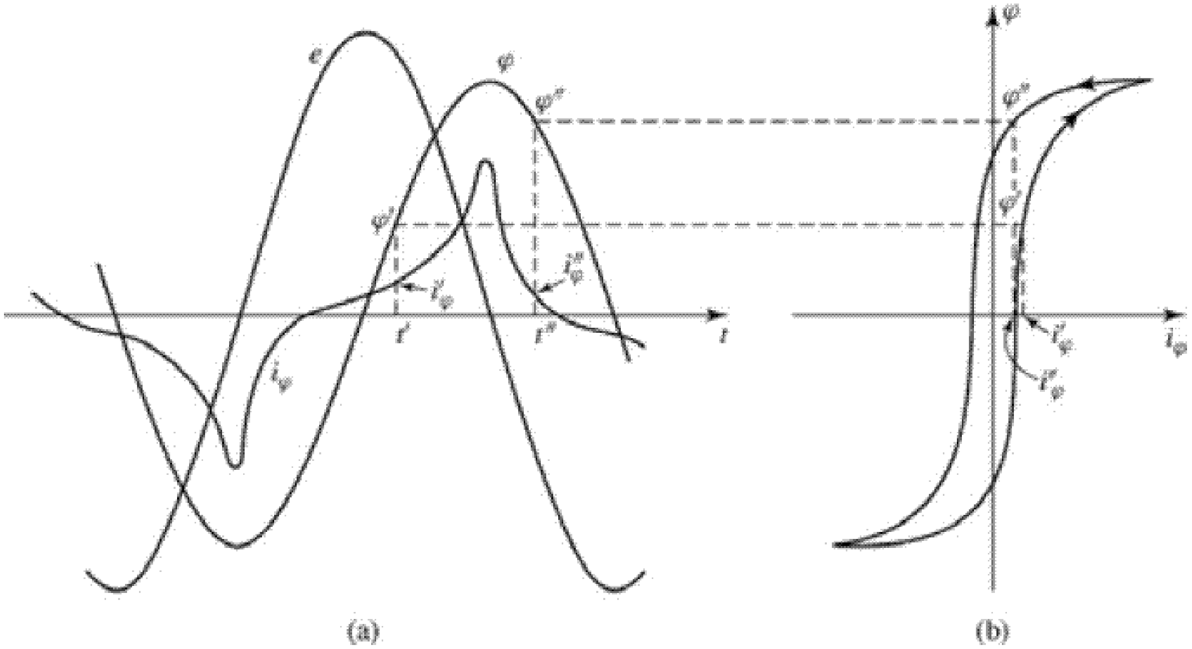
$\phi = \varphi_{\mathrm{max}} \mathrm{sin}\omega t$ (2.3)

$e_1 = N_1 \frac{d\phi}{\mathrm{dt}} = \omega\varphi_{\mathrm{max}} \mathrm{cos}\omega t$ (2.4)

$E_1 = \frac{2\pi}{\sqrt{2}} \mathrm{fN_1} \varphi_{\mathrm{max}} = \sqrt{2}\pi \mathrm{fN_1} \varphi_{\mathrm{max}}$ (2.5)

$\varphi_{\mathrm{max}} = \frac{V_1}{\sqrt{2}\pi \mathrm{fN_1}}$ (2.6)

- The core flux is determined by the applied voltage, its frequency, and the number of turns in the winding. The core flux is fixed by the applied voltage, and the required exciting current is determined by the magnetic properties of the core; the exciting current must adjust itself so as to produce the mmf required to create the flux demanded by (2.6).
- A curve of the exciting current as a function of time can be found graphically from the ac hysteresis loop as shown in Fig. 2.5.



(a)

(b)

Figure 2.5 Excitation phenomena. (a) Voltage, flux, and exciting current;

(b) corresponding hysteresis loop.

- If the exciting current is analyzed by Fourier-series methods, its is found to consist of a fundamental component and a series of odd harmonics.

- The fundamental component can, in turn, be resolved into two components, one in phase with the counter emf and the other lagging the counter emf by 90o.

    - Core-loss component: the in-phase component supplies the power absorbed by hysteresis and eddy-current losses in the core.
    - Magnetizing current: It comprises a fundamental component lagging the counter emf by 90^o , together with all the harmonics, of which the principal is the third (typically 40%).

- The peculiarities of the exciting-current waveform usually need not by taken into account, because the exciting current itself is small, especially in large transformers. (typically about 1 to 2 percent of full-load current)
- Phasor diagram in Fig. 2.6.

$\hat{E}_1 =$the rms value of the induced emf

$\hat{\Phi} =$the rms value of the flux

$\hat{I}_\phi =$the rms value of the equivalent sinusoidal exciting current

- I_ϕ lags E_1 by a phase angle θ_c.

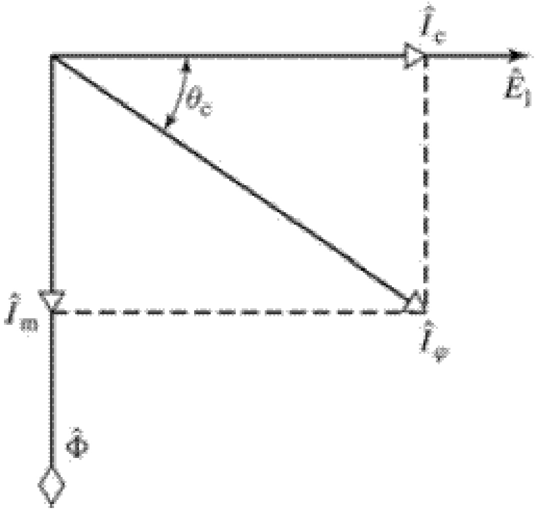


Figure 2.6 No-load phasor diagram.

- The core loss P_c equals the product of the in-phase components of the $\hat{E}_1$and I_ϕ :

$P_c = E_1 I_\phi \cos\theta_c$ (2.7)

- $I_c =$ core-loss current, $I_m =$ magnetizing current

§2.3 Effect of Secondary Current; Ideal Transformer

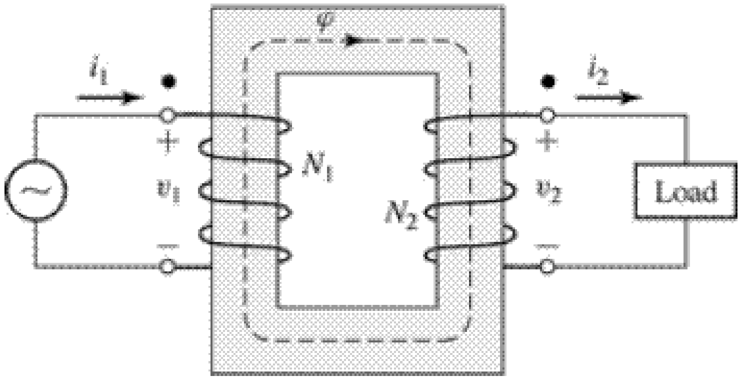


Figure 2.7 Ideal transformer and load.

- Ideal Transformer (Fig. 2.7)

- Assumptions:

1. Winding resistances are negligible.
2. Leakage flux is assumed negligible.
3. There are no losses in the core.
4. Only a negligible mmf is required to establish the flux in the core.

- The impressed voltage, the counter emf, the induced emf, and the terminal voltage:

$v_1 = e_1 = N_1 \frac{d\phi}{\mathrm{dt}}$ (2.8)

$v_2 = e_2 = N_2 \frac{d\phi}{\mathrm{dt}}$ (2.9)

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$ (2.10)

-
  - An ideal transformer transforms voltages in the direct ratio of the turns in its windings.

- Let a load be connected to the secondary.

$N_1 i_1 - N_2 i_2 = 0$ (2.11)

$N_1 i_1 = N_2 i_2$ (2.12)

$\frac{i_1}{i_2} = \frac{N_2}{N_1}$ (2.13)

- An ideal transformer transforms currents in the inverse ratio of the turns in its windings.

- From (2.10) and (2.13),

$v_1 i_1 = v_2 i_2$ (2.14)

-
    - Instantaneous power input to the primary equals the instantaneous power output from the secondary.

- Impedance transformation properties: Fig. 2.8.

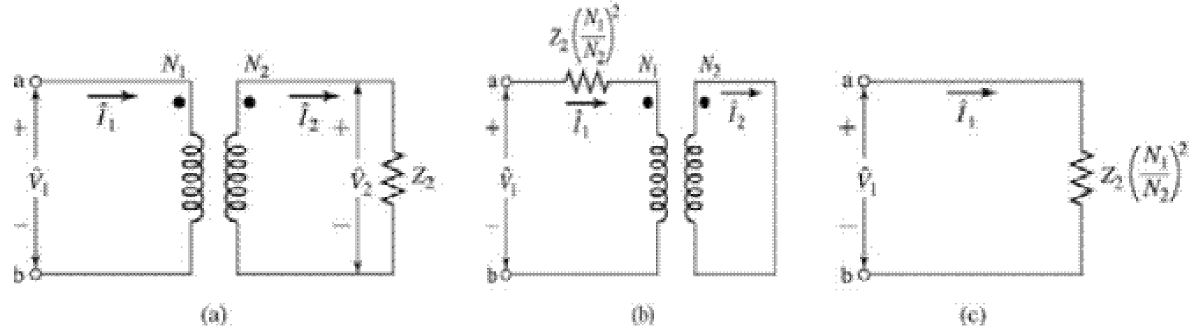


Figure 2.8 Three circuits which are identical at terminals ab when the transformer is ideal.

$\hat{v}_1 = \frac{N_1}{N_2}\hat{v}_2\text{and }\{\hat{v}_2 = \frac{N_2}{N_1}\hat{v}_1$ (2.15)

$\hat{I}_1 = \frac{N_1}{N_2}\hat{I}_2\text{and }\{\hat{I}_2 = \frac{N_2}{N_1}\hat{I}_1$ (2.16)

$\frac{\hat{V}_1}{\hat{I}_1} = \frac{N_1}{N_2}^2 \frac{\hat{V}_2}{\hat{I}_2}$ (2.17)

$Z_2 = \frac{\hat{V}_2}{\hat{I}_2}$ (2.18)

$Z_1 = \frac{N_1}{N_2}^2 Z_2$ (2.19)

- Transferring an impedance from one side to the other is called “referring the impedance to the other side.” Impedances transform as the square of the turns ratio.

    - Summary for the ideal transformer:

- Voltages are transformed in the direct ratio of turns.
- Currents are transformed in the inverse ratio of turns.
- Impedances are transformed in the direct ratio squared.
- Power and voltamperes are unchanged.

§2.4 Transformer Reactances and Equivalent Circuits

- A more complete model must take into account the effects of winding resistances, leakage fluxes, and finite exciting current due to the finite and nonlinear permeability of the core.

- Note that the capacitances of the windings will be neglected.
- Method of the equivalent circuit technique is adopted for analysis.

- Development of the transformer equivalent circuit

- Leakage flux: Fig. 2.9.

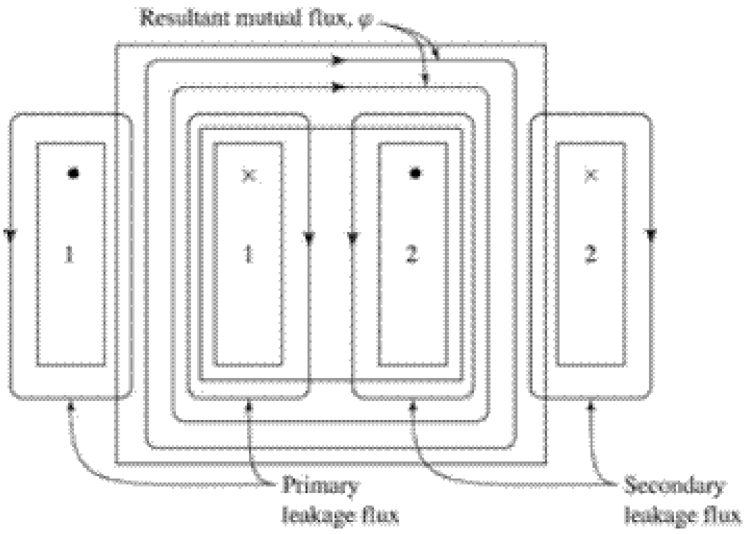


Figure 2.9 Schematic view of mutual and leakage fluxes in a transformer.

- L_{1_1}= primary leakage inductance, X_{1_1} = primary leakage reactance

$X_{1_1} = 2\pi\mathrm{fL}_{1_1}$ (2.20)

- Effect of the primary winding resistance: R_1
- Effect of the exciting current:

$$\begin{aligned} N_1\hat{I}_\phi &= N_1\hat{I}_1 - N_2\hat{I}_2 \\ &= N_1(\hat{I}_\phi + \hat{I}_2^{'}) - N_2\hat{I}_2 \end{aligned}$$ (2.21)- (2.22)

$\hat{I}_2^{'} = \frac{N_2}{N_1}\hat{I}_2$ (2.23)

- L_m = magnetizing inductance, X_m = magnetizing reactance

$X_m = 2\pi\mathrm{fL}_m$ (2.24)

- Ideal transformer:

$\frac{\hat{E}_1}{\hat{E}_2} = \frac{N_1}{N_2}$ (2.25)

- Secondary resistance, secondary leakage reactance
- Equivalent-T circuit for a transformer:

$\hat{X}_{1_2} = \frac{N_1}{N_2}\ ^2 X_{1_2}, \{R_2^{'} = \frac{N_1}{N_2}\ ^2 R_2, \{V_2^{'} = \frac{N_1}{N_2}\ ^2 V_2$ (2.26)

- Steps in the development of the transformer equivalent circuit: Fig.2.10

- The actual transformer can be seen to be equivalent to an ideal transformer plus external impedances
- Refer to the assumptions for an ideal transformer to understand the definitions and meanings of these resistances and reactances.

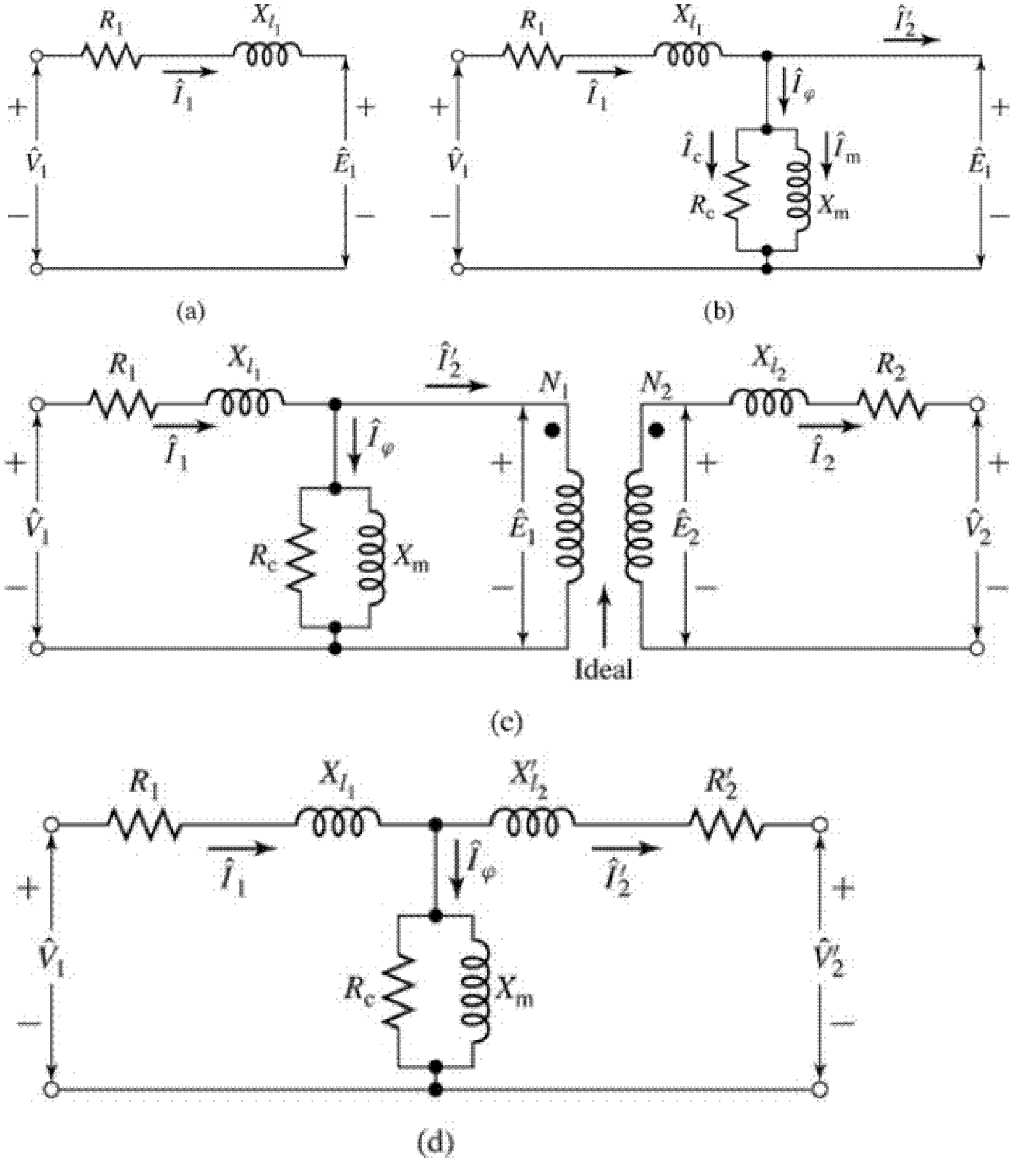


Figure 2.10 Steps in the development of the transformer equivalent circuit.

§2.5 Engineering Aspects of Transformer Analysis

- Approximate forms of the equivalent circuit:

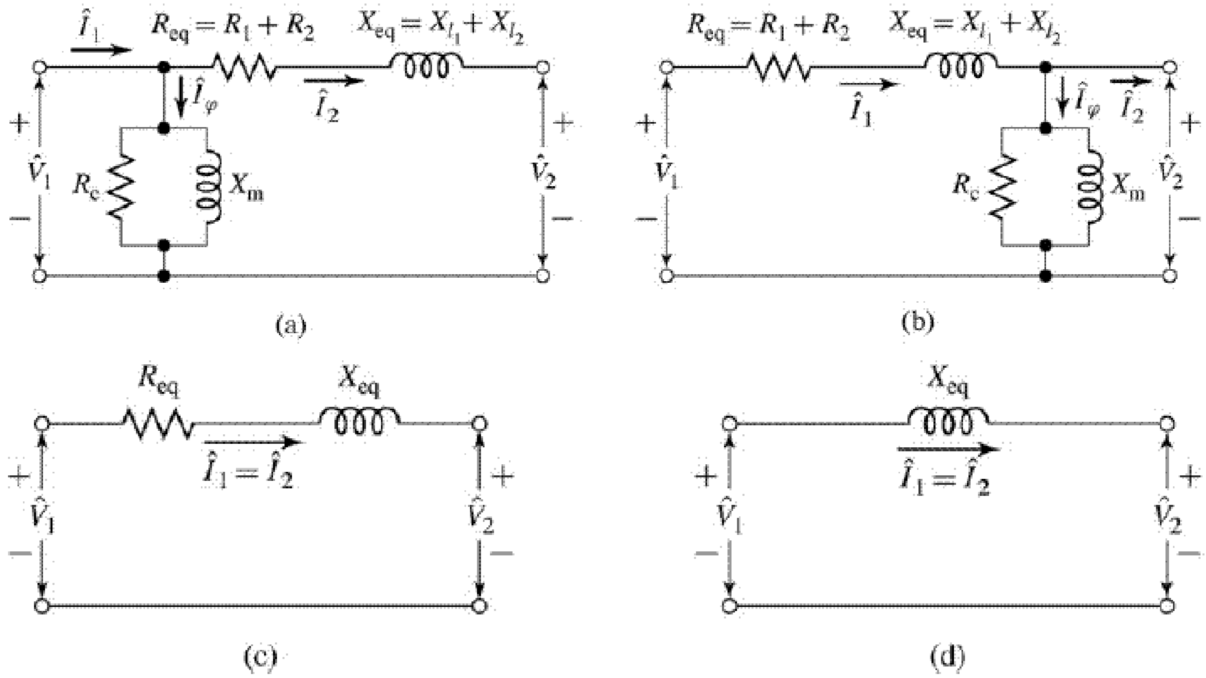


Figure 2.11 Approximate transformer equivalent circuits.

- Two tests serve to determine the parameters of the equivalent circuits of Figs. 2.10 and 2.11.

- Short-circuit test and open-circuit test

- Short-Circuit Test

- The test is used to find the equivalent series impedance $R_{\text{eq}} + \text{jX}_{\text{eq}}$.
- The high voltage side is usually taken as the primary to which voltage is applied.
- The short circuit is applied to the secondary
- Typically an applied voltage on the order of 10 to 15 % or less of the rated value will result in rated current.
- See Fig. 2.12.Note that $Z_\phi = R_c // \text{jX}_m$.

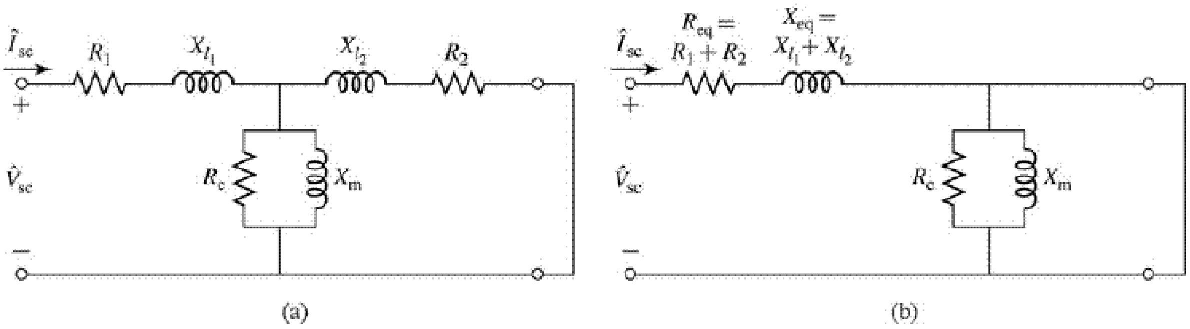


Figure 2.12 Equivalent circuit with short-circuited secondary. (a) Complete equivalent circuit.(b) Cantilever equivalent circuit with the exciting branch at the transformer secondary.

$Z_{\mathrm{sc}} = R_1 + \mathrm{jX}_{1_1} + \frac{Z_\phi(R_2 + \mathrm{jX}_{1_2})}{Z_\phi + R_2 + \mathrm{jX}_{1_2}}$ (2.27)

$Z_{\mathrm{sc}} \approx R_1 + \mathrm{jX}_{1_1} + R_2 + \mathrm{jX}_{1_2} = R_{\mathrm{eq}} + \mathrm{jX}_{\mathrm{eq}}$ (2.28)

Typically the instrumentation will measure the rms magnitude of the applied voltage V_{sc} , the short-circuit current I_{sc} , and the power P_{sc}. The circuit parameters (referred to the primary) can be found as (2.29)-(2.31).

$\mid Z_{\mathrm{eq}} \mid = \mid Z_{\mathrm{sc}} \mid = \frac{V_{\mathrm{sc}}}{I_{\mathrm{sc}}}$ (2.29)

$R_{\mathrm{eq}} = R_{\mathrm{sc}} = \frac{P_{\mathrm{sc}}}{I_{\mathrm{sc}}^2}$ (2.30)

$X_{\mathrm{eq}} = X_{\mathrm{sc}} = \sqrt{\mid Z_{\mathrm{sc}} \mid^2 - R_{\mathrm{sc}}^2}$ (2.31)

- The equivalent impedance can be referred from one side to the other.
- Approximate values of the individual primary and secondary resistances and leakage reactances can be obtained by assuming that $R_1 = R_2 = 0.5\mathrm{R}_{\mathrm{eq}}$ and $X_{l_1} = X_{l_2} = 0.5\mathrm{X}_{\mathrm{eq}}$ when all impedances are referred to the same side.
- Note that it is possible to measure $R_1$and R_2 directly by a dc resistance measurement on each winding. However, no such simple test exists for X_{l_1} and X_{l_2}.

- Open-Circuit Test

- The test is used to find the equivalent shunt impedance $R_c // \text{jX}_m$.
- The test is performed with the secondary open-circuited and rated voltage impressed on the primary. If the transformer is to be used at other than its rated voltage, the test should be done at that voltage.
- An exciting current of a few percent of full-load current is obtained.
- See Fig. 2.16. Note that $Z_\phi = R_c // \text{jX}_m$.

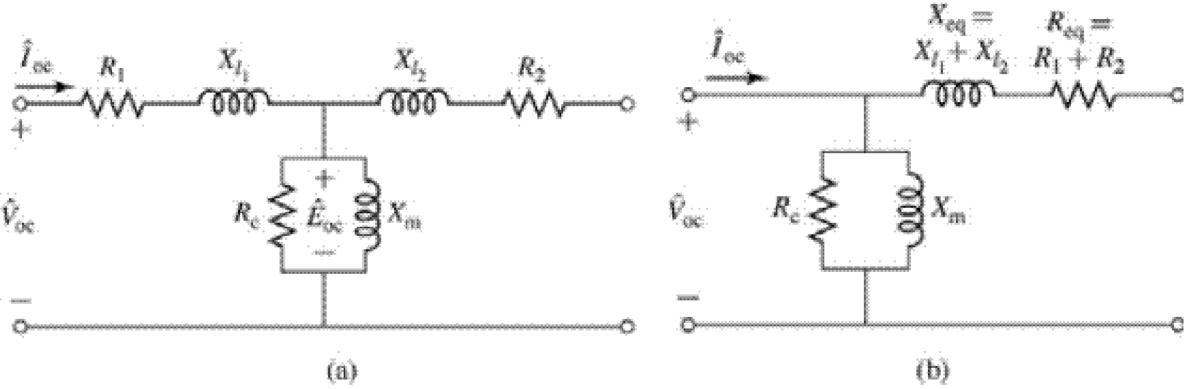


Figure 2.13 Equivalent circuit with open-circuited secondary. (a) Complete equivalent circuit.(b) Cantilever equivalent circuit with the exciting branch at the transformer primary.

$$Z_{\text{oc}} = R_1 + \text{jX}_{1_1} + Z_\phi = R_1 + \text{jX}_{1_1} + \frac{R_c(\text{jX}_m)}{R_c + \text{jX}_m} \quad (2.32)$$

$$Z_{\text{oc}} \approx Z_\phi = \frac{R_c(\text{jX}_m)}{R_c + \text{jX}_m} \quad (2.33)$$

- Typically the instrumentation will measure the rms magnitude of the applied voltage V_{oc}, the open-circuit current I_{oc} , and the power P_{oc} . The circuit parameters (referred to the primary) can be found as (2.34)-(2.36).

$$R_c = \frac{V_{\text{oc}}^2}{P_{\text{oc}}} \quad (2.34)$$

$$\mid Z_\phi \mid = \frac{V_{\text{oc}}}{P_{\text{oc}}} \quad (2.35)$$

$$X_m = \frac{1}{\sqrt{(1/|Z_\phi|)^2 - (1/R_c)^2}} \quad (2.36)$$

- The open-circuit test can be used to obtain the core loss for efficiency computations and to check the magnitude of the exciting current.

- Note the term “Voltage Regulation” which is to be discussed in Example 2.6.

§2.6 Autotransformers; Multiwinding Transformers

- Two-winding   Other winding configurations.

§2.6.1 Autotransformers

- Autotransformer connection: Fig. 2.14.

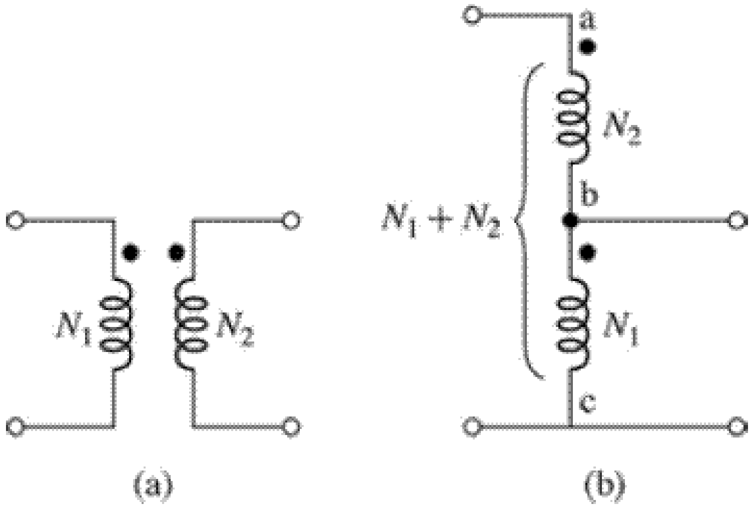


Figure 2.14 (a) Two-winding transformer. (b) Connection as an autotransformer.

- The windings of the two-winding transformer are electrically isolated whereas those of the autotransformer are connected directly together.
- In the transformer connection, winding ab must be provided with extra insulation.
- Autotransformer have lower leakage reactances, lower losses, and smaller exciting current and cost less than two-winding transformers when the voltage ration does not differ too greatly from 1:1.
- The rated voltages of the transformer can be expressed in terms of those of the two-winding transformer as

$V_{L_{\text{rated}}} = V_{1_{\text{rated}}}$ (2.37)

$V_{H_{\text{rated}}} = V_{1_{\text{rated}}} + V_{2_{\text{rated}}} = \frac{N_1+N_2}{N_1} V_{L_{\text{rated}}}$ (2.38)

- The effective turns ratio of the autotransformer is thus $(N_1 + N_2)/N_1$.
- The power rating of the autotransformer is equal to $(N_1 + N_2)/N_2$ times that of the two winding transformer.

§2.6.2 Multiwinding Transformers

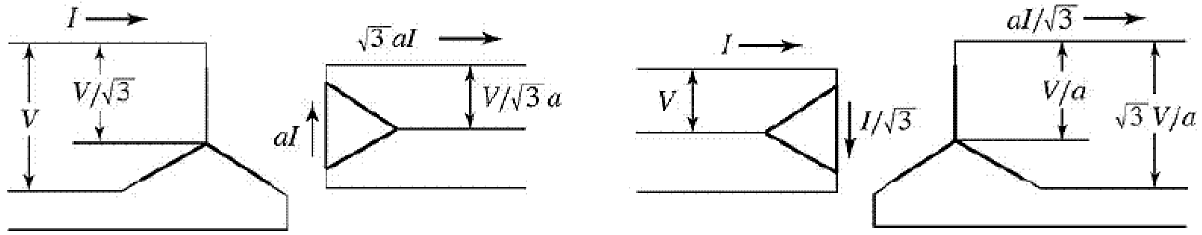
- Transformers having three or more windings, known as multiwinding or multicircuit transformers, are often used to interconnect three or more circuits which may have different voltages.

- Transformers having a primary and multiple secondaries are frequently found in multiple-output dc power supplies.
- Distribution transformers used to supply power for domestic purposes usually have two 120V secondaries connected in series.
- The three-phase transformer banks used to interconnect two transmission system of different voltages often have a third, or tertiary, set of windings to provide voltage for auxiliary power purposes in substation or to supply a local distribution system.

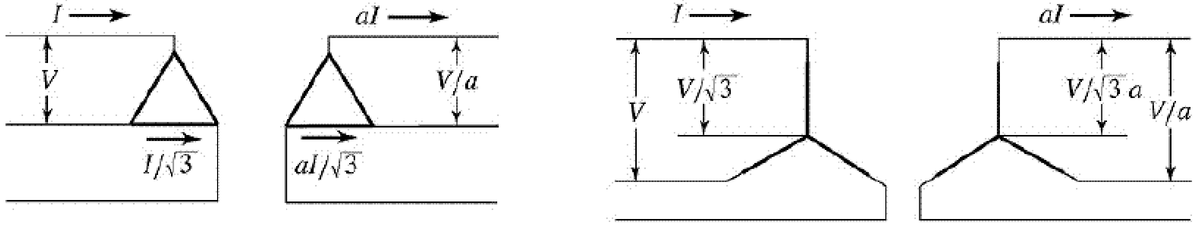
    - Static capacitors or synchronous condensers may be connected to the tertiary windings for power factor correction or voltage regulation.
    - Sometimes $\mathit{\Delta}$-connected tertiary windings are put on three-phase banks to provide a low-impedance path for third harmonic components of the exciting current to reduce third-harmonic components of the neutral voltage.

§2.7 Transformers in Three-Phase Circuits

- Three single-phase transformers can be connected to form a three-phase transformer bank in any of the four ways shown in Fig. 2.15. Note that $a = N_1/N_2$.



(a) Y-Δ connection (b) Δ-Y connection



(c) Δ-Δ connection (d) Y-Y connection

Figure 2.15 Common three-phase transformer connections;

the transformer windings are indicated by the heavy lines.

- The $Y-\Delta$ connection is commonly used in stepping down from a high voltage to a medium or low voltage.
- The $\Delta-Y$connection is commonly used for stepping up to a high voltage.
- The $\Delta-\Delta$ connection has the advantage that one transformer can be removed for repair or maintenance while the remaining two continue to function as a three-phase bank with the rating reduced to 58 percent of that of the original bank.(Open-delta, or V, connection)
- The Y-Y connection is seldom used because of difficulties with exciting-current phenomenon.
    - Because there is no neutral connection to carry harmonics of the exciting current and harmonic voltages are produced which significantly distort the transformer voltages.
- A three-phase bank may consist of one three-phase transformer having all six windings on a common multi-legged core and contained in a single tank.

- They cost less, weigh less, require less floor space, and have somewhat higher efficiency.

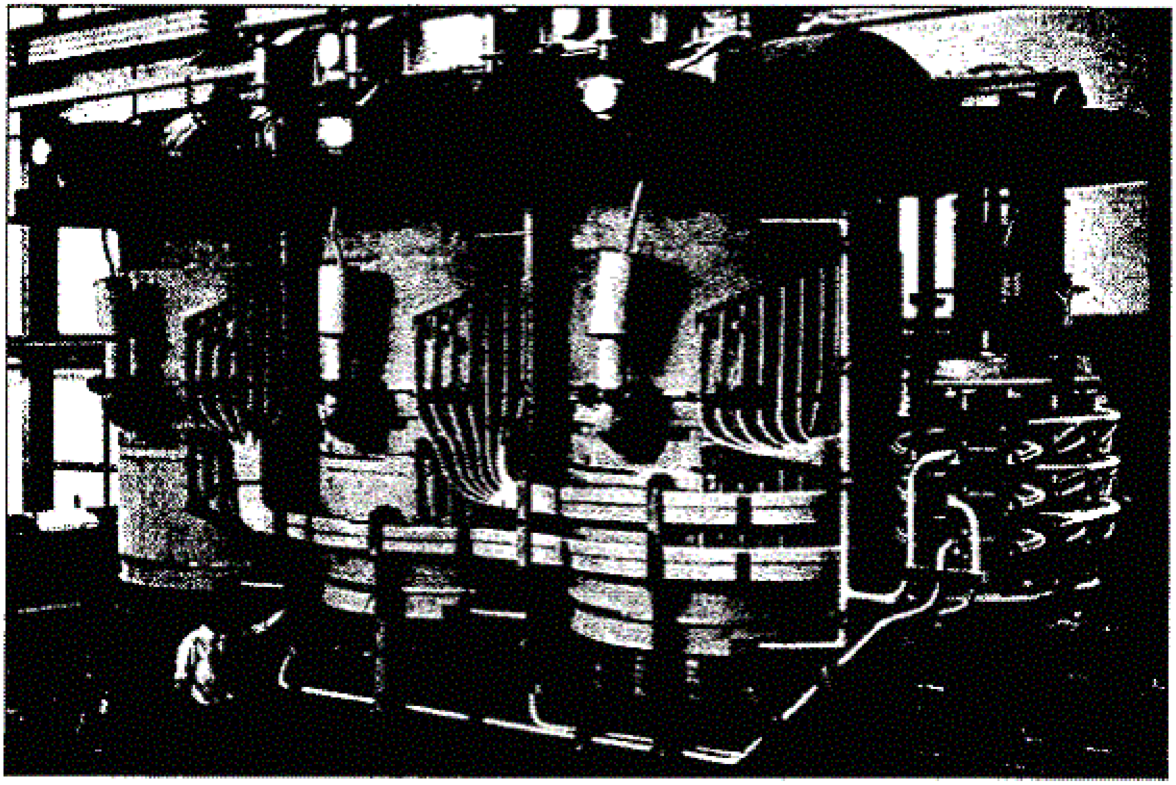


Figure 2.16 A 200-MVA, three-phase, 50-Hz, three-winding, 210/80/10.2-kV transformer removed from its tank. The 210-kV winding has an on-load tap changer for adjustment of the voltage. (*Brown Boveri Corporation.*)

- It is usually convenient to carry out circuit computations involving three-phase transformer banks under balanced conditions on a single-phase (per-phase-Y, line-to-neutral) basis.

- $Y-\mathit{\Delta}$, $\mathit{\Delta}-Y$, and $\mathit{\Delta}-\mathit{\Delta}$connections equivalent Y-Y connections
- A balanced $\mathit{\Delta}$-connected circuit of $Z_{\mathit{\Delta}}\Omega/$phase is equivalent to a balanced Y-connected circuit of $Z_Y\Omega/$phase if

$Z_Y=\frac{1}{3}Z_{\mathit{\Delta}}$ (2.39)

§2.8 Voltage and Current Transformers

- Transformers are often used in instrumentation applications to match the magnitude of a voltage or current to the range of a meter or other instrumentation.

- Most 60-Hz power-systems’ instrumentation is based upon voltages in the range of 0-120V rms and currents in the range of 0-5 A rms.
- Power system voltages range up to 765-kV line-to-line and currents can be 10’s of kA.

- Some method of supplying an accurate, low-level representation of these signals to the instrumentation is required.

- Potential Transformer (PT) and Current Transformer (CT), also referred to as Instrumentation Transformer, are designed to approximate the ideal transformer as closely as is practically possible.

- The load on an instrumentation transformer is frequently referred to as the burden on that transformer.
- A potential transformer should ideally accurately measure voltage while appearing as an open circuit to the system under measurement, i.e. drawing negligible current and power.

- Its load impedance should be “large” in some sense.

- An ideal current transformer would accurately measure current while appearing as a short circuit to the system under measurement, i.e. developing negligible voltage drop and drawing negligible power.

- Its load impedance should be “small” in some sense.

§2.9 The Per-Unit System

- Computations relating to machines, transformers, and systems of machines are often carried out in per-unit system.

- All pertinent quantities are expressed as decimal fractions of appropriately chose base values.

- All the usual computations are then carried out in these per unit values instead of the familiar volts, amperes, ohms, and so on.
- Advantages:

- The parameter values typically fall in a reasonably narrow numerical range when expressed in a per-unit system based upon their rating.
- When transformer equivalent-circuit parameters are converted to their per-unit values, the ideal transformer turns ratio becomes 1:1 and hence the ideal transformer can be eliminated.

- Actual quantities: V , I , P , Q ,VA , R , X , Z , G , B , Y

$$\mathrm{Quantityinperunit} = \frac{\mathrm{Actualquantity}}{\mathrm{Basevalueofquantity}} \quad (2.40)$$

- To a certain extent, base values can be chosen arbitrarily, but certain relations between them must be observed. For a single-phase system:

$$P_{\mathrm{base}}, Q_{\mathrm{base}}, \mathrm{VA}_{\mathrm{base}} = V_{\mathrm{base}} I_{\mathrm{base}} \quad (2.41)$$

$$R_{\mathrm{base}}, X_{\mathrm{base}}, Z_{\mathrm{base}} = \frac{V_{\mathrm{base}}}{I_{\mathrm{base}}} \quad (2.42)$$

- Only two independent base quantities can be chose arbitrarily; the remaining quantities are determined by (2.48) and (2.49).
- In typical usage, values of $\mathrm{VA}_{\mathrm{base}}$ and V_{base} and are chosen first; values of I_{base} and all other quantities in (2.48) and (2.49) are then uniquely established.
- The value of $\mathrm{VA}_{\mathrm{base}}$ must be the same over the entire system under analysis.
- When a transformer is encountered, the values of V_{base}differ on each side and should be chosen in the same ratio as the turns ratio of the transformer.

- The per-unit ideal transformer will have a unity turns ratio and hence can be eliminated.
- Usually the rated or nominal voltages of the respective sides are chosen.

- The procedure for performing system analyses in per-unit is summarized as follows:
  - Select a VA base and a base voltage at some point in the system.
  - Convert all quantites toper unit on the chosen VA base and with a voltage base that transforms as the turns ratio of any transformer which is encountered as one moves through the system.
  - Perform a standard electrical analysis with all quantities in per unit.
  - When the analysis is completed, all quantities can be converted back to real unit(e.g., volts, amperes, watts, etc.) by multiplying their per-unit values by their corresponding base values.

- Machine Ratings as Bases

When expressed in per-unit form on their rating as a base, the per-unit values of machine parameters fall within a relatively narrow range.

- The physics behind each type of device is the same and, in a crude sense, they can each be considered to be simply scaled versions of the same basic device.
- When normalized to their own rating, the effect of the scaling is eliminated and the result is a set of per-unit parameter values which is quite similar over the whole size range of that device. For power and distribution transformers, $I_\phi =$ 0.02- 0.06pu , R   0.005 - 0.02pu , and X   0.015 - 0.10pu.

- Manufacturers often supply device parameters in per unit on the device base.

- When performing a system analysis, it may be necessary to convert the supplied per-unit parameter values to per-unit values on the base chosen for the analysis.

$$(P,Q,\mathrm{VA})_{\text{pu on base2}} = (P,Q,\mathrm{VA})_{\text{pu on base1}} \left[\frac{\mathrm{VA}_{\text{base1}}}{\mathrm{VA}_{\text{base2}}} \right] \quad (2.43)$$

$$(P,X,Z)_{\text{pu on base2}} = (P,X,Z)_{\text{pu on base1}} \left[\frac{(V_{\text{base1}})^2 \mathrm{VA}_{\text{base2}}}{(V_{\text{base2}})^2 \mathrm{VA}_{\text{base1}}} \right] \quad (2.44)$$

$$V_{\text{pu on base2}} = V_{\text{pu on base1}} \left[\frac{V_{\text{base1}}}{V_{\text{base2}}} \right] \text{ (2.45)}$$

$$I_{\text{pu on base2}} = I_{\text{pu on base1}} \left[\frac{V_{\text{base2}} \mathrm{VA}_{\text{base1}}}{V_{\text{base1}} \mathrm{VA}_{\text{base2}}} \right] \text{ (2.46)}$$

Balanced Three-Phase System:

- Relations for base values:

$$(P_{\text{base}}, Q_{\text{base}}, \mathrm{VA}_{\text{base}})_{3-\text{phase}} = 3\mathrm{VA}_{\text{base, per phase}} \text{ (2.47)}$$

- The three-phase volt-ampere base $(\mathrm{VA}_{\text{base},3-\text{phase}})$ and the line-to-line voltage base $(V_{\text{base},3-\text{phase}} = V_{\text{base},1-1})$are usually chosen first.
- The base values for the phase (line-to-neutral) voltage then is

$$V_{\text{base},1-n} = \frac{1}{\sqrt{3}} V_{\text{base},1-1} \text{ (2.48)}$$

- The base current for three-phase system is equal to the phase current, which is the same as the base current for a single-phase (per-phase) analysis.

$$I_{\text{base},3-\text{phase}} = I_{\text{base,perphase}} = \frac{\mathrm{VA}_{\text{base},3-\text{phase}}}{\sqrt{3} V_{\text{base},3-\text{phase}}} \text{ (2.49)}$$

- The three-phase base impedance is chosen to be the single-phase base impedance.

$$\begin{aligned} Z_{\text{base},3-\text{phase}} &= Z_{\text{base,perphase}} \\ &= \frac{V_{\text{base},1-n}}{I_{\text{base,perphase}}} \\ &= \frac{V_{\text{base},3-\text{phase}}}{\sqrt{3} I_{\text{base},3-\text{phase}}} \\ &= \frac{(V_{\text{base},3-\text{phase}})^2}{\mathrm{VA}_{\text{base},3-\text{phase}}} \end{aligned} \quad \text{((2.50); (2.51); (2.52);(2.53)}$$

Note that the factors of 3 and 3 are automatically taken care of in per unit by the base values. Three-phase problems can thus be solved in per unit as if they were single-phase problems.

PROBLEMS - chapter 2

PROBLEMS

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

2.1 A transformer is made up of a 1200-turn primary coil and an open-circuited 75-turn secondary coil wound around a closed core of cross-sectional area 42 cm^2. The core material can be considered to saturate when the rms applied flux density reaches 1.45T. What maximum 60-Hz rms primary voltage is possible without reaching this saturation level? What is the corresponding secondary voltage? How are these values modified if the applied frequency is lowered to 50 Hz?

2.2 A magnetic circuit with a cross-sectional area of 15 cm^2 is to be operated at 60 Hz from a 120-V rms supply. Calculate the number of turns required to achieve a peak magnetic flux density of 1.8 T in the core.

2.3 A transformer is to be used to transform the impedance of a 8 Ω resistor to an impedance of 75 Ω. Calculate the required turns ratio, assuming the transformer to be ideal.

2.4 A 100 Ω resistor is connected to the secondary of an idea transformer with a turns ratio of 1:4 (primary to secondary). A 10-V rms, 1-kHz voltage source is connected to the primary. Calculate the primary current and the voltage across the 100 Ω resistor.

2.5 A source which can be represented by a voltage source of 8 V rms in series with an internal resistance of 2 $k\Omega$ is connected to a 50 Ω load resistance through an ideal transformer. Calculate the value of turns ratio for which maximum power is supplied to the load and the corresponding load power? Using MATLAB, plot the the power in milliwatts supplied to the load as a function of the transformer ratio, covering ratios from 1.0 to 10.0.

2.6 Repeat Problem 2.5 with the source resistance replaced by a 2- $k\Omega$ reactance.

2.7 A single-phase 60-Hz transformer has a nameplate voltage rating of 7.97 kV:266 V, which is based on its winding turns ratio. The manufacturer calculates that the primary (7.97-kV) leakage inductance is 165 mH and the primary magnetizing inductance is 135 H. For an applied primary voltage of 7970 V at 60 Hz, calculate the resultant open-circuit secondary voltage.

2.8 The manufacturer calculates that the transformer of Problem 2.7 has a secondary leakage inductance of 0.225 mH.

a. Calculate the magnetizing inductance as referred to the secondary side.

b. A voltage of 266 V, 60 Hz is applied to the secondary. Calculate (i) the resultant open-circuit primary voltage and (ii) the secondary current which would result if the primary were short-circuited.

2.9 A 120-V:2400-V, 60-Hz, 50-kVA transformer has a magnetizing reactance (as measured from the 120-V terminals) of 34.6 Ω. The 120-V winding has a leakage reactance of 27.4 $m\Omega$ and the 2400-V winding has a leakage reactance of 11.2 Ω.

a. With the secondary open-circuited and 120 V applied to the primary (120-V) winding, calculate the primary current and the secondary voltage.

b. With the secondary short-circuited, calculate the primary voltage which will result in rated current in the primary winding. Calculate the corresponding current in the secondary winding.

2.10 A 460-V:2400-V transformer has a series leakage reactance of 37.2 Ω as referred to the high-voltage side. A load connected to the low-voltage side is observed to be absorbing 25 kW, unity power factor, and the voltage is measured to be 450 V. Calculate the corresponding voltage and power factor as measured at the high-voltage terminals.

2.11 The resistances and leakage reactances of a 30-kVA, 60-Hz, 2400-V:240-V distribution transformer are

R1 = 0.68 Ω R2 = 0.0068 Ω

X_{1_1} = 7.8 ΩX_{1_2} = 0.0780 Ω

where subscript 1 denotes the 2400-V winding and subscript 2 denotes the 240-V winding. Each quantity is referred to its own side of the transformer.

a. Draw the equivalent circuit referred to (i) the high- and (ii) the low-voltage sides. Label the impedances numerically.

b. Consider the transformer to deliver its rated kVA to a load on the low-voltage side with 230 V across the load. (i) Find the high-side terminal voltage for a load power factor of 0.85 power factor lagging. (ii) Find the high-side terminal voltage for a load power factor of 0.85 power factor leading.

c. Consider a rated-kVA load connected at the low-voltage terminals operating at 240V. Use MATLAB to plot the high-side terminal voltage as a function of the power-factor angle as the load power factor varies from 0.6 leading through unity power factor to 0.6 pf lagging.

2.12 Repeat Problem 2.11 for a 75-kVA, 60-Hz, 4600-V:240-V distribution transformer whose resistances and leakage reactances are

R1 = 0.846 Ω R2 = 0.00261 Ω

X_{1_1} = 26.8 ΩX_{1_2}= 0.0745 Ω

where subscript 1 denotes the 4600-V winding and subscript 2 denotes the 240-V winding. Each quantity is referred to its own side of the transformer.

2.13 A single-phase load is supplied through a 35-kV feeder whose impedance is 95+j360 Ω and a 35-kV:2400-V transformer whose equivalent impedance is 0.23 + j1.27 Ω referred to its low-voltage side. The load is 160 kW at 0.89 leading power factor and 2340 V.

a. Compute the voltage at the high-voltage terminals of the transformer.

b. Compute the voltage at the sending end of the feeder.

c. Compute the power and reactive power input at the sending end of the feeder.

2.14 The nameplate on a 50-MVA, 60-Hz single-phase transformer indicates that it has a voltage rating of 8.0-kV:78-kV. An open-circuit test is conducted from the low-voltage side, and the corresponding instrument readings are 8.0 kV, 62.1 A, and 206 kW. Similarly, a short-circuit test from the low-voltage side gives readings of 674 V, 6.25 kA, and 187 kW.

a. Calculate the equivalent series impedance, resistance, and reactance of the transformer as referred to the low-voltage terminals.

b. Calculate the equivalent series impedance of the transformer as referred to the high-voltage terminals.

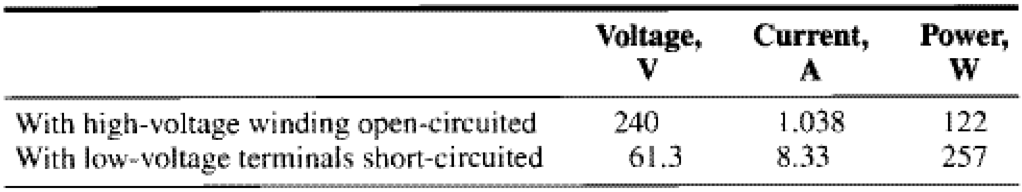
c. Making appropriate approximations, draw a T equivalent circuit for the transformer.

d. Determine the efficiency and voltage regulation if the transformer is operating at the rated voltage and load (unity power factor).

e. Repeat part (d), assuming the load to be at 0.9 power factor leading.

2.15 A 550-kVA, 60-Hz transformer with a 13.8-kV primary winding draws 4.93 A and 3420 W at no load, rated voltage and frequency. Another transformer has a core with all its linear dimensions $\sqrt{2}$ times as large as the corresponding dimensions of the first transformer. The core material and lamination thickness are the same in both transformers. If the primary windings of both transformers have the same number of turns, what no-load current and power will the second transformer draw with 27.6 kV at 60 Hz impressed on its primary?

2.16 The following data were obtained for a 20-kVA, 60-Hz, 2400:240-V distribution transformer tested at 60 Hz:



	Voltage, V	Current, A	Power, W
With high-voltage winding open-circuited	240	1.038	122
With low-voltage terminals short-circuited	61.3	8.33	257

a. Compute the efficiency at full-load current and the rated terminal voltage at 0.8 power factor.

b. Assume that the load power factor is varied while the load current and secondary terminal voltage are held constant. Use a phasor diagram to determine the load power factor for which the regulation is greatest. What is this regulation?

2.17 A 75-kVa, 240-V:7970-V, 60-Hz single-phase distribution transformer has the following parameters referred to the high-voltage side:

R1 = 5.93 Ω X1 = 43.2 Ω

R2 = 3.39 Ω X2 = 40.6 Ω

Rc = 244 $k\Omega$ X m = 114 $k\Omega$

Assume that the transformer is supplying its rated kVA at its low-voltage terminals. Write a MATLAB script to determine the efficiency and regulation of the transformer for any specified load power factor (leading or lagging). You may use reasonable engineering approximations to simplify your analysis. Use your MATLAB script to determine the efficiency and regulation for a load power factor of 0.87 leading.

2.18 The transformer of Problem 2.11 is to be connected as an autotransformer. Determine (a) the voltage ratings of the high- and low-voltage windings for this connection and (b) the kVA rating of the autotransformer connection.

2.19 A 120:480-V, 10-kVA transformer is to be used as an autotransformer to supply a 480-V circuit from a 600-V source. When it is tested as a two-winding transformer at rated load, unity power factor, its efficiency is 0.979.

a. Make a diagram of connections as an autotransformer.

b. Determine its kVA rating as an autotransformer.

c. Find its efficiency as an autotransformer at full load, with 0.85 power factor lagging.

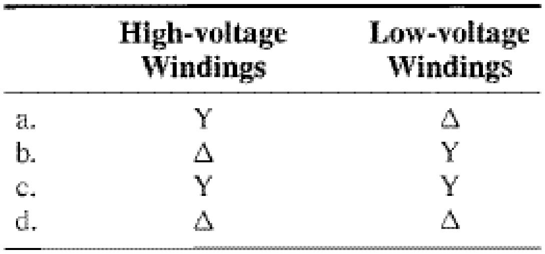
2.20 Consider the 8-kV:78-kV, 50-MVA transformer of Problem 2.14 connected as an autotransformer.

a. Determine the voltage ratings of the high- and low-voltage windings for this connection and the kVA rating of the autotransformer connection.

b. Calculate the efficiency of the transformer in this connection when it is supplying its rated load at unity power factor.

2.21 Write a MATLAB script whose inputs are the rating (voltage and kVA) and rated-load, unity-power-factor efficiency of a single-transformer and whose output is the transformer rating and rated-load, unity-power-factor efficiency when connected as an autotransformer.

2.22 The high-voltage terminals of a three-phase bank of three single-phase transformers are supplied from a three-wire, three-phase 13.8-kV (line-to-line) system. The low-voltage terminals are to be connected to a three-wire, three-phase substation load drawing up to 4500kVA at 2300 V line-to-line. Specify the required voltage, current, and kVA ratings of each transformer (both high- and low-voltage windings) for the following connections:



	High-voltage Windings	Low-voltage Windings
a.	Y	Δ
b.	Δ	Y
c.	Y	Y
d.	Δ	Δ

2.23 Three 100-MVA single-phase transformers, rated at 13.8 kV:66.4 kV, are to be connected in a three-phase bank. Each transformer has a series impedance of 0.0045 + j0.19 Ω referred to its 13.8-kV winding.

a. If the transformers are connected Y-Y, calculate (i) the voltage and power rating of the three-phase connection, (ii) the equivalent impedance as referred to its low-voltage terminals, and (iii) the equivalent impedance as referred to its high-voltage terminals.

b. Repeat part (a) if the transformer is connected Y on its low-voltage side and $\varDelta$ on its high-voltage side.

2.24 A three-phase $\varUpsilon - \Omega$ transformer is rated 225-kV:24-kV, 400 MVA and has a series reactance of 11.7 Ω as referred to its high-voltage terminals. The transformer is supplying a load of 325 MVA, with 0.93 power factor lagging at a voltage of 24 kV (line-to-line) on its low-voltage side. It is supplied from a feeder whose impedance is 0.11 + j 2.2 Ω connected to its high-voltage terminals. For these conditions, calculate (a) the line-to-line voltage at the high-voltage terminals of the transformer and (b) the line-to-line voltage at the sending end of the feeder.

2.25 Assume the total load in the system of Problem 2.24 to remain constant at 325 MVA. Write a MATLAB script to plot the line-to-line voltage which must be applied to the sending end of the feeder to maintain the load voltage at 24 kV line-to-line for load power factors in range from 0.75 lagging to unity to 0.75 leading. Plot the sending-end voltage as a function of power factor angle.

2.26 A Δ-Y-connected bank of three identical 100-kVA, 2400-V:120-V, 60-Hz transformers is supplied with power through a feeder whose impedance is 0.065 + j0.87 Ω per phase. The voltage at the sending end of the feeder is held constant at 2400 V line-to-line. The results of a single-phase short-circuit test on one of the transformers with its low-voltage terminals short-circuited are

VH=53.4V f=60Hz IH=41.7A P=832W

a. Determine the line-to-line voltage on the low-voltage side of the transformer when the bank delivers rated current to a balanced three-phase unity power factor load.

b. Compute the currents in the transformer's high- and low-voltage windings and in the feeder wires if a solid three-phase short circuit occurs at the secondary line terminals.

2.27 A 7970-V: 120-V, 60-Hz potential transformer has the following parameters as seen from the high-voltage (primary) winding:

X1 = 1721 $\Omega X_2^{'}$ = 1897 Ω Xm = 782 $k\Omega$

R1 = 1378 $\Omega R_2^{'}$ = 1602 Ω

a. Assuming that the secondary is open-circuited and that the primary is connected to a 7.97-kV source, calculate the magnitude and phase angle (with respect to the high-voltage source) of the voltage at the secondary terminals.

b. Calculate the magnitude and phase angle of the secondary voltage if a 1 $k\Omega$ resistive load is connected to the secondary terminals.

c. Repeat part (b) if the burden is changed to a 1 ***SORRY, THIS MEDIA TYPE IS NOT SUPPORTED.*** reactance.

2.28For the potential transformer of Problem 2.27, find the maximum reactive burden (mimimum reactance) which can be applied at the secondary terminals such that the voltage magnitude error does not exceed 0.5 percent.

2.29Consider the potential transformer of Problem 2.27.

a. Use MATLAB to plot the percentage error in voltage magnitude as a function of the magnitude of the burden impedance (i) for a resistive burden of 100 $\Omega \leq R_b \leq$ 3000 Ω and (ii) for a reactive burden of 100 $\Omega \leq X_b \leq$ 3000 Ω. Plot these curves on the same axis.

b. Next plot the phase error in degrees as a function of the magnitude of the burden impedance (i) for a resistive burden of 100 $\Omega \leq R_{b\leq}$ ≤ 3000 Ω and (ii) for a reactive burden of 100 $\Omega \leq X_b \leq$ 3000 Ω. Again, plot these curves on the same axis.

2.30 A 200-A:5-A, 60-Hz current transformer has the following parameters as seen from the 200-A (primary) winding:

X1 = 745 $\mu\Omega X_2^{'}$= 813 $\mu\Omega$ Xm = 307 $m\Omega$

R1 = 136 $\mu\Omega R_2^{'}$= 128 $\mu\Omega$

a. Assuming a current of 200 A in the primary and that the secondary is short-circuited, find the magnitude and phase angle of the secondary current.

b. Repeat the calculation of part (a) if the CT is shorted through a 250 $\mu\Omega$ burden.

2.31 Consider the current transformer of Problem 2.30.

a. Use MATLAB to plot the percentage error in current magnitude as a function of the magnitude of the burden impedance (i) for a resistive burden of 100 $\Omega \leq R_{b\leq}$ 1000 Ω and (ii) for a reactive burden of 100 $\Omega \leq X_b \leq$ 1000 Ω. Plot these curves on the same axis.

b. Next plot the phase error in degrees as a function of the magnitude of the burden impedance (i) for a resistive burden of $100\Omega \leq R_b \leq 1000\Omega$and (ii) for a reactive burden of $100\Omega \leq X_b \leq 1000\Omega$. Again, plot these curves on the same axis.

2.32 A 15-kV: 175-kV, 125-MVA, 60-Hz single-phase transformer has primary and secondary impedances of 0.0095 + j0.063 per unit each. The magnetizing impedance is j148 per unit. All quantities are in per unit on the transformer base. Calculate the primary and secondary resistances and reactances and the magnetizing inductance (referred to the low-voltage side) in ohms and henrys.

2.33 The nameplate on a 7.97-kV:460-V, 75-kVA, single-phase transformer indicates that it has a series reactance of 12 percent (0.12 per unit).

a. Calculate the series reactance in ohms as referred to (i) the low-voltage terminal and (ii) the high-voltage terminal.

b. If three of these transformers are connected in a three-phase Y-Y connection, calculate (i) the three-phase voltage and power rating, (ii) the per unit impedance of the transformer bank, (iii) the series reactance in ohms as referred to the high-voltage terminal, and (iv) the series reactance in ohms as referred to the low-voltage terminal.

c. Repeat part (b) if the three transformers are connected in Y on their HV

side and Δ on their low-voltage side.

2.34 a. Consider the Y-Y transformer connection of Problem 2.33, part (b). If the rated voltage is applied to the high-voltage terminals and the three low-voltage terminals are short-circuited, calculate the magnitude of the phase current in per unit and in amperes on (i) the high-voltage side and (ii) the low-voltage side.

b. Repeat this calculation for the Y- Δ connection of Problem 2.33, part (c).

2.35 A three-phase generator step-up transformer is rated 26-kV:345-kV, 850 MVA and has a series impedance of 0.0035 + j0.087 per unit on this base. It is connected to a 26-kV, 800-MVA generator, which can be represented as a voltage source in series with a reactance of j1.57 per unit on the generator base.

a. Convert the per unit generator reactance to the step-up transformer base.

b. The unit is supplying 700 MW at 345 kV and 0.95 power factor lagging to the system at the transformer high-voltage terminals. (i) Calculate the transformer low-side voltage and the generator internal voltage behind its reactance in kV. (ii) Find the generator output power in MW and the power factor.

Chapter 3:Electromechanical-Energy-Conversion

Chapter 3: Electromechanical-Energy-Conversion Principles

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- The electromechanical-energy-conversion process takes place through the medium of the electric or magnetic field of the conversion device of which the structures depend on their respective functions.

- Transducers: microphone, pickup, sensor, loudspeaker
- Force producing devices: solenoid, relay, electromagnet
- Continuous energy conversion equipment: motor, generator

This chapter is devoted to the principles of electromechanical energy conversion and the analysis of the devices accomplishing this function. Emphasis is placed on the analysis of systems that use magnetic fields as the conversion medium.

- The concepts and techniques can be applied to a wide range of engineering situations involving electromechanical energy conversion.
- Based on the energy method, we are to develop expressions for forces and torques in magnetic-field-based electromechanical systems.

§3.1 Forces and Torques in Magnetic Field Systems

- The Lorentz Force Law gives the force F on a particle of charge q in the presence of electric and magnetic fields.

$F = q(E + v \times B)$ (3.1)

F : newtons, q : coulombs, E : volts/meter, B : telsas, v : meters/second

- In a pure electric-field system,

F   qE (3.2)

- In pure magnetic-field systems,

$F = q(v \times B)$ (3.3)

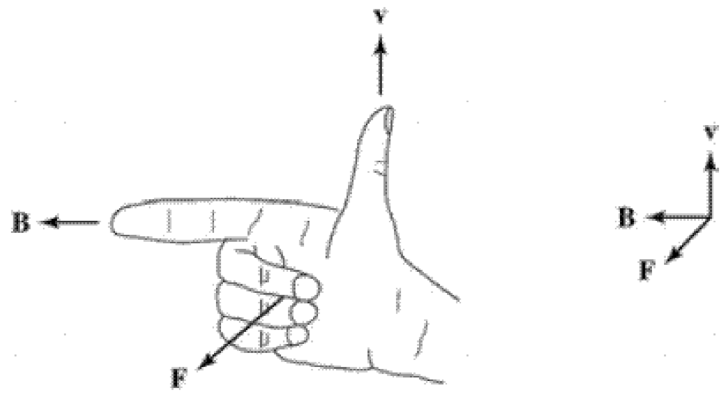


Figure 3.1 Right-hand rule for F=(q x v) B .

- For situations where large numbers of charged particles are in motion,

$F_v = \rho(E + v \times B)$ (3.4)

$J = \rho v$ (3.5)

$F_v = J \times B$ (3.6)

(charge density): coulombs/ m^3, F (force density): newtons/ m^3,

$J = \rho v$ (current density): amperes/ m^2.

- Most electromechanical-energy-conversion devices contain magnetic material.

- Forces act directly on the magnetic material of these devices which are constructed of rigid, nondeforming structures.
- The performance of these devices is typically determined by the net force, or torque, acting on the moving component. It is rarely necessary to calculate the details of the internal force distribution.
- Just as a compass needle tries to align with the earth's magnetic field, the two sets of fields associated with the rotor and the stator of rotating machinery attempt to align, and torque is associated with their displacement from alignment.

    - In a motor, the stator magnetic field rotates ahead of that of the rotor, pulling on it and performing work.
    - For a generator, the rotor does the work on the stator.

        - The Energy Method

- Based on the principle of conservation of energy: energy is neither created nor destroyed; it is merely changed in form.
- Fig. 3.2(a): a magnetic-field-based electromechanical-energy-conversion device.

  - A lossless magnetic-energy-storage system with two terminals
  - The electric terminal has two terminal variables: e (voltage), i (current).
  - The mechanical terminal has two terminal variables: f_{fld}(force), x (position)
  - The loss mechanism is separated from the energy-storage mechanism.

–Electrical losses: ohmic losses…

–Mechanical losses: friction, windage…

Fig. 3.2(b): a simple force-producing device with a single coil forming the electric terminal, and a movable plunger serving as the mechanical terminal.

- The interaction between the electric and mechanical terminals, i.e. the electromechanical energy conversion, occurs through the medium of the magnetic stored energy.

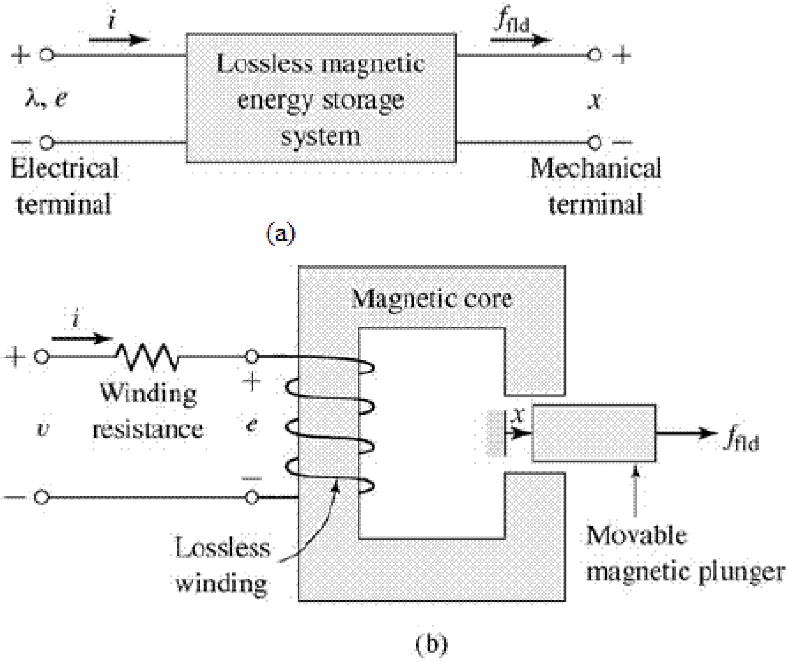


Figure 3.2(a) Schematic magnetic-field electromechanical-energy-conversion device;

(b) simple force-producing device.

- W_{fld}: the stored energy in the magnetic field

$\frac{\mathrm{dW_{fld}}}{\mathrm{dt}} = \mathrm{ei} - f_{\mathrm{fld}}\frac{\mathrm{dx}}{\mathrm{dt}}$ (3.7)

$e = \frac{d\lambda}{\mathrm{dt}}$ (3.8)

$\mathrm{dW_{fld}} = \mathrm{id}\lambda - f_{\mathrm{fld}}\mathrm{dx}$ (3.9)

- Equation (3.9) permits us to solve for the force simply as a function of the flux and the mechanical terminal position x .
- Equations (3.7) and (3.9) form the basis for the energy method.

§3.2 Energy Balance

- Consider the electromechanical systems whose predominant energy-storage mechanism is in magnetic fields. For motor action, we can account for the energy transfer as

$$\begin{matrix} \text{Energy input} & & \text{Mechanical} & & \text{Increase in energy} & & \text{Energy} \\ \text{form electric} & = & \text{energy} & + & \text{stored in magnetic} & + & \text{converted} \\ \text{source} & & \text{ouput} & & \text{field} & & \text{into heat} \end{matrix} \quad (3.10)$$

- Note the generator action.

- The ability to identify a lossless-energy-storage system is the essence of the energy method.

- This is done mathematically as part of the modeling process.
- For the lossless magnetic-energy-storage system of Fig. 3.3(a), rearranging (3.9) in form of (3.10) gives

$\mathrm{dW_{elec}} = \mathrm{dW_{mech}} + \mathrm{dW_{fld}}$ (3.11)

where

$\mathrm{dW_{elec}} = \mathrm{id}\lambda$differential electric energy input

$\mathrm{dW_{mech}} = f_{\mathrm{fld}}\mathrm{dx}$differential mechanical energy output

$\mathrm{dW_{fld}}$differential change in magnetic stored energy

- Here e is the voltage induced in the electric terminals by the changing magnetic stored energy. It is through this reaction voltage that the external electric circuit supplies power to the coupling magnetic field and hence to the mechanical output terminals.

$\mathrm{dW_{elec}} = \mathrm{eidt}$ (3.12)

- The basic energy-conversion process is one involving the coupling field and its action and reaction on the electric and mechanical systems.
- Combining (3.11) and (3.12) results in

$\mathrm{dW_{elec}} = \mathrm{eidt} = \mathrm{dW_{mech}} + \mathrm{dW_{fld}}$ (3.13)

§3.3 Energy in Singly-Excited Magnetic Field Systems

- We are to deal energy-conversion systems: the magnetic circuits have air gaps between the stationary and moving members in which considerable energy is stored in the magnetic field.

- This field acts as the energy-conversion medium, and its energy is the reservoir between the electric and mechanical system.

- Fig. 3.3 shows an electromagnetic relay schematically. The predominant energy storage occurs in the air gap, and the properties of the magnetic circuit are determined by the dimensions of the air gap.

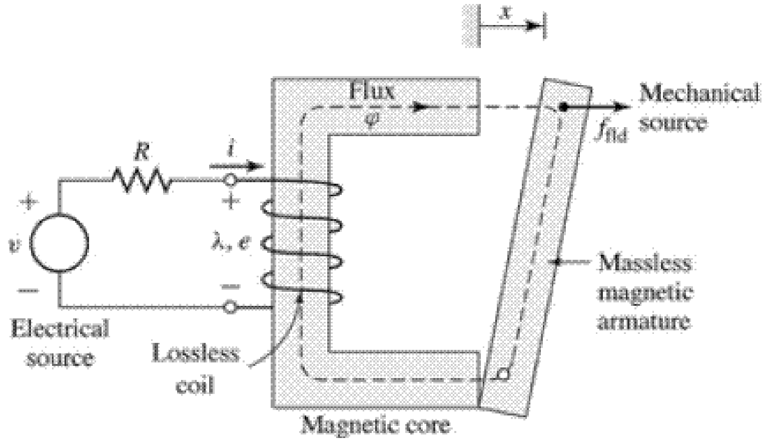


Figure 3.3Schematic of an electromagnetic relay.

$\lambda = L(x)I$(3.14)

$\mathrm{dW}_{\mathrm{mech}} = f_{\mathrm{fld}}\mathrm{dx}$(3.15)

$\mathrm{dW}_{\mathrm{fld}} = \mathrm{id}\lambda - f_{\mathrm{fld}}\mathrm{dx}$(3.16)

- W_{fld} is uniquely specified by the values of λ and x . Therefore, $\lambda$and x are referred to as state variables.
- Since the magnetic energy storage system is lossless, it is a conservative system. W_{fld} is the same regardless of how λ and x are brought to their final values. See Fig. 3.4 where tow separate paths are shown.

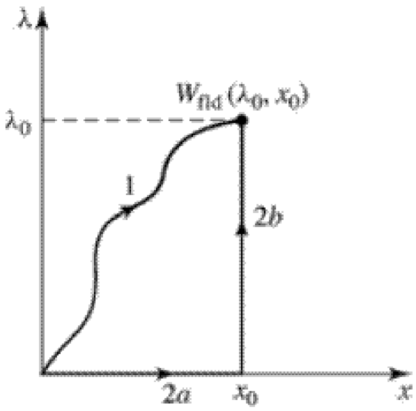


Figure 3.4Integration paths for Wfld

$W_{\text{fld}}(\lambda_0{,}x_0) = \int_{\text{path 2a}} \mathrm{dW}_{\text{fld}} + \int_{\text{path 2b}} \mathrm{dW}_{\text{fld}}$ (3.17)

On path 2a, $d\lambda = 0$ and $f_{\text{fld}} = 0$. Thus, $\mathrm{dW}_{\text{fld}} = 0$ on path 2a.

On path 2b, dx 0 .

Therefore, (3.17) reduces to the integral of $\mathrm{i}d\lambda$ over path 2b.

$W_{\text{fld}}(\lambda_0{,}x_0) = \int_0^{\lambda_0} i(\lambda{,}x_0)d\lambda$ (3.18)

For a linear system in which λ is proportional to i , (3.18) gives

$W_{\text{fld}}(\lambda{,}x) = \int_0^{\lambda} i(\lambda'{,}x)d\lambda' = \int_0^{\lambda} \frac{\lambda'}{L(x)} d\lambda' = \frac{1}{2}\frac{\lambda^2}{L(x)}$ (3.19)

- V : the volume of the magnetic field

$W_{\text{fld}} = \int_v (\int_0^B H.dB')\mathrm{dV}$ (3.20)

If $B = \mu H$,

$W_{\text{fld}} = \int_v (\frac{B^2}{2\mu})\mathrm{dV}$ (3.21)

§3.4 Determination of Magnetic Force and Torque form Energy

- The magnetic stored energy W_{fld} is a state function, determined uniquely by the values of the independent state variables λ and x.

$\mathrm{dW}_{\text{fld}}(\lambda{,}x) = \mathrm{id}\lambda - f_{\text{fld}}\mathrm{dx}$ (3.22)

$\mathrm{dF}(x_1{,}x_2) = \frac{\partial F}{\partial x_1}\ |_{x_2}\ \mathrm{dx}_1 + \frac{\partial F}{\partial x_2}\ |_{x_1}\ \mathrm{dx}_2$ (3.23)

$\mathrm{dW}_{\mathrm{fld}}(\lambda{,}x) = \frac{\partial W_{\mathrm{fld}}}{\partial \lambda}\ |_x\ d\lambda + \frac{\partial W_{\mathrm{fld}}}{\partial x}\ |_\lambda$ dx (3.24)

Comparing (3.22) with (3.24) gives (3.25) and (3.26):

$i = \frac{\partial W_{\mathrm{fld}}(\lambda{,}x)}{\partial \lambda}\ |_x$ (3.25)

$f_{\mathrm{fld}} = -\frac{\partial W_{\mathrm{fld}}(\lambda{,}x)}{\partial x}\ |_\lambda$(3.26)

- Once we know W_{fld} as a function of λ and as a function of λ and i(λ, x) .
- Equation (3.26) can be used to solve for the mechanical force $f_{\mathrm{fld}}(\lambda{,}x)$.The partial derivative is taken while holding the flux linkages λ constant.
- For linear magnetic systems for which $\lambda = L(x)i$, the force can be found as

$f_{\mathrm{fld}} = -\frac{\partial}{\partial x}\ \frac{1}{2}\ \frac{\lambda^2}{L(x)}\ \ |_\lambda = \frac{\lambda^2}{2\mathrm{L}(x)^2}\frac{\mathrm{dL}(x)}{\mathrm{dx}}$ (3.27)

$f_{\mathrm{fld}} = \frac{i^2}{2}\frac{\mathrm{dL}(x)}{\mathrm{dx}}$ (3.28)

- For a system with a rotating mechanical terminal, the mechanical terminal variables become the angular displacement θ and the torque T_{fld} .

$\mathrm{dW}_{\mathrm{fld}}(\lambda{,}\theta) = \mathrm{id}\lambda - T_{\mathrm{fld}}d\theta$ (3.29)

$T_{\mathrm{fld}} = -\frac{\partial W_{\mathrm{fld}}(\lambda{,}\theta)}{\partial \theta}\ |_\lambda$ (3.30)

- For linear magnetic systems for which $\lambda = L(\theta)i$:

$W_{\mathrm{fld}}(\lambda{,}\theta) = \frac{1}{2}\ \frac{\lambda^2}{L(\theta)}$ (3.31)

$T_{\mathrm{fld}} = -\frac{\partial}{\partial \theta}\ \frac{1}{2}\ \frac{\lambda^2}{L(\theta)}\ \ |_\lambda = \frac{1}{2}\ \frac{\lambda^2}{L(\theta)^2}\frac{\mathrm{dL}(\theta)}{d\theta}$ (3.32)

$T_{\mathrm{fld}} = \frac{i^2}{2}\frac{\mathrm{dL}(\theta)}{d\theta}$ (3.33)

§3.5 Determination of Magnetic Force and Torque from Coenergy

- Recall that in §3.4, the magnetic stored energy W_{fld} is a state function, determined uniquely by the values of the independent state variables λ and x .

$\mathrm{dW}_{\mathrm{fld}}(\lambda{,}x) = \mathrm{id}\lambda - f_{\mathrm{fld}}\mathrm{dx}$ (3.34)

$\mathrm{dW}_{\mathrm{fld}}(\lambda{,}x) = \frac{\partial W_{\mathrm{fld}}}{\partial \lambda}\ |_x\ d\lambda + \frac{\partial W_{\mathrm{fld}}}{\partial x}\ |_\lambda$ dx (3.35)

$i = \frac{\partial W_{\text{fld}}(\lambda,x)}{\partial \lambda} \ |_{x}$ (3.36)

$f_{\text{fld}} = \frac{\partial W_{\text{fld}}(\lambda,x)}{\partial x} \ |_{\lambda}$ (3.37)

- Coenergy: from which the force can be obtained directly as a function of the current.The selection of energy or coenergy as the state function is purely a matter of convenience.

- The coenergy $W_{\text{fld}}^{'}(i,x)$ is defined as a function of i and x such that

$W_{\text{fld}}^{'}(i,x) = i\lambda - W_{\text{fld}}(\lambda,x)$ (3.38)

$d(i\lambda) = \mathrm{id}\lambda = \lambda \mathrm{di}$ (3.39)

$dW_{\text{fld}}^{'}(i,x) = d(i\lambda) - \mathrm{dW}_{\text{fld}}(\lambda,x)$ (3.40)

$dW_{\text{fld}}^{'}(i,x) = \lambda \mathrm{di} + f_{\text{fld}}\mathrm{dx}$ (3.41)

- From (3.37), the coenergy $W_{\text{fld}}^{'}(i,x)$can be seen to be a state function of the two independent variables i and x .

$dW_{\text{fld}}^{'}(i,x) = \frac{\partial W_{\text{fld}}^{'}}{\partial i} \ |_{x} \ \mathrm{di} + \frac{\partial W_{\text{fld}}^{'}}{\partial x} \ |_{i} \ \mathrm{dx}$ (3.42)

$\lambda = \frac{\partial W_{\text{fld}}^{'}(i,x)}{\partial i} \ |_{x}$ (3.43)

$f_{\text{fld}} = \frac{\partial W_{\text{fld}}^{'}(i,x)}{\partial x} \ |_{i}$ (3.44)

- For any given system, (3.26) and (3.40) will give the same result.
- By analogy to (3.18) in §3.3, the coenergy can be found as (3.41)

$W_{\text{fld}}(\lambda_0,x_0) = \int_0^{\lambda_0} i(\lambda,x_0)d\lambda$ (3.42)

$W_{\text{fld}}^{'}(i,x) = \int \lambda(i^{'},x)di^{'}$ (3.43)

For linear magnetic systems for which $\lambda = L(x)i$,

$W_{\text{fld}}^{'}(i,x) = \frac{1}{2}L(x)i^2$ (3.44)

$f_{\text{fld}} = \frac{i^2}{2}\frac{\mathrm{dL}(x)}{\mathrm{dx}}$ (3.45)

- For a system with a rotating mechanical displacement,

$$W_{\text{fld}}^{'}(i,\theta) = \int_0^i \lambda(i^{'},\theta)di^{'} \text{ (3.46)}$$

$$T_{\text{fld}} = \frac{\partial W_{\text{fld}}^{'}(i,\theta)}{\partial\theta} \ |_i \text{ (3.47)}$$

If the system is magnetically linear,

$$W_{\text{fld}}^{'}(i,\theta) = \frac{1}{2}L(\theta)i^2 \text{ (3.48)}$$

$$T_{\text{fld}} = \frac{i^2}{2}\frac{\text{dL}(\theta)}{d\theta} \text{ (3.49)}$$

(3.47) is identical to the expression given by (3.33).

- In field-theory terms, for soft magnetic materials

$$W_{\text{fld}}^{'} = \int_V \int_0^{H_0} \text{BdH dV (3.50)}$$

$$W_{\text{fld}}^{'} = \int_v \frac{\mu H^2}{2}\text{dV (3.51)}$$

For permanent-magnet (hard) materials

$$W_{\text{fld}}^{'} = \int_v \int_{H_c}^{H_0} \text{BdH dV (3.52)}$$

- For a magnetically-linear system, the energy and coenergy (densities) are numerically equal: $\lambda^2/2\text{L} = \frac{1}{2}\text{Li}^2, B^2/2\text{μ} = \frac{1}{2}\mu H^2$. For a nonlinear system in which λ and i or B and H are not linearly proportional, the two functions are not even numerically equal.

$$W_{\text{fld}} + W_{\text{fld}}^{'} = \lambda i \text{ (3.53)}$$

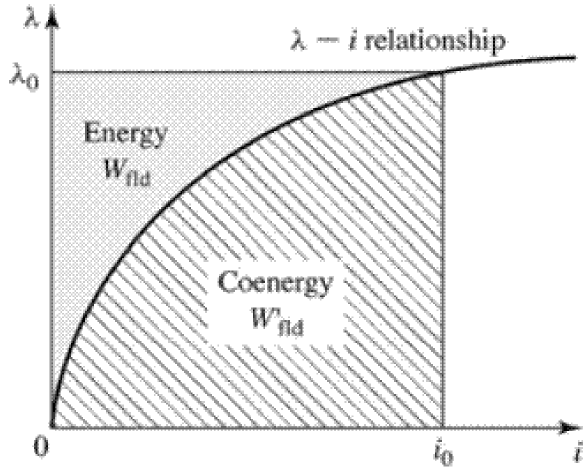


Figure 3.5Graphical interpretation of energy and coenergy in a singly-excited system.

- Consider the relay in Fig. 3.3. Assume the relay armature is at position x so that the device operating at point a in Fig. 3.6. Note that

$$f_{\text{fld}} = -\frac{\partial W_{\text{fld}}(\lambda,x)}{\partial x}\ \Big|_{\lambda} \simeq \lim_{\Delta x\to 0}\frac{-\Delta W_{\text{fld}}}{\Delta x}\ \Big|_{\lambda}\ \text{and} f_{\text{fld}} = \frac{\partial W_{\text{fld}}^{'}(i,x)}{\partial x}\ \Big|_{i} \simeq \lim_{\Delta x\to 0}\frac{\Delta W_{\text{fld}}^{'}}{\Delta x}\ \Big|_{i}$$

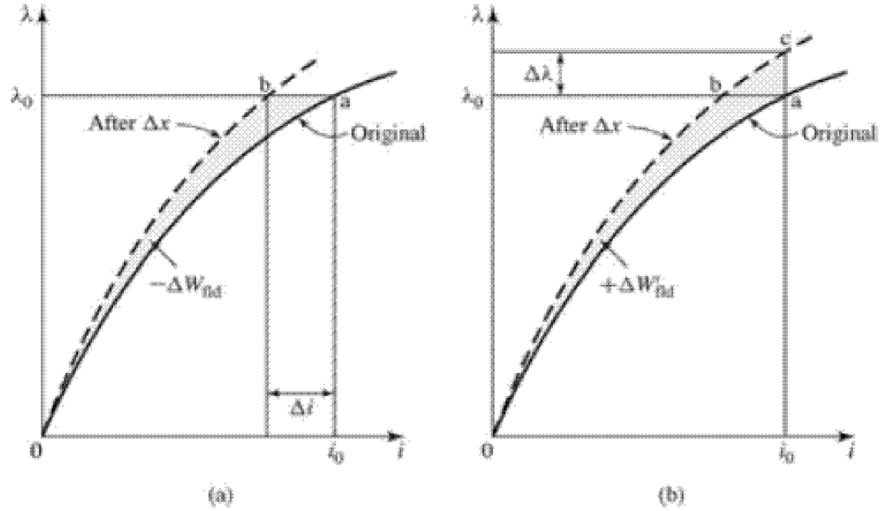


Figure 3.6Effect of Δx on the energy and coenergy of a singly-excited device:

(a) change of energy with λ held constant; (b) change of coenergy with i held constant.

- The force acts in a direction to decrease the magnetic field stored energy at constant flux or to increase the coenergy at constant current.
- In a singly-excited device, the force acts to increase the inductance by pulling on members so as to reduce the reluctance of the magnetic path linking the winding.

§3.6 Multiply-Excited Magnetic Field Systems

- Many electromechanical devices have multiple electrical terminals.

- Measurement systems: torque proportional to two electric signals; power as the product of voltage and current.
- Energy conversion devices: multiply-excited magnetic field system.
- A simple system with two electrical terminals and one mechanical terminal: Fig.3.7.

    - Three independent variables: $\{\theta,\lambda_1,\lambda_2\}$, $\{\theta,i_1,i_2\}$, $\{\theta,\lambda_1,i_2\}$, or $\{\theta,i_1,\lambda_2\}$

dW$_{\mathrm{fld}}(\lambda_1,\lambda_2,\theta) = i_1 d\lambda_1 + i_2 d\lambda_2 - T_{\mathrm{fld}} d\theta$ (3.54)

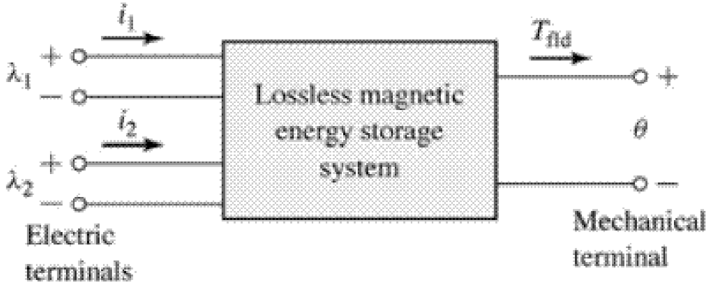


Figure 3.7Multiply-excited magnetic energy storage system.

$i_1 = \frac{\partial W_{\mathrm{fld}}(\lambda_1,\lambda_2,\theta)}{\partial \lambda_1} \,|_{\lambda_2,\theta}$ (3.55)

$i_2 = \frac{\partial W_{\mathrm{fld}}(\lambda_1,\lambda_2,\theta)}{\partial \lambda_2} \,|_{\lambda_1,\theta}$(3.56)

$T_{\mathrm{fld}} = -\frac{\partial W_{\mathrm{fld}}(\lambda_1,\lambda_2,\theta)}{\partial \theta} \,|_{\lambda_1,\lambda_2}$ (3.57)

To find W_{fld} , use the path of integration in Fig. 3.14.

$W_{\mathrm{fld}}(\lambda_{1_0},\lambda_{2_0},\theta_0) = \int_0^{\lambda_{2_0}} i_2(\lambda_1 = 0,\lambda_2,\theta = \theta_0) d\lambda_2 + \int_0^{\lambda_{1_0}} i_1(\lambda_1,\lambda_2 = \lambda_{2_0},\theta = \theta_0) d\lambda_1$ (3.58)

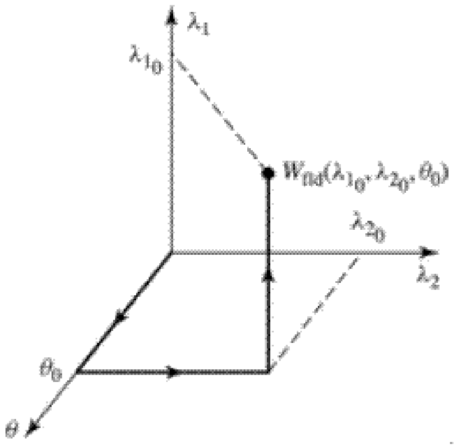


Figure 3.8Integration path to obtain $W_{\mathrm{fld}}(\lambda_{1_0},\lambda_{2_0},\theta_0)$.

- In a magnetically-linear system,

$\lambda_1 = L_{11} i_1 + L_{12} i_2$ (3.59)

$\lambda_2 = L_{21} i_1 + L_{22} i_2$ (3.60)

$L_{12} = L_{21}$ (3.61)

Note that $L_{\mathrm{ij}} = L_{\mathrm{ij}}(\theta)$

$i_1 = \frac{L_{22}\lambda_1 - L_{12}\lambda_2}{D}$ (3.62)

$i_2 = \frac{-L_{21}\lambda_1 + L_{11}\lambda_2}{D}$ (3.63)

$D = L_{11}L_{22} - L_{12}L_{21}$ (3.64)

The energy for this linear system is

$$\begin{aligned} W_{\mathrm{fld}}(\lambda_{1_0},\lambda_{2_0},\theta_0) &= \int_0^{\lambda_{2_0}} \frac{L_{11}(\theta_0)\lambda_2}{D(\theta_0)} d\lambda_2 + \int \frac{(L_{22}(\theta_0)\lambda_1 - L_{12}(\theta_0)\lambda_{2_0})}{D(\theta_0)} \\ &= \frac{1}{2\mathrm{D}(\theta_0)} L_{11}(\theta_0)\lambda_{2_0}^2 + \frac{1}{2\mathrm{D}(\theta_0)} L_{22}(\theta_0)\lambda_{1_0}^2 - \frac{L_{12}(\theta_0)}{D(\theta_0)}\lambda_{1_0}\lambda_{2_0} \end{aligned} \quad (3.65)$$

- Coenergy function for a system with two windings can be defined as:

$W_{\mathrm{fld}}^{'}(i_1,i_2,\theta) = \lambda_1 i_1 + \lambda_2 i_2 - W_{\mathrm{fld}}$ (3.66)

$dW_{\mathrm{fld}}^{'}(i_1,i_2,\theta) = \lambda_1 \mathrm{di}_1 + \lambda_2 \mathrm{di}_2 + T_{\mathrm{fld}} d\theta$ (3.67)

$\lambda_1 = \frac{\partial W_{\mathrm{fld}}(i_1,i_2,\theta)}{\partial i_1} \; |_{i_2,\theta}$ (3.68)

$$\lambda_2 = \frac{\partial W^{'}_{\mathrm{fld}}(i_1,i_2,\theta)}{\partial i_2} \Big|_{i_1,i_2} \text{ (3.69)}$$

$$T_{\mathrm{fld}} = \frac{\partial W^{'}_{\mathrm{fld}}(i_1,i_2,\theta)}{\partial \theta} \Big|_{i_1,i_2} \text{ (3.70)}$$

$$W^{'}_{\mathrm{fld}}(i_1,i_2,\theta_0) = \int_0^{i_{2_0}} \lambda_2(i_1 = 0,i_2,\theta = \theta_0)\mathrm{di}_2 + \int_0^{\lambda_{1_0}} \lambda_1(i_1,i_2 = i_{2_0},\theta = \theta_0)\mathrm{di}_1 \text{ (3.71)}$$

- For the linear system:

$$W^{'}(i_1,i_2,\theta_0) = \frac{1}{2}L_{11}(\theta)i_1^2 + \frac{1}{2}L_{22}(\theta)i_2^2 + L_{12}(\theta)i_1i_2 \text{ (3.72)}$$

$$T_{\mathrm{fld}} = \frac{\partial W^{'}_{\mathrm{fld}}(i_1,i_2,\theta_0)}{\partial \theta} \Big|_{i_1,i_2} = \frac{i_1^2}{2}\frac{\mathrm{dL}_{11}(\theta)}{d\theta} + \frac{i_2^2}{2}\frac{\mathrm{dL}_{22}(\theta)}{d\theta} + i_1i_2\frac{\mathrm{dL}_{12}(\theta)}{d\theta} \text{ (3.73)}$$

- Note that (3.70) is simpler than (3.57). That is, the coenergy function is a relatively simple function of displacement.
- The use of a coenergy function of the terminal currents simplifies the determination of torque or force.
- Systems with more than two electrical terminals are handled in analogous fashion.
    - System with linear displacement:

$$W_{\mathrm{fld}}(\lambda_{1_0},\lambda_{2_0},x_0) = \int_0^{\lambda_{2_0}} i_2(\lambda_1 = 0,\lambda_2,x = x_0)d\lambda_2 + \int_0^{\lambda_{1_0}} i_1(\lambda_1,\lambda_2 = \lambda_{2_0},x = x_0)d\lambda_1 \text{ (3.74)}$$

$$W^{'}_{\mathrm{fld}}(i_{1_0},i_{2_0},x_0) = \int_0^{\lambda_{2_0}} \lambda_2(i_1,i_2,x = x_0)\mathrm{di}_2 + \int_0^{\lambda_{1_0}} \lambda_1(i_1,i_2 = i_{2_0},x = x_0)\mathrm{di}_1$$

$$f_{\mathrm{fld}} = -\frac{\partial W_{\mathrm{fld}}(\lambda_1,\lambda_2,x)}{\partial x} \Big|_{\lambda_1,\lambda_2} \text{ (3.76)}$$

$$f_{\mathrm{fld}} = -\frac{\partial W^{'}_{\mathrm{fld}}(i_1,i_2,x)}{\partial x} \Big|_{i_1,i_2} \text{ (3.77)}$$

For a magnetically-linear system,

$$W^{'}_{\mathrm{fld}}(i_1,i_2,x) = \frac{1}{2}L_{11}(x)i_1^2 + L_{22}(x)i_2^2 + L_{12}(x)i_1i_2 \text{ (3.78)}$$

$$f_{\mathrm{fld}} = \frac{i_1^2}{2}\frac{\mathrm{dL}_{11}(x)}{\mathrm{dx}} + \frac{i_2^2}{2}\frac{\mathrm{dL}_{22}(x)}{\mathrm{dx}} + i_1i_2\frac{\mathrm{dL}_{12}(x)}{\mathrm{dx}} \text{ (3.79)}$$

PROBLEMS - chapter 3

PROBLEMS

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

3.1 An inductor has an inductance which is found experimentally to be of the form

$$L = \frac{2\mathrm{L}_0}{1+x/x_0}$$

where L0 = 30 mH, x0 = 0.87 mm, and x is the displacement of a movable element. Its winding resistance is measured and found to equal 110 m Ω.

a. The displacement x is held constant at 0.90 mm, and the current is increased from 0 to 6.0 A. Find the resultant magnetic stored energy in the inductor.

b. The current is then held constant at 6.0 A, and the displacement is increased to 1.80 mm. Find the corresponding change in magnetic stored energy.

3.2 Repeat Problem 3.1, assuming that the inductor is connected to a voltage source which increases from 0 to 0.4 V (part [a]) and then is held constant at 0.4 V (part [b]). For both calculations, assume that all electric transients can be ignored.

3.3 The inductor of Problem 3.1 is driven by a sinusoidal current source of the form

i(t) = $I_0 \sin \omega t$

where I0 = 5.5 A and $\omega = 100\pi$ (50 Hz). With the displacement held fixed at x = x_0, calculate (a) the time-averaged magnetic stored energy (W_{fld}) in the inductor and (b) the time-averaged power dissipated in the winding resistance.

3.4 An actuator with a rotating vane is shown in Fig. 3.1. You may assume that the permeability of both the core and the vane are infinite ($\mu \to \infty$). The total air-gap length is 2g and shape of the vane is such that the effective area of the air gap can be assumed to be of the form

$$A_g = A_0\left[1 - \left[\frac{4\theta}{\pi}\right]^2\right]$$

(valid only in the range $\mid \theta \mid \leq \frac{\pi}{6}$). The actuator dimensions are g = 0.8 mm, $A_0 = 6.0\mathrm{mm}^2$, and N = 650 turns.

a. Assuming the coil to be carrying current i, write an expression for the magnetic stored energy in the actuator as a function of angle θ for $\mid \theta \mid \leq \frac{\pi}{6}$.

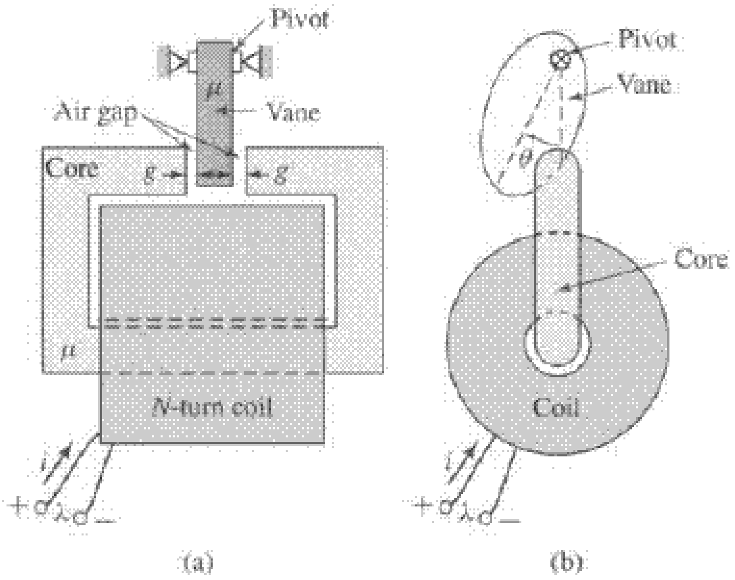


Figure 3.1 Actuator with rotating vane for

Problem 3.7. (a) Side view. (b) End view.

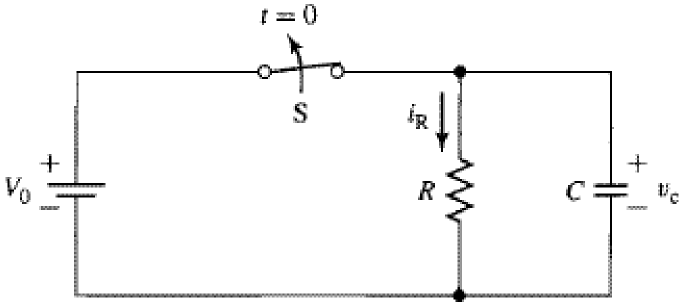


Figure 3.2 An RC circuit for Problem 3.8.

b. Find the corresponding inductance L(θ). Use MATLAB to plot this inductance as a function of θ.

3.5 An RC circuit is connected to a battery, as shown in Fig. 3.2. Switch S is initially closed and is opened at time t = 0.

a. Find the capacitor voltage V_C(t) for t 0

b. What are the initial and final (t = ∞) values of the stored energy in the capacitor? (Hint: $W_{\text{fld}} = \frac{1}{2} \cdot \frac{q^2}{C}$, where q = C V_0.) What is the energy stored in the capacitor as a function of time?

c. What is the power dissipated in the resistor as a function of time? What is the total energy dissipated in the resistor?

3.6 An RL circuit is connected to a battery, as shown in Fig. 3.3. Switch S is initially closed and is opened at time t = 0.

a. Find the inductor current i_L(t) for t 0. (Hint: Note that while the switch is closed, the diode is reverse-biased and can be assumed to be an open circuit. Immediately after the switch is opened, the diode becomes forward-biased and can be assumed to be a short circuit.)

b. What are the initial and final (t = ∞) values of the stored energy in the inductor? What is the energy stored in the inductor as a function of time?

c. What is the power dissipated in the resistor as a function of time? What is the total energy dissipated in the resistor?

3.7 The L/R time constant of the field winding of an 500-MVA synchronous generator is 4.8 s. At normal operating conditions, the field winding is known to be dissipating 1.3 MW. Calculate the corresponding magnetic stored energy.

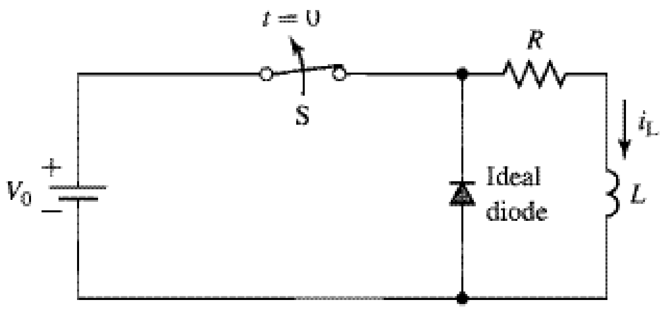


Figure 3.23 An RL circuit for Problem 3.9.

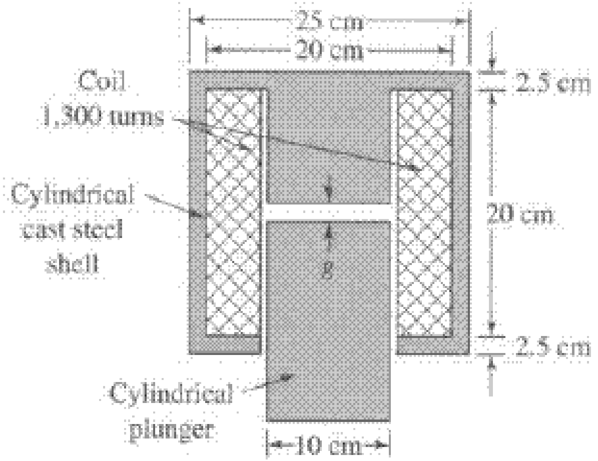


Figure 3.4 Plunger actuator for Problem 3.12.

3.8 The inductance of a phase winding of a three-phase salient-pole motor is measured to be of the form

L(
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) = + $L_2\cos 2\theta_m$

where θ_m is the angular position of the rotor.

a. How many poles are on the rotor of this motor?

b. Assuming that all other winding currents are zero and that this phase is excited by a constant current I_0, find the torque $T_{\text{nd}}(\theta)$ acting on the rotor.

3.9 Cylindrical iron-clad solenoid actuators of the form shown in Fig. 3.4 are used for tripping circuit breakers, for operating valves, and in other applications in which a relatively large force is applied to a member which moves a relatively short distance. When the coil current is zero, the plunger drops against a stop such that the gap g is 2.25 cm. When the coil is energized by a direct current of sufficient magnitude, the plunger is raised until it hits another stop set so that g is 0.2 cm. The plunger is supported so that it can move freely in the vertical direction. The radial air gap between the shell and the plunger can be assumed to be uniform and 0.05 cm in length. For this problem neglect the magnetic leakage and fringing in the air gaps. The exciting coil has 1300 turns and carries a constant current of 2.3 A. Assume that the mmf in the iron can be neglected and use MATLAB to

a. plot the flux density in the variable gap between the yoke and the plunger for the range of travel of the plunger,

b. plot the corresponding values of the total energy stored in the magnetic field in μJ, and

c. plot the corresponding values of the coil inductance in μH.

3.10 Consider the plunger actuator of Fig. 3.4. Assume that the plunger is initially fully opened (g = 2.25 cm) and that a battery is used to supply a current of 2.5 A to the winding.

a. If the plunger is constrained to move very slowly (i.e., slowly compared to the electrical time constant of the actuator), reducing the gap g from 2.25 to 0.20 cm, how much mechanical work in joules will be supplied to the plunger?

b. For the conditions of part (a), how much energy will be supplied by the battery (in excess of the power dissipated in the coil)?

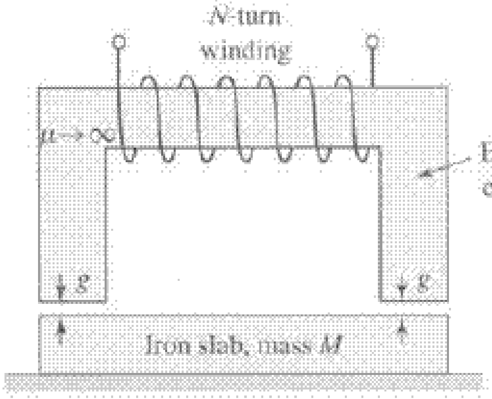
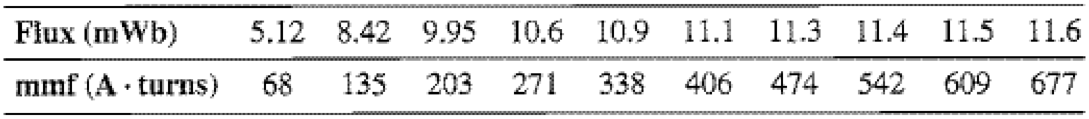


Figure 3.5 Electromagnet lifting an iron slab (Problem 3.14).

3.11 As shown in Fig.3.5, an N-turn electromagnet is to be used to lift a slab of iron of mass M. The surface roughness of the iron is such that when the iron and the electromagnet are in contact, there is a minimum air gap of $g_{\min}$ = 0.18 mm in each leg. The electromagnet cross-sectional area A_C = 32 cm and coil resistance is 2.8 Ω. Calculate the minimum coil voltage which must be used to lift a slab of mass 95 kg against the force of gravity. Neglect the reluctance of the iron.

3.12 Data for the magnetization curve of the iron portion of the magnetic circuit of the plunger actuator of Problem 3.12 are given below:



Flux (mWb)	5.12	8.42	9.95	10.6	10.9	11.1	11.3	11.4	11.5	11.6
mmf (A · turns)	68	135	203	271	338	406	474	542	609	677

a. Use the MATLAB polyfit function to obtain a 3'rd-order fit of reluctance and total flux versus mmf for the iron portions of the magnetic circuit.

Your fits will be of the form:

$$R_{\text{iron}} = a_1 F_{\text{iron}}^3 + a_2 F_{\text{iron}}^2 + a_3 F_{\text{iron}} + a_4$$

$$\varphi_{\text{iron}} = b_1 F_{\text{iron}}^3 + b_2 F_{\text{iron}}^2 + b_3 F_{\text{iron}} + b_4$$

List the coefficients.

b. (i) Using MATLAB and the functional forms found in part (a), plot the magnetization curve for the complete magnetic circuit (flux linkages λ versus winding current i) for a variable-gap length of g = 0.2 cm. On the same axes, plot the magnetization curve corresponding to the assumption that the iron is of infinite permeability. The maximum current in your plot should correspond to a flux in the magnetic circuit of 600 mWb.

(ii) Calculate the magnetic field energy and coenergy for each of these cases corresponding to a winding current of 2.0 A.

c. Repeat part (b) for a variable-gap length of g = 2.25 cm. In part (ii), calculate the magnetic field energy and coenergy corresponding to a winding current of 20 A.

3.13 An inductor is made up of a 525-turn coil on a core of 14 cm^2 cross-sectional area and gap length 0.16 mm. The coil is connected directly to a 120-V 60-Hz voltage source. Neglect the coil resistance and leakage inductance. Assuming the coil reluctance to be negligible, calculate the time-averaged force acting on the core tending to close the air gap. How would this force vary if the air-gap length were doubled?

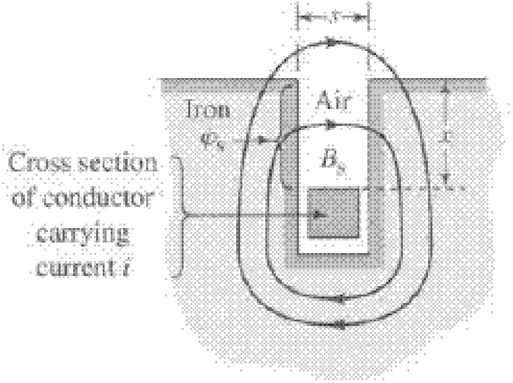


Figure 3.6 Conductor in a slot (Problem 3.17).

3.14 Figure 3.6 shows the general nature of the slot-leakage flux produced by current i in a rectangular conductor embedded in a rectangular slot in iron. Assume that the iron reluctance is negligible and that the slot leakage flux goes straight across the slot in the region between the top of the conductor and the top of the slot.

a. Derive an expression for the flux density B_S in the region between the top of the conductor and the top of the slot.

b. Derive an expression for the slot-leakage ϕ_S sits crossing the slot above the conductor, in terms of the height x of the slot above the conductor, the slot width s, and the embedded length l perpendicular to the paper.

c. Derive an expression for the force f created by this magnetic field on a conductor of length l. In what direction does this force act on the conductor?

d. When the conductor current is 850 A, compute the force per meter on a conductor in a slot 2.5 cm wide.

3.15 A long, thin solenoid of radius r0 and height h is shown in Fig.3.7. The magnetic field inside such a solenoid is axially directed, essentially uniform and equal to H = N i/h. The magnetic field outside the solenoid can be shown to be negligible. Calculate the radial pressure in newtons per square meter acting on the sides of the solenoid for constant coil current i = I_0.

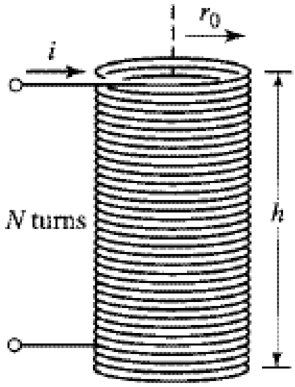


Figure 3.7 Solenoid coil (Problem 3.18).

3.16 An electromechanical system in which electric energy storage is in electric fields can be analyzed by techniques directly analogous to those derived in this chapter for magnetic field systems. Consider such a system in which it is possible to separate the loss mechanism mathematically from those of energy storage in electric fields. Then the system can be represented as in Fig. 3.8. For a single electric terminal, Eq. $\mathrm{dW_{elec} = dW_{mech} + dW_{fld}}$ applies, where dWelec = vi dt = v dq

where v is the electric terminal voltage and q is the net charge associated with electric energy storage. Thus, by analogy to Eq.3.16,

d Wfld = v dq - ffld dx

a. Derive an expression for the electric stored energy W_{fld} (q, x) analogous to that for the magnetic stored energy in Eq. $W_{\mathrm{fld}}(\lambda_0,x_0) = \int_0^{\lambda_0} i(\lambda,x_0) d\lambda$.

b. Derive an expression for the force of electric origin ffld analogous to that of Eq. $f_{\mathrm{fld}} = \frac{\partial W_{\mathrm{fld}}(\lambda,x)}{\partial x} |_{\lambda}$. State clearly which variable must be held constant when the derivative is taken.

c. By analogy to the derivation of Eqs.3.34 to 3.41, derive an expression for the coenergy $W'_{\mathrm{fld}}(v,x)$ and the corresponding force of electric origin.

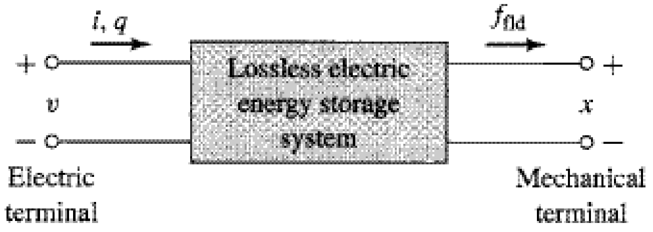


Figure 3.8 Lossless electric energy storage system.

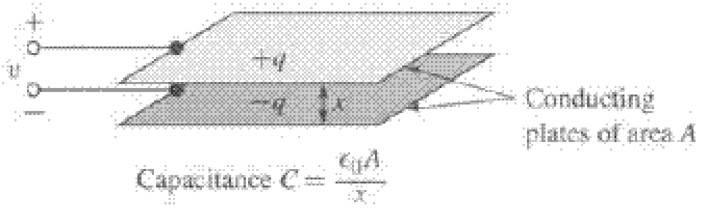

+
v
−
$+q$
$-q$
x
Conducting plates of area A
Capacitance $C = \frac{\epsilon_0 A}{x}$

Figure 3.9 Capacitor plates (Problem 3.20).

3.17 A capacitor (Fig.3.9) is made of two conducting plates of area A separated in air by a spacing x. The terminal voltage is v, and the charge on the plates is q. The capacitance C, defined as the ratio of charge to voltage, is

$$C = \frac{q}{v} = \frac{\varepsilon_0 A}{x}$$

where ε_0 is the dielectric constant of free space (in SI units ε_0 = 8.85x 10^{-12} F/m).

a. Using the results of Problem 3.16, derive expressions for the energy W_{fld} (q, x) and the coenergy W'_{fld}(v, x).

b. The terminals of the capacitor are connected to a source of constant voltage V_0. Derive an expression for the force required to maintain the plates separated by a constant spacing x = δ.

3.18 Figure 3.10 shows in schematic form an electrostatic voltmeter, a capacitive system consisting of a fixed electrode and a moveable electrode. The moveable electrode is connected to a vane which rotates on a pivot such that the air gap between the two electrodes remains fixed as the vane rotates. The capacitance of this system is given by

$$C(\theta) = \frac{\varepsilon_0 \mathrm{Rd}(\alpha - |\theta|)}{g} (|\ \theta\ | \le \alpha)$$

A torsional spring is connected to the moveable vane, producing a torque

Tspring = - K ($\theta - \theta_0$)

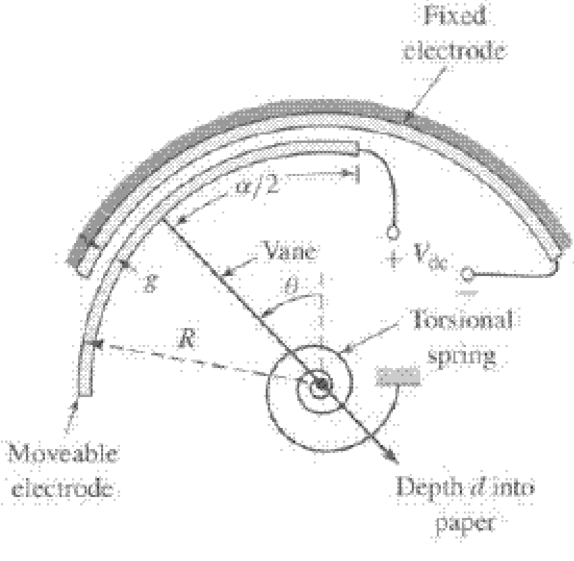


Figure 3.10 Schematic electrostatic voltmeter (Problem 3.21).

a. For $0 \leq \theta \leq \alpha$, using the results of Problem 3.16, derive an expression for the electromagnetic torque T_{nd} in terms of the applied voltage V_{dc}.

b. Find an expression for the angular position of the moveable vane as a function of the applied voltage V_{dc}.

c. For a system with

R = 12 cm, d = 4 cm, g = 0.2 mm

$\alpha - \frac{\pi}{3}$ rad , θ_0 = 0 rad , K=3.65 N.m/rad

Plot the vane position in degrees as a function of applied voltage for 0 $V_{\mathrm{dc}\leq}$ 1500V.

3.19 The two-winding magnetic circuit of Fig. 3.11 has a winding on a fixed yoke and a second winding on a moveable element. The moveable element is constrained to motion such that the lengths of both air gaps remain equal.

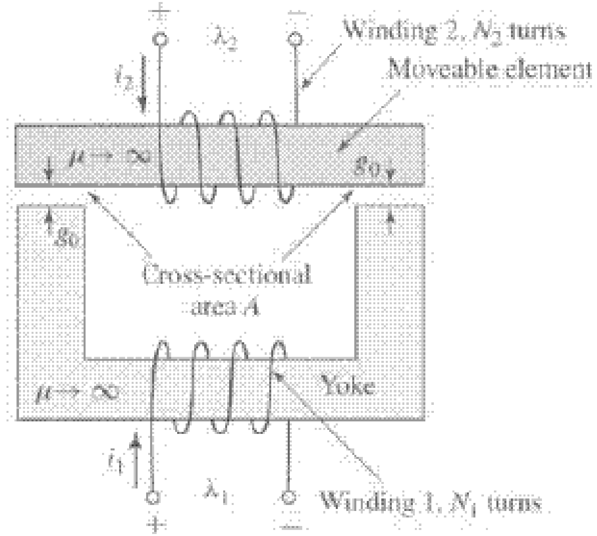


Figure 3.11 Two-winding magnetic circuit for Problem 3.22.

a. Find the self-inductances of windings 1 and 2 in terms of the core dimensions and the number of turns.

b. Find the mutual inductance between the two windings.

c. Calculate the coenergy $W_{\text{fld}}^{'}(i_1,i_2)$.

d. Find an expression for the force acting on the moveable element as a function of the winding currents.

3.20 Two coils, one mounted on a stator and the other on a rotor, have self- and mutual inductances of

L_{11} = 3.5 mH L_{22} = 1.8 mH L_{12} = 2.1cos θ mH

where θ is the angle between the axes of the coils. The coils are connected in series and carry a current

$$i = \sqrt{2}I\text{sin}\omega t$$

a. Derive an expression for the instantaneous torque T on the rotor as a function of the angular position θ.

b. Find an expression for the time-averaged torque T_{avg} as a function of θ.

c. Compute the numerical value of T_{avg} for I = 10 A and θ = 90^0.

d. Sketch curves of T_{avg} versus 0 for currents I = 5, 7.07, and 10 A.

e. A helical restraining spring which tends to hold the rotor at $\theta = 90^0$ is now attached to the rotor. The restraining torque of the spring is proportional to the angular deflection from $\theta = 90^0$ and is -0.1 N. m when the rotor is turned to $\theta = 0^0$. Show on the curves of part (d) how you could find the angular position of the rotor-plus-spring combination for coil currents I = 5, 7.07, and 10 A. From your curves, estimate the rotor angle for each of these currents.

f. Write a MATLAB script to plot the angular position of the rotor as a function of rms current for 0 $I \leq$ 10 A. (Note that this problem illustrates the principles of the dynamometer-type ac ammeter.)

3.21 Two windings, one mounted on a stator and the other on a rotor, have self- and mutual inductances of

L_{11} = 4.5H L_{22} = 2.5H L_{12}= 2.8cosθ H

where 0 is the angle between the axes of the windings. The resistances of the windings may be neglected. Winding 2 is short-circuited, and the current in winding 1 as a function of time is i_1 = 10sin ωt A.

a. Derive an expression for the numerical value in newton-meters of the instantaneous torque on the rotor in terms of the angle θ.

b. Compute the time-averaged torque in newton-meters when $\theta = 45^0$.

c. If the rotor is allowed to move, will it rotate continuously or will it tend to come to rest? If the latter, at what value of θ_0 ?

3.22 A loudspeaker is made of a magnetic core of infinite permeability and circular symmetry, as shown in Figs. 3.12a and b. The air-gap length g is much less than the radius r0 of the central core. The voice coil is constrained to move only in the x direction and is attached to the speaker cone, which is not shown in the figure. A constant radial magnetic field is produced in the air gap by a direct current in coil l, $i_1 = I_1$. An audio-frequency signal $i_2 = I_2\text{cos}\omega t$ is then applied to the voice coil. Assume the voice coil to be of negligible thickness and composed of N_2 turns uniformly distributed over its height h. Also assume that its displacement is such that it remains in the air gap (0 $x \leq$ l-h).

a. Calculate the force on the voice coil, using the Lorentz Force Law (Eq. 3.1).

b. Calculate the self-inductance of each coil.

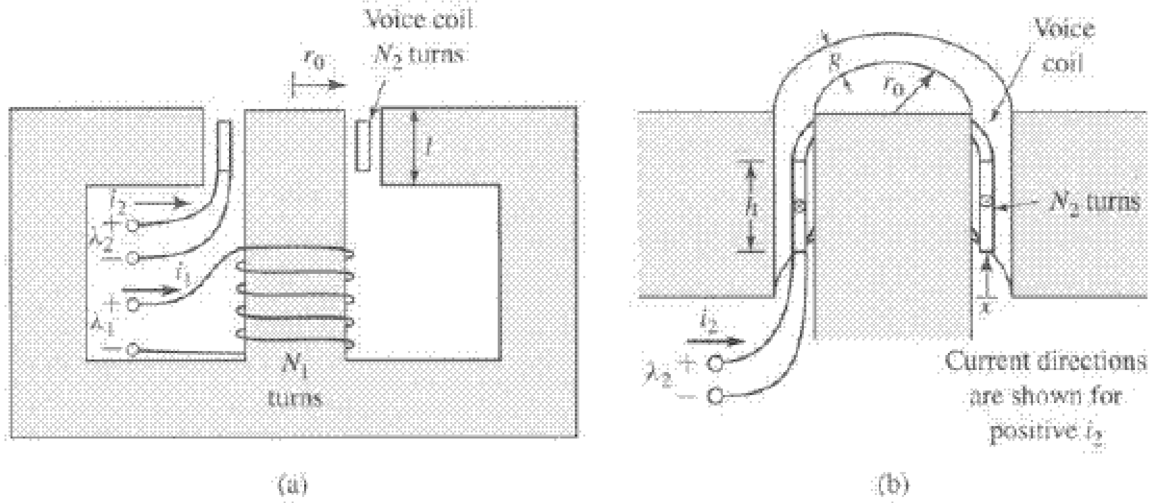


Figure 3.12 Loudspeaker for Problem 3.25.

c. Calculate the mutual inductance between the coils. (Hint: Assume that current is applied to the voice coil, and calculate the flux linkages of coil 1. Note that these flux linkages vary with the displacement x.)

d. Calculate the force on the voice coil from the coenergy $W_{\text{fld}}^{'}$.

3.23 The magnetic structure of Fig. 3.13 is a schematic view of a system designed to support a block of magnetic material ($\mu \rightarrow \infty$) of mass M against the force of gravity. The system includes a permanent magnet and a winding. Under normal conditions, the force is supplied by the permanent magnet alone. The function of the winding is to counteract the field produced by the magnet so that the mass can be removed from the device. The system is designed such that the air gaps at each side of the mass remain constant at length g_0/2. Assume that the permanent magnet can be represented by a linear characteristic of the form

$B_m = \mu_R(H_m - H_C)$and that the winding direction is such that positive winding current reduces the air-gap flux produced by the permanent magnet. Neglect the effects of magnetic fringing.

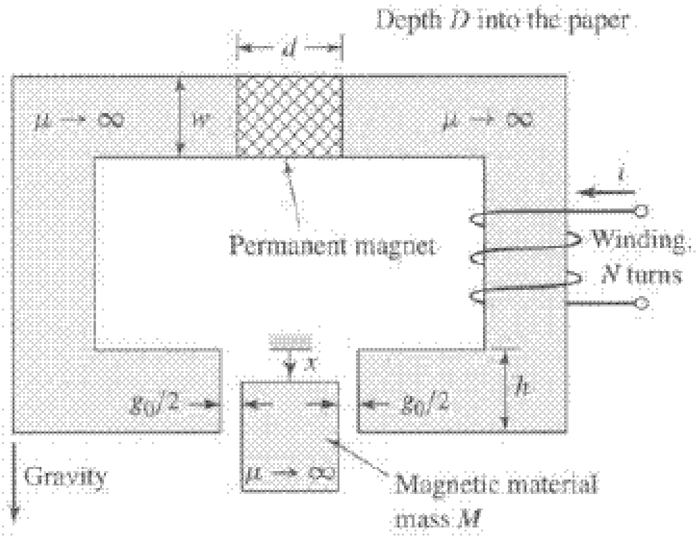


Figure 3.13 Magnetic support system for Problem 3.27.

a. Assume the winding current to be zero. (i) Find the force fnd acting on the mass in the x direction due to the permanent magnet alone as a function of x (0 $x \le$ h). (ii) Find the maximum mass $M_{\max}$ that can be supported against gravity for 0 $x \le$ h.

b. For M = $M_{\max}$/2, find the minimum current required to ensure that the mass will fall out of the system when the current is applied.

3.24 Winding 1 in the loudspeaker of Problem 3.22 (Fig.3.12) is replaced by a permanent magnet as shown in Fig.3.14. The magnet can be represented by the linear characteristic $B_m = \mu_R(H_m - H_c)$.

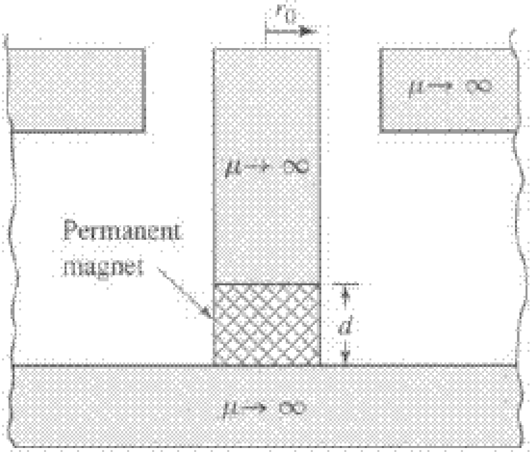


Figure 3.14 Central core of loudspeaker of Fig. 3.37

with winding 1 replaced by a permanent magnet (Problem 3.28).

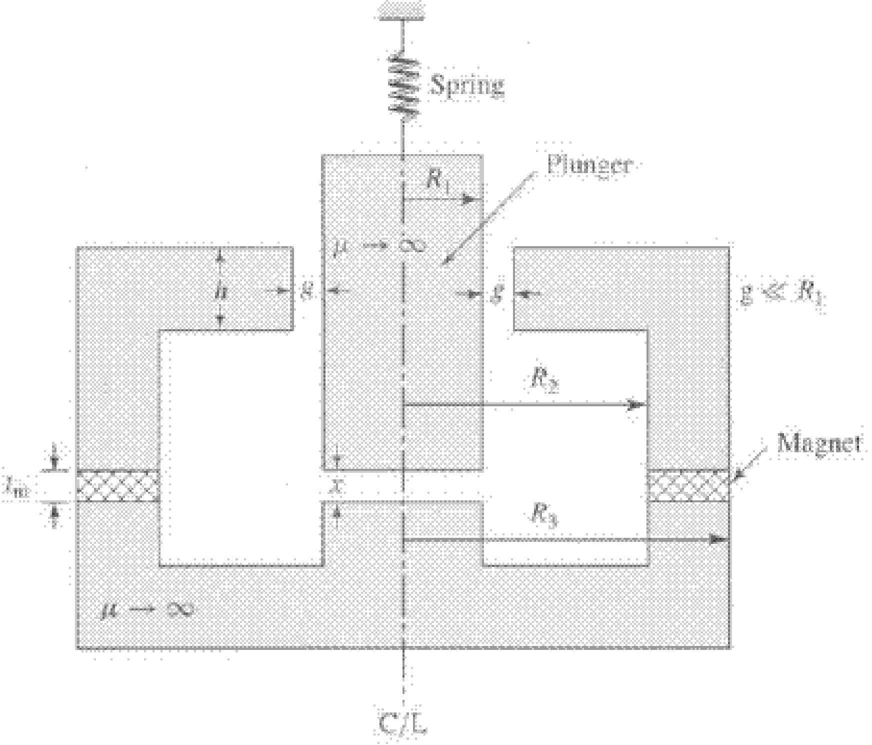


Figure 3.15 Permanent-magnet system for Problem 3.29.

a. Assuming the voice coil current to be zero, (i_2 = 0), calculate the magnetic flux density in the air gap.

b. Calculate the flux linkage of the voice coil due to the permanent magnet as a function of the displacement x.

c. Calculate the coenergy $W'_{\mathrm{fld}}(i_2, x)$ assuming that the voice coil current is sufficiently small so that the component of W'_{fld} due to the voice coil self inductance can be ignored.

d. Calculate the force on the voice coil.

3.25 Figure 3.15 shows a circularly symmetric system in which a moveable plunger (constrained to move only in the vertical direction) is supported by a spring of spring constant K = 5.28 N/m. The system is excited by a samarium-cobalt permanent-magnet in the shape of a washer of outer radius R_3, inner radius R_2, and thickness t_m. The system dimensions are:

R_1 = 2.1 cm, R_2 = 4 cm, R_3 = 4.5 cm

h = 1 cm, g = 1 mm, t_m = 3 mm

The equilibrium position of the plunger is observed to be x = 1.0 mm.

a. Find the magnetic flux density B_g in the fixed gap and B_x in the variable gap.

b. Calculate the x-directed magnetic force pulling down on the plunger.

c. The spring force is of the form f_{spring}= K (X_0 - x). Find X_0.

3.26 The plunger of a solenoid is connected to a spring. The spring force is given by f = K_0(0.9a - x), where x is the air-gap length. The inductance of the solenoid is of the form L = L_0(1 - x/a), and its winding resistance is R. The plunger is initially stationary at position x = 0.9a when a dc voltage of magnitude V_0 is applied to the solenoid.

a. Find an expression for the force as a function of time required to hold the plunger at position a/2.

b. If the plunger is then released and allowed to come to equilibrium, find the equilibrium position X_0. You may assume that this position falls in the range 0 $X_0 \leq$ a.

3.27 Consider the solenoid system of Problem 3.26. Assume the following parameter values:

L_0 = 4.0mH a = 2.2cm R = 1.5 ΩK_0=3.5 N/cm

The plunger has mass M = 0.2 kg. Assume the coil to be connected to a dc source of magnitude 4 A. Neglect any effects of gravity.

a. Find the equilibrium displacement X_0.

b. Write the dynamic equations of motion for the system.

c. Linearize these dynamic equations for incremental motion of the system around its equilibrium position.

d. If the plunger is displaced by an incremental distance E from its equilibrium position X_0 and released with zero velocity at time t = 0, find (i) The resultant motion of the plunger as a function of time, and (ii) The corresponding time-varying component of current induced across the coil terminals.

3.28 The solenoid of Problem 3.27 is now connected to a dc voltage source of magnitude 6V.

a. Find the equilibrium displacement X_0.

b. Write the dynamic equations of motion for the system.

c. Linearize these dynamic equations for incremental motion of the system around its equilibrium position.

Chapter 4: Introduction to Rotating Machines

Chapter 4: Introduction to Rotating Machines

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- The objective of this chapter is to introduce and discuss some of the principles underlying the performance of electric machinery, both ac and dc machines.

§4.1 Elementary Concepts

- Voltages can be induced by time-varying magnetic fields. In rotating machines, voltages are generated in windings or groups of coils by rotating these windings mechanically through a magnetic field, by mechanically rotating a magnetic field past the winding, or by designing the magnetic circuit so that the reluctance varies with rotation of the rotor.

- The flux linking a specific coil is changed cyclically, and a time-varying voltage is generated.
- Electromagnetic energy conversion occurs when changes in the flux linkage result from mechanical motion.
- A set of such coils connected together is typically referred to as an armature winding, a winding or a set of windings carrying ac currents.
    - In ac machines such as synchronous or induction machines, the armature winding is typically on the stator. (the stator winding)
    - In dc machines, the armature winding is found on the rotor. (the rotor winding)

- Synchronous and dc machines typically include a second winding (or set of windings), referred to as the field winding, which carrys dc current and which are used to produce the main operating flux in the machine.
    - In dc machines, the field winding is found on the stator.
    - In synchronous machines, the field winding is found on the rotor.
    - Permanent magnets can be used in the place of field windings.

- In most rotating machines, the stator and rotor are made of electrical steel, and the windings are installed in slots on these structures.The stator and rotor structures are typically built from thin laminations of electrical steel, insulated from each other, to reduce eddy-current losses.

§4.2 Introduction to AC And DC Machines

§4.2.1 AC Machines

- Traditional ac machines fall into one of two categories: synchronous and induction.

- In synchronous machines, rotor-winding currents are supplied directly from the stationary frame through a rotating contact.
- In induction machines, rotor currents are induced in the rotor windings by a combination of the time-variation of the stator currents and the motion of the rotor relative to the stator.

- Synchronous Machines

Fig. 4.1: a simplified salient-pole ac synchronous generator with two poles.

- The armature winding is on the stator, and the field winding is on the rotor.
- The field winding is excited by direct current conducted to it by means of stationary carbon brushes that contact rotating slip rings or collector rings.
- It is advantages to have the single, low-power field winding on the rotor while having the high-power, typically multiple-phase, armature winding on the stator.
- Armature winding (a, a) consists of a single coil of N turns.
- Conductors forming these coil sides are connected in series by end connections.
- The rotor is turned at a constant speed by a source of mechanical power connected to its shaft. Flux paths are shown schematically by dashed lines.

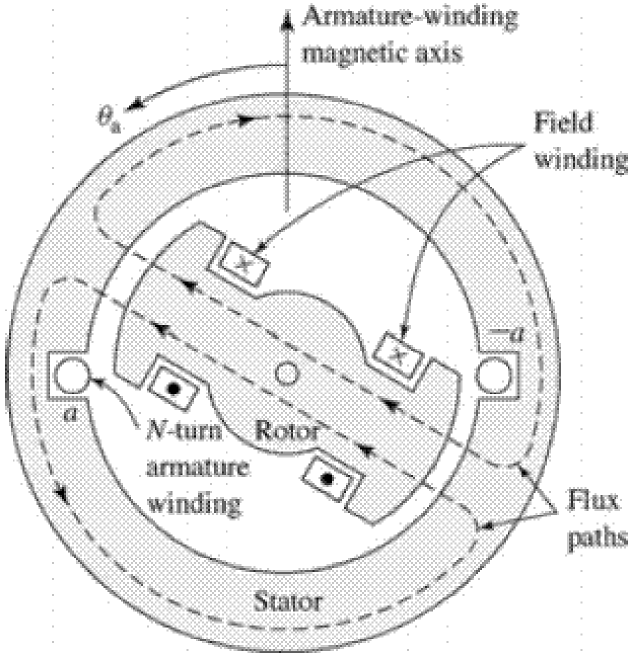


Figure 4.1 Schematic view of a simple, two-pole, single-phase synchronous generator.

Assume a sinusoidal distribution of magnetic flux in the air gap of the machine in Fig.4.1.

- The radial distribution of air-gap flux density B is shown in Fig. 4.2(a) as a function of the spatial angle θ around the rotor periphery.
- As the rotor rotates, the flux –linkages of the armature winding change with time and the resulting coil voltage will be sinusoidal in time as shown in Fig 4.2(b). The frequency in cycles per second (Hz) is the same as the speed of the rotor in revolutions in second (rps).
- A two-pole synchronous machine must revolve at 3600 rpm to produce a 60Hz voltage.
- Note the terms “rpm” and “rps”.

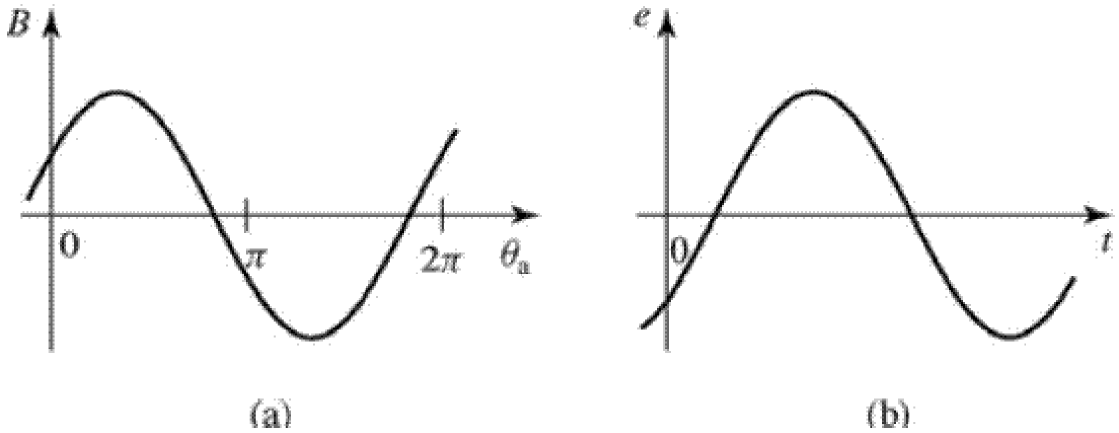


Figure 4.2 (a) Space distribution of flux density and (b) corresponding waveform of

the generated voltage for the single-phase generator of Fig. 4.1.

A great many synchronous machines have more than two poles. Fig 4.3 shows in schematic form a four-pole single-phase generator.

- The field coils are connected so that the poles are of alternate polarity.
- The armature winding consists of two coils $(a_1, -a_1)$and $(a_2, -a_2)$ connected in series by their end connections.
- There are two complete wavelengths, or cycles, in the flux distribution around the periphery, as shown in Fig. 4.4.
- The generated voltage goes through two complete cycles per revolution of the rotor.
- The frequency in Hz is thus twice the speed in rps.

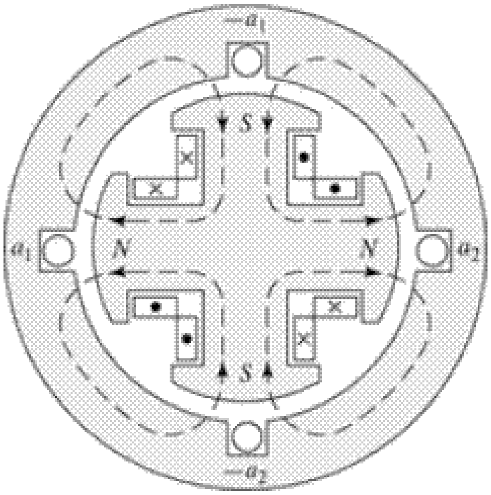


Figure 4.3 Schematic view of a simple, four-pole, single-phase synchronous generator.

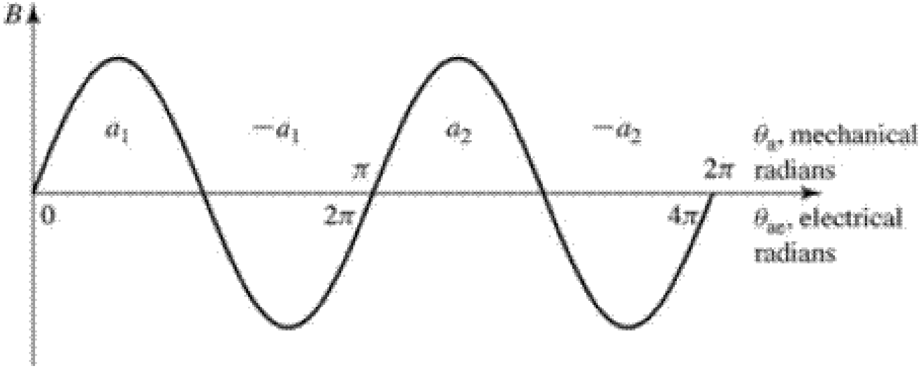


Figure 4.4 Space distribution of the air-gap flux density in an idealized,

four-pole synchronous generator.

When a machine has more than two poles, it is convenient to concentrate on a single pair of poles and to express angles in electrical degrees or electrical radians rather than in physical units.

- One pair of poles equals 360 electrical degrees or 2 π electrical radians.
- Since there are poles/2 wavelengths, or cycles, in one revolution, it follows that

$\theta_{\mathrm{ae}} = (\frac{\mathrm{poles}}{2})\theta_a$ (4.1)

Where $\theta_{\mathrm{ae}}$is the angle in electrical units and θ_a is the spatial angle.

- The coil voltage of a multipole machine passes through a complete cycle every time a pair of poles sweeps by, or (poles/2) times each revolution. The electrical frequency f_e of the voltage generated is therefore

$$f_e = \left(\frac{\text{poles}}{2}\right)\frac{n}{60}\,\text{Hz} \quad (4.2)$$

where n is the mechanical speed in rpm.Note that $\omega_e = (\text{poles}/2)\omega_m$

-
  - The rotors shown in Figs.4.1 and 4.3 have salient, or projecting, poles with concentrated windings. Fig.4.5 shows diagrammatically a nonsalient-pole, or cylindrical, rotor.

- The field winding is a two-pole distributed winding; the coil sides are distributed in multiple slots around the rotor periphery and arranged to produce an approximately sinusoidal distribution of radial air-gap flux.
- Most power systems in the world operate at frequencies of either 50 or 60 Hz.
- A salient-pole construction is characteristic of hydroelectric generators because hydraulic turbines operate at relatively low speeds, and hence a relatively large number of poles is required to produce the desired frequency.
- Steam turbines and gas turbines operate best at relatively high speeds, and turbine- driven alternators or turbine generators are commonly two- or four-pole cylindrical- rotor machines.

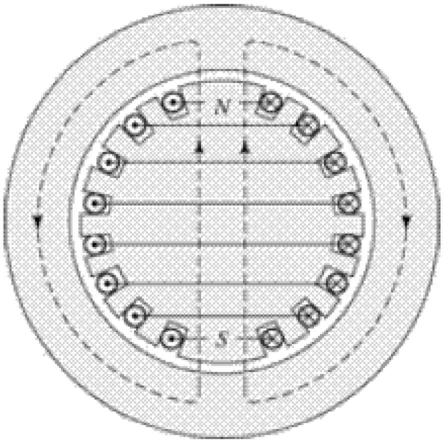


Figure 4.5 Elementary two-pole cylindrical-rotor field winding.

Most of the world’s power systems are three-phase systems. With very few exceptions, synchronous generators are three-phase machines.

- A simplified schematic view of a three-phase, two-pole machine with one coil per phase is shown in Fig. 4.6 (a)
- Fig. 4.6(b) depicts a simplified three-phase, four-pole machine. Note that a minimum of two sets of coils must be used. In an elementary multipole machine, the minimum number of coils sets is given by one half the number of poles.
- Note that coils (a, a) and $(a', -a')$ can be connected in series or in parallel. Then the coils of the three phases may then be either Y- or Δ-connected. See Fig. 4.6(c).

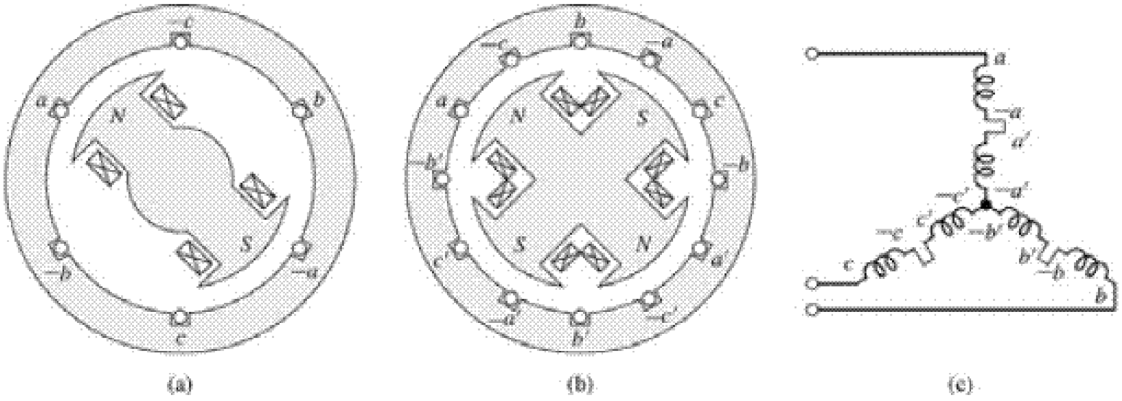


Figure 4.6 Schematic views of three-phase generators: (a) two-pole, (b) four-pole, and

(c) Y connection of the windings.

- The electromechanical torque is the mechanism through which a synchronous generator converts mechanical to electric energy.
  - When a synchronous generator supplies electric power to a load, the armature current creates a magnetic flux wave in the air gap that rotates at synchronous speed.
  - This flux reacts with the flux created by the field current, and an electromechanical torque results from the tendency of these two magnetic fields to align.
  - In a generator this torque opposes rotation, and mechanical torque must be applied from the prime mover to sustain rotation.

- The counterpart of the synchronous generator is the synchronous motor.
    - Ac current supplied to the armature winding on the stator, and dc excitation is supplied to the field winding on the rotor. The magnetic field produced by the armature currents rotates at synchronous speed.
    - To produce a steady electromechanical torque, the magnetic fields of the stator and rotor must be constant in amplitude and stationary with respect to each other.
    - In a motor the electromechanical torque is in the direction of rotation and balances the opposing torque required to drive the mechanical load.
    - In both generators and motors, an electromechanical torque and a rotational voltage are produced which are the essential phenomena for electromechanical energy conversion.
    - Note that the flux produced by currents in the armature of a synchronous motor rotates ahead of that produced by the field, thus pulling on the field (and hence on the rotor) and doing work. This is the opposite of the situation in a synchronous generator, where the field does work as its flux pulls on that of the armature, which is lagging behind.
- Induction Machines
- Alternating currents are applied directly to the stator windings. Rotors currents are then produced by induction, i.e., transformer action.
    - Alternating currents flow in the rotor windings of an induction machine, in contrast to a synchronous machine in which a field winding on the rotor is excited with dc current.
    - The induction machine may be regarded as a generalized transformer in which electric power is transformed between rotor and stator together with a change of frequency and a flow of mechanical power.
- The induction motor is the most common of all motors.
    - The induction machine is seldom used as a generator.
    - In recent years it has been found to be well suited for wind-power applications.
    - It may also be used as a frequency changer.

- In the induction motor, the stator windings are essentially the same as those of a synchronous machine.The rotor windings are electrically short-circuited.
    - The rotor windings frequently have no external connections.
    - Currents are induced by transformer action from the stator winding.
    - Squirrel-cage induction motor: relatively expensive and highly reliable.
- The armature flux in the induction motor leads that of the rotor and produces an electromechanical torque.
    - The rotor does not rotate synchronously.
    - It is the slipping of the rotor with respect to the synchronous armature flux that gives rise to the induced rotor currents and hence the torque.
    - Induction motors operate at speeds less than the synchronous mechanical speed.
    - A typical speed-torque characteristic for an induction motor is shown in Fig.4.7.

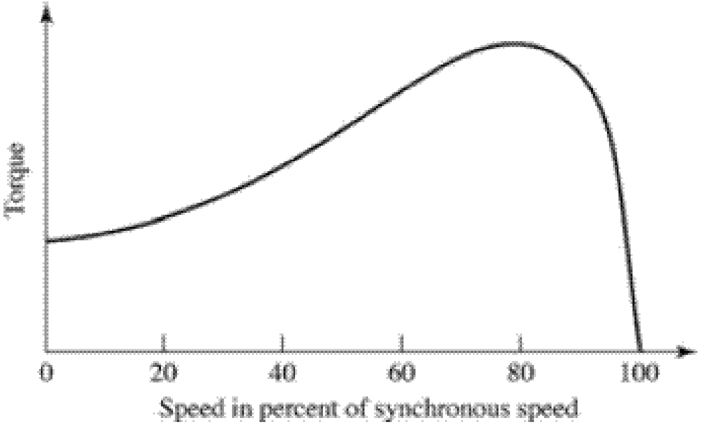


Figure 4.7 Typical induction-motor speed-torque characteristic.

§4.2.2 DC Machines

- DC Machines
- There are two sets of windings in a dc machine.

-
  - The armature winding is on the rotor with current conducted from it by means of carbon brushes.
  - The field winding is on the stator and is excited by direct current.

- An elementary two-pole dc generator is shown in Fig. 4.8.

- Armature winding: (a, a) , pitch 180^o
- The rotor is normally turned at a constant speed by a source of mechanical power connected the shaft.

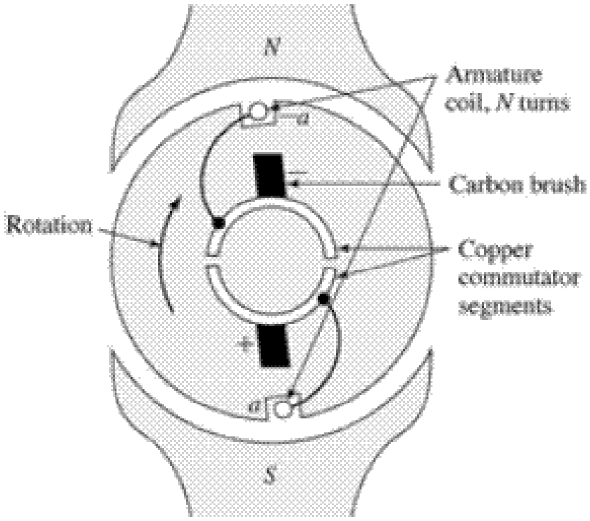


Figure 4.8 Elementary dc machine with commutator.

- The air-gap flux distribution usually approximates a flat-topped wave, rather than the sine wave found in ac machines, and is shown in Fig. 4.9(a).
- Rotation of the coil generates a coil voltage which is a time function having the same waveform as the spatial flux-density distribution.
- The voltage induced in an individual armature coil is an alternating voltage and rectification is produced mechanically by means of a commutator. Stationary carbon brushes held against the commutator surface connect the winding to the external armature terminal.
- The need for commutation is the reason why the armature windings are placed on the rotor.
- The commutator provides full-wave rectification, and the voltage waveform between brushes is shown in Fig. 4.9(b).

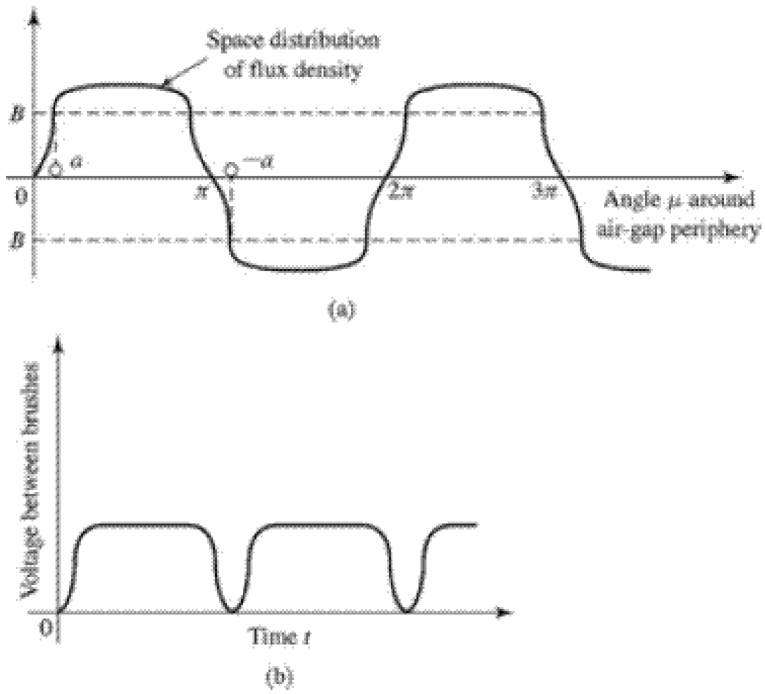


Figure 4.9 (a) Space distribution of air-gap flux density in an elementary dc machine;

(b) waveform of voltage between brushes.

It is the interaction of the two flux distributions created by the direct currents in the field and the armature windings that creates an electromechanical torque.

- If the machine is acting as a generator, the torque opposes rotation.
- If the machine is acting as a motor, the torque acts in the direction of the rotation.

§4.3 MMF of Distributed Windings

- Most armatures have distributed windings, i.e. windings which are spread over a number of slots around the air-gap periphery.

- The individual coils are interconnected so that the result is a magnetic field having the same number of poles as the field winding.
- Consider Fig. 4.10(a).
  - Full-pitch coil: a coil which spans 180 electrical degrees.

  - In Fig. 4.10(b), the air gap and winding are in developed form (laid out flat) and the air-gap mmf distribution is shown by the steplike distribution of amplitude

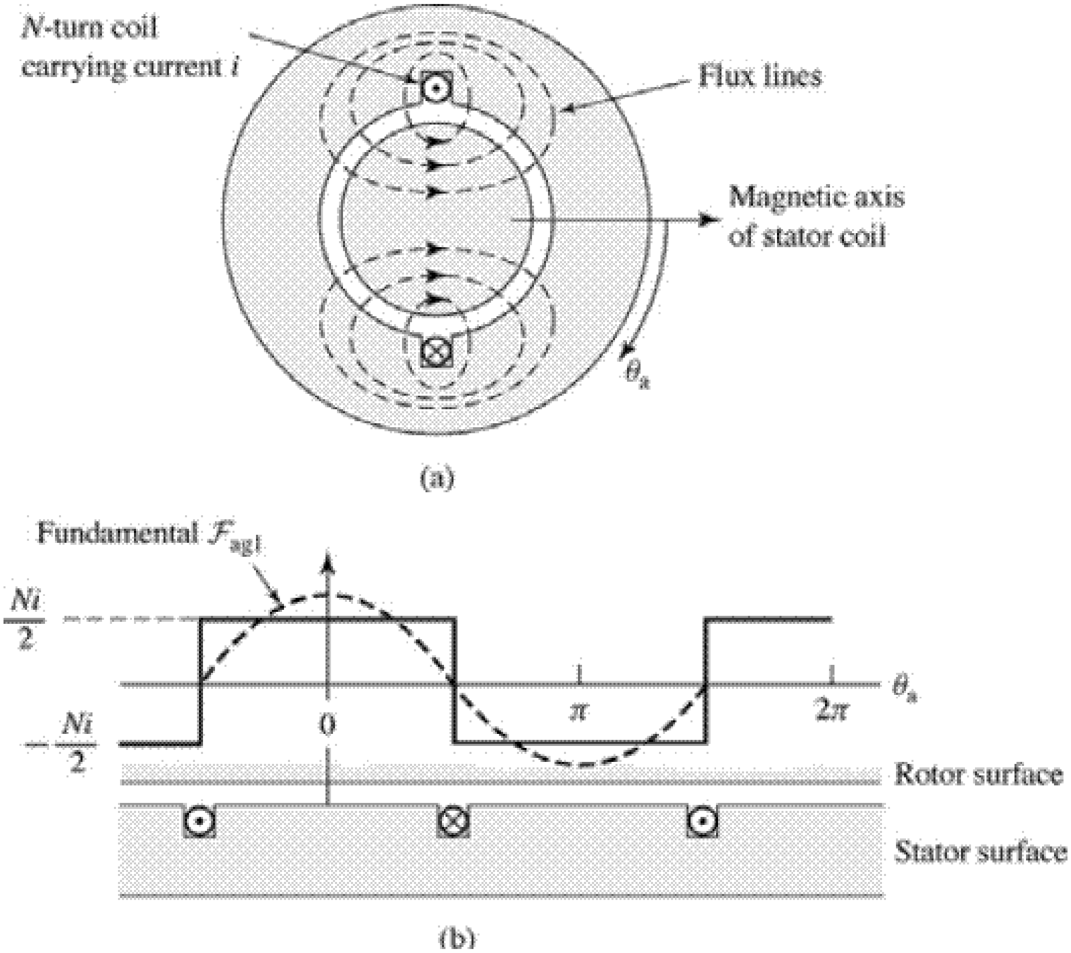


Figure 4.10 (a) Schematic view of flux produced by a concentrated, full-pitch winding in a machine with a uniform air gap. (b) The air-gap mmf produced by current in this winding.

§4.3.1 AC Machines

- It is appropriate to focus our attention on the space-fundamental sinusoidal component of the air-gap mmf.

- In the design of ac machines, serious efforts are made to distribute the coils making up the windings so as to minimize the higher-order harmonic components.

- The rectangular air-gap mmf wave of the concentrated two-pole, full-pitch coil of Fig.4.10(b) can be resolved to a Fourier series comprising a fundamental component and a series of odd harmonics.

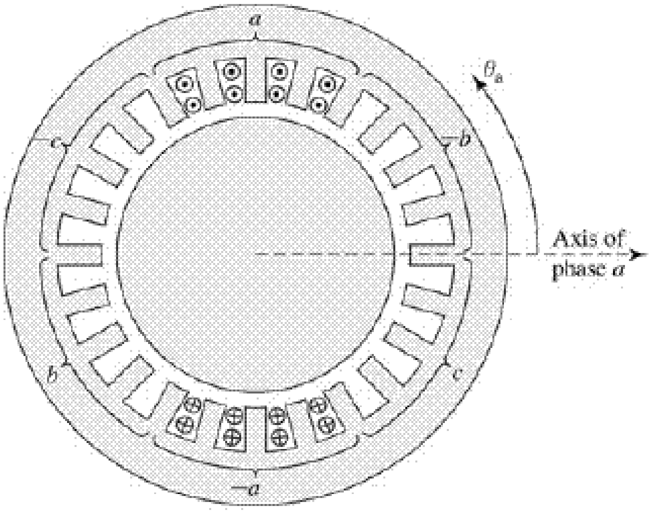
- The fundamental component $F_{\text{agl}}$and its amplitude $(F_{\text{agl}})_{\text{peak}}$are

$F_{\text{agl}} = \frac{4}{\pi}(\frac{\text{Ni}}{2})\text{cos}\theta_a$ (4.3)

$(F_{\text{agl}})_{\text{peak}} = \frac{4}{\pi}(\frac{\text{Ni}}{2})$ (4.4)

Consider a distributed winding, consisting of coils distributed in several slots.

- Fig. 4.11(a) shows phase a of the armature winding of a simplified two-pole, three-phase ac machine and phases b and c occupy the empty slots.
- The windings of the three phases are identical and are located with their magnetic axes 120 degrees apart.The winding is arranged in two layers, each full-pitch coil of N_c turns having one side in the top of a slot and the other coil side in the bottom of a slot a pole pitch away.
- Fig. 4.11(b) shows that the mmf wave is a series of steps each of height $2\text{N}_c i_a$. It can be seen that the distributed winding produces a closer approximation to a sinusoidal mmf wave than the concentrated coil of Fig.4.10 does.



(a)

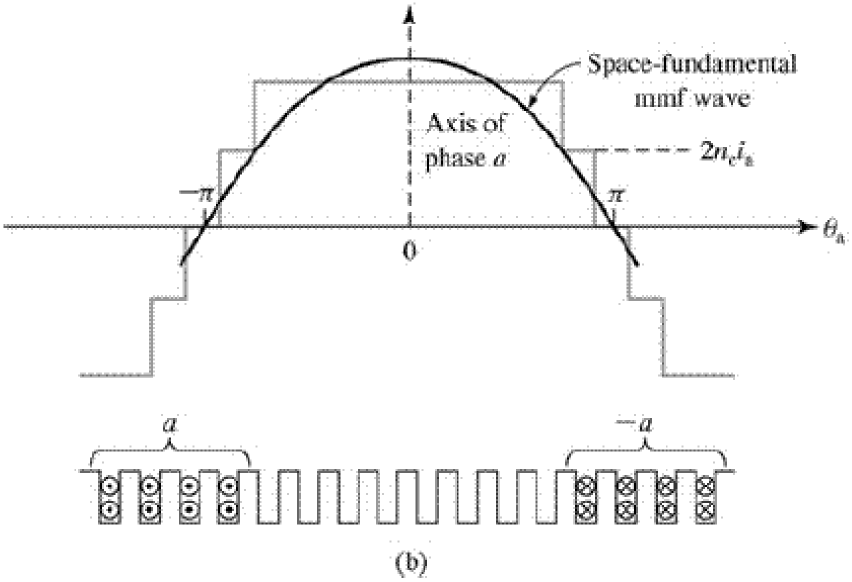


Figure 4.11 The mmf of one phase of a distributed two-pole,

three-phase winding with full-pitch coils.

- The modified form of (4.3) for a distributed multipole winding is

$F_{\text{agl}} = \frac{4}{\pi}(\frac{k_w N_{\text{ph}}}{\text{poles}})i_a \cos(\frac{\text{poles}}{2}\theta_a)$ (4.5)

N_{ph}: number of series turns per phase,

k_w: winding factor, a reduction factor taking into account the distribution of the winding, typically in the range of 0.85 to 0.95, $k_w = k_b k_p (\text{or} k_d k_p)$.

- The peak amplitude of this mmf wave is

$(F_{\text{agl}})_{\text{peak}} = \frac{4}{\pi}(\frac{k_w N_{\text{ph}}}{\text{poles}})i_a$ (4.6)

- Eq. (4.5) describes the space-fundamental component of the mmf wave produced by current in phase a of a distributed winding.

- If $i_a = I_m \cos \omega t$ the result will be an mmf wave which is stationary in space and varies sinusoidally both with respect to θ_a and in time.
- The application of three-phase currents will produce a rotating mmf wave.

- Rotor windings are often distributed in slots to reduce the effects of space harmonics.

- Fig. 4.12(a) shows the rotor of a typical two-pole round-rotor generator.
- As shown in Fig. 4.12(b), there are fewer turns in the slots nearest the pole face.
- The fundamental air-gap mmf wave of a multipole rotor winding is

$$F_{\text{agl}} = \frac{4}{\pi}\left(\frac{k_r N_r}{\text{poles}}\right) I_r \cos\left(\frac{\text{poles}}{2}\theta_r\right) \quad (4.7)$$

$$(F_{\text{agl}})_{\text{peak}} = \frac{4}{\pi}\left(\frac{k_r N_r}{\text{poles}}\right) I_r \quad (4.8)$$

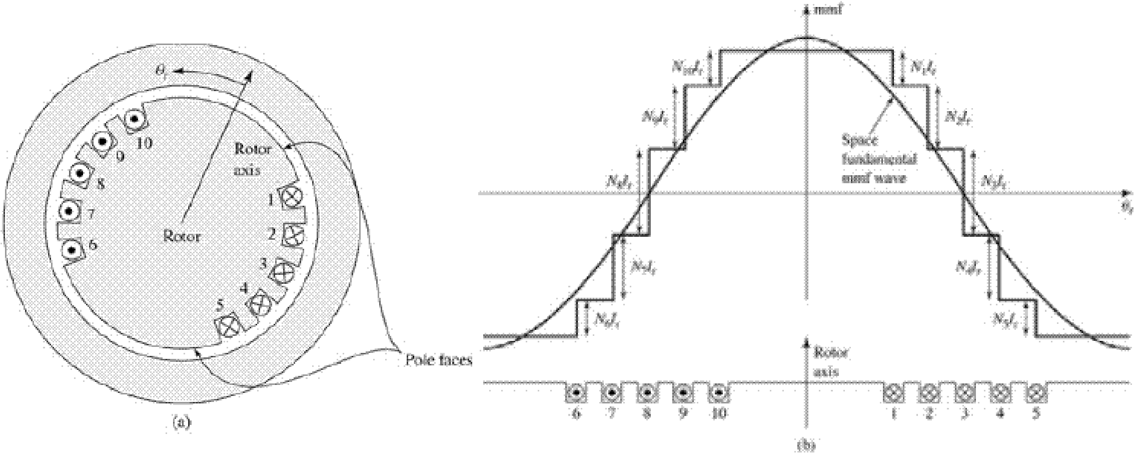


Figure 4.12 The air-gap mmf of a distributed winding on the rotor of a round-rotor generator.

§4.3.2 DC Machines

- Because of the restrictions imposed on the winding arrangement by the commutator, the mmf wave of a dc machine armature approximates a sawtooth waveform more nearly than the sine wave of ac machines.

- Fig. 4.13 shows diagrammatically in cross section the armature of a two-pole dc machine.

- The armature coil connections are such that the armature winding produces a magnetic field whose axis is vertical and thus is perpendicular to the axis of the field winding.
- As the armature rotates, the magnetic field of the armature remains vertical due to commutator action and a continuous unidirectional torque results.

- The mmf wave is illustrated and analyzed in Fig. 4.14.

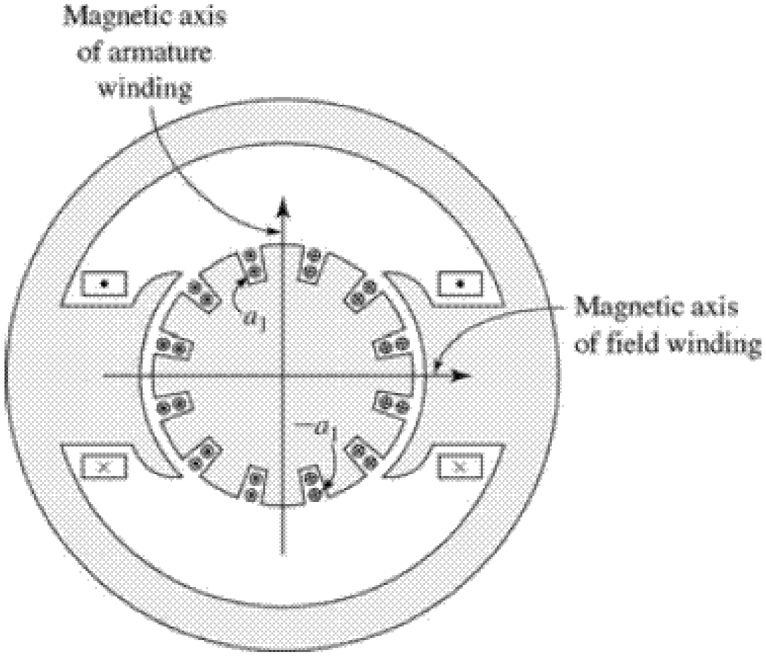
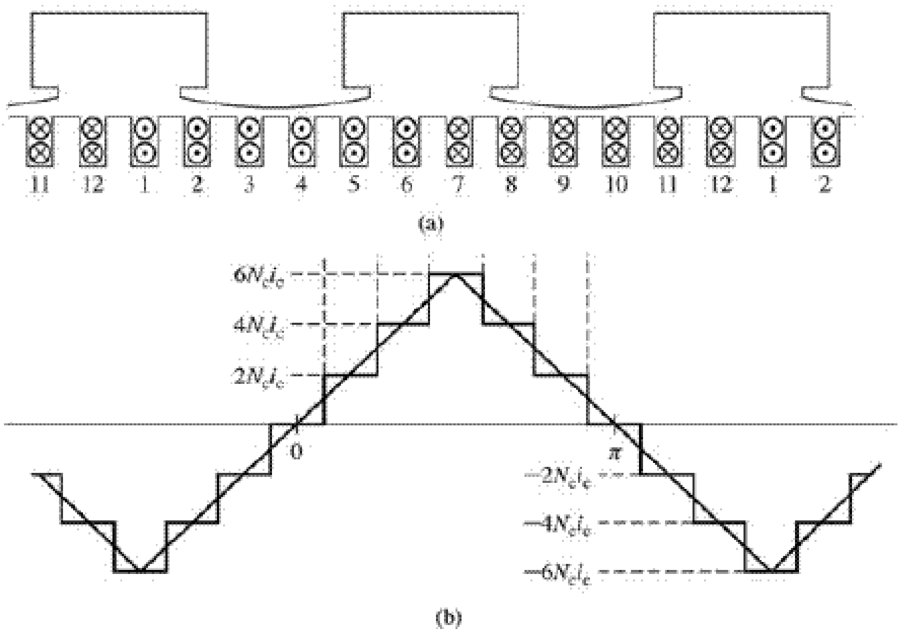

Magnetic axis of armature winding
a_1
$-a_1$
Magnetic axis of field winding

Figure 4.13 Cross section of a two-pole dc machine.



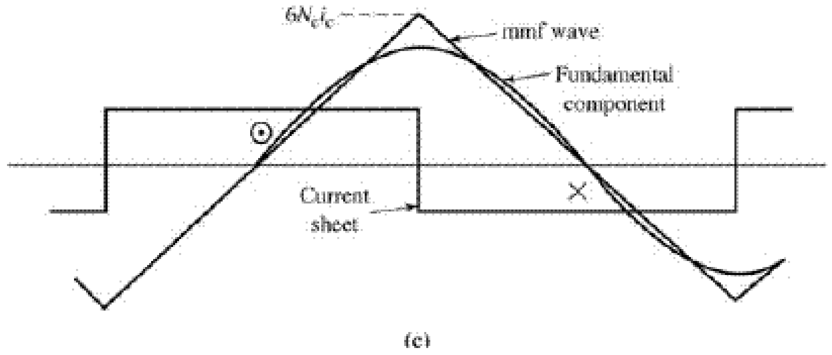


Figure 4.14 (a) Developed sketch of the dc machine of Fig. 4.22; (b) mmf wave; (c) equivalent sawtooth mmf wave, its fundamental component, and equivalent rectangular current sheet.

DC machines often have a magnetic structure with more than two poles.

- Fig. 4.15(a) shows schematically a four-pole dc machine.
- The machine is shown in laid-out form in Fig. 4.15(b).

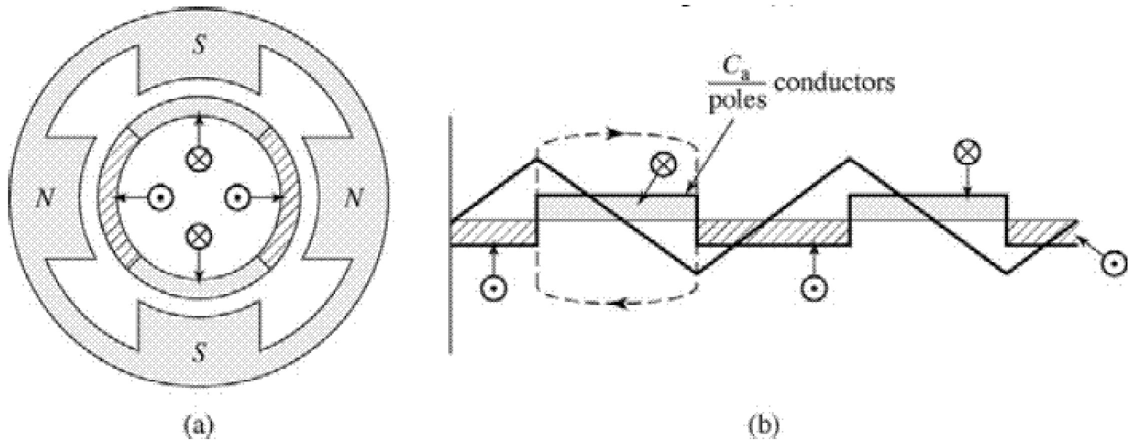


Figure 4.15 (a) Cross section of a four-pole dc machine; (b) development of current sheet and mmf wave.

- The peak value of the sawtooth armature mmf wave can be written as

$$(F_{\text{ag}})_{\text{peak}} = \left(\frac{C_a}{\text{2m.poles}}\right) i_a \text{A.turn/ pole} \quad (4.9)$$

Ca = total number of conductors in armature winding

m = number of parallel paths through armature winding ia = armature current, A

$(F_{\text{ag}})_{\text{peak}} = (\frac{N_a}{\text{poles}})i_a$,$N_a = C_a/(2\text{m})$:no.of series armature turns (4.10)

$(F_{\text{ag}})_{\text{peak}} = \frac{8}{\pi^2}(\frac{N_a}{\text{poles}})i_a$ (4.11)

§4.4 Magnetic Fields In Rotating Machinery

- The behavior of electric machinery is determined by the magnetic fields created by currents in the various windings of the machine.

- The investigations of both ac and dc machines are based on the assumption of sinusoidal spatial distribution of mmf.
- Results from examining a two-pole machine can immediately be extrapolated to a multipole machine.

§4.4.1 Magnetic with Uniform Air Gaps

- Consider machines with uniform air gaps.

- Fig. 4.16(a) shows a single full-pitch, N-turn coil in a high-permeability magnetic structure $\mu \rightarrow \infty$, with a concentric, cylindrical rotor.

- In Fig. 4.16(b) the air-gap mmf $F_{\text{ag}}$is plotted versus angle θ_a.
- Fig. 4.16(c) demonstrates the air-gap constant radial magnetic field H_{ag}.

$H_{\text{ag}} = \frac{F_{\text{ag}}}{g}$ (4.12)

$(H_{\text{agl}}) = \frac{F_{\text{agl}}}{g} = \frac{4}{\pi}(\frac{\text{Ni}}{2\text{g}})\text{cos}\theta_a$ (4.13)

$(H_{\text{agl}})_{\text{peak}} = \frac{4}{\pi}(\frac{\text{Ni}}{2\text{g}})$ (4.14)

- For a distributed winding, the air-gap magnetic field intensity is

$H_{\text{agl}} = \frac{4}{\pi}(\frac{k_w N_{\text{ph}}}{g.\text{poles}})i_a\text{cos}(\frac{\text{poles}}{2}\theta_a)$ (4.15)

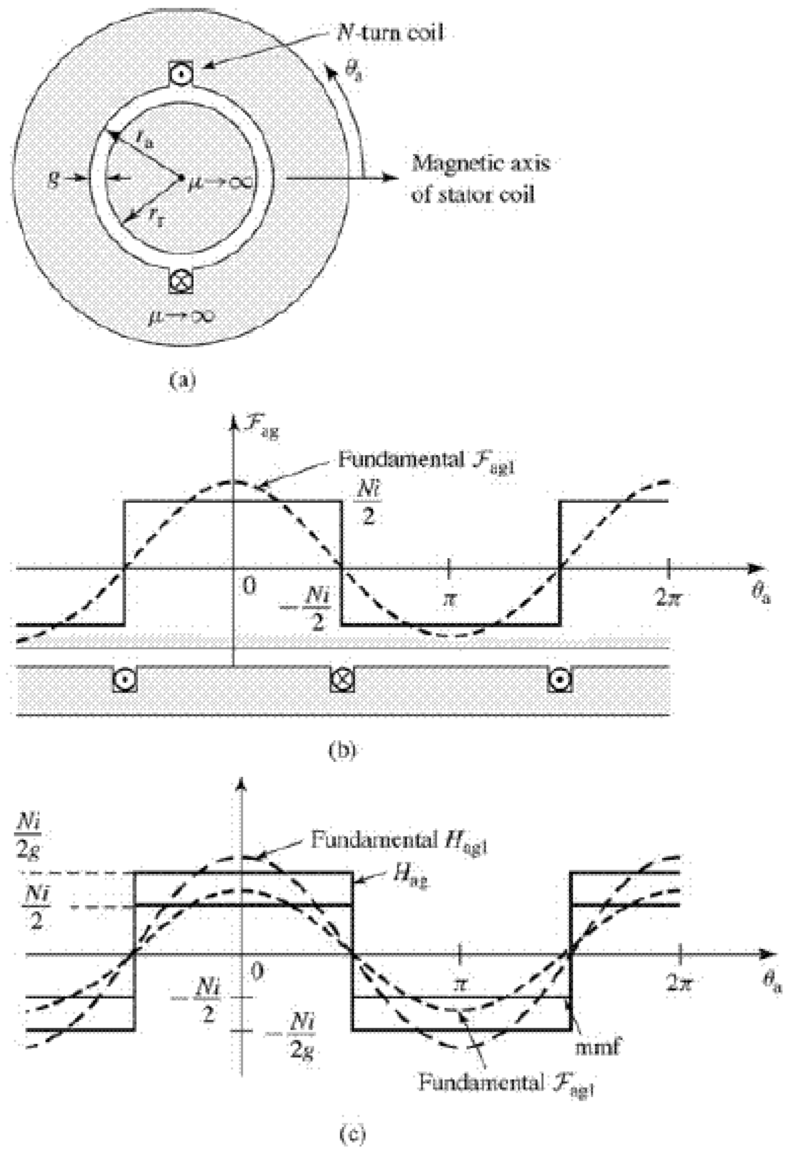


Figure 4.16 The air-gap mmf and radial component of H_{ag} for a concentrated full-pitch winding.

§4.4.2 Machines with Nonuniform Air Gaps

- The air-gap magnetic-field distribution of machines with nonuniform air gaps is more complex than that of uniform-air-gap machines.
- Fig. 4.17(a) shows the structure of a typical dc machine and Fig. 4.17 (b) shows the structure of a typical salient-pole synchronous machine.

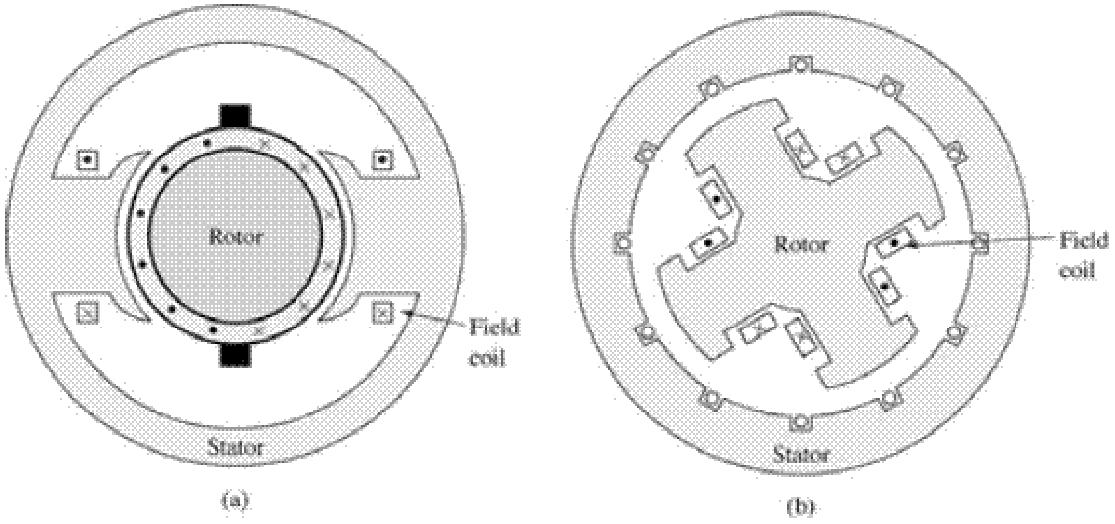


Figure 4.17 Structure of typical salient-pole machines:

(a) dc machine and (b) salient-pole synchronous machine.

- Detailed analysis of the magnetic field distributions requires complete solutions of the field problem.
- Fig. 4.18 shows the magnetic field distribution in a salient-pole dc generator (obtained by finite-element solution).

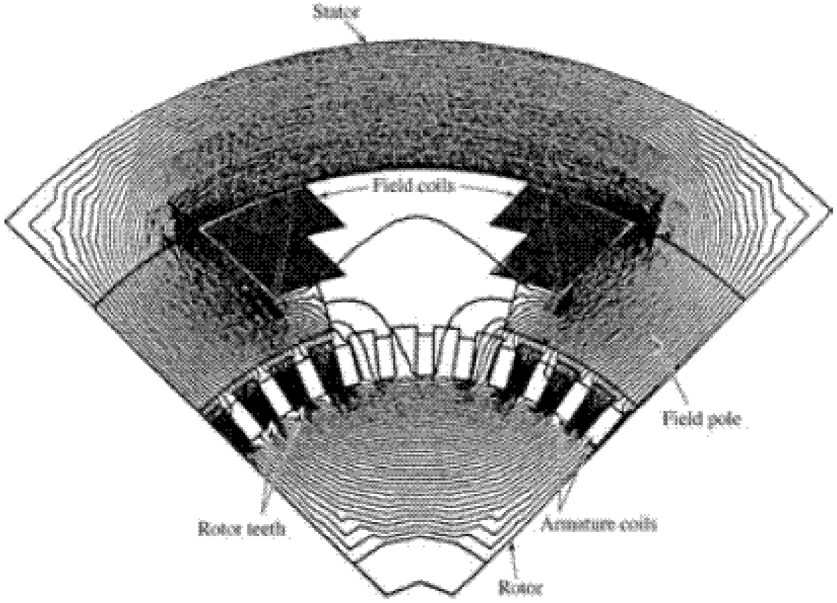


Figure 4.18 Finite-element solution of the magnetic field distribution in a salient-pole dc generator. Field coils excited; no current in armature coils.

(General Electric Company.)

§4.5 Rotating MMF Waves in AC Machines

- To understand the theory and operation of polyphase ac machines, it is necessary to study the nature of the mmf wave produced by a polyphase winding.

§4.5.1 MMF Wave of a Single-Phase Winding

- Fig. 4.19(a) shows the space-fundamental mmf distribution of a single-phase winding.

- Note that from Eq. (4.5), F_{agl} is

$$F_{\text{agl}} = \frac{4}{\pi}\left(\frac{k_w N_{\text{ph}}}{\text{poles}}\right) i_a \cos\left(\frac{\text{poles}}{2}\theta_a\right) \quad (4.16)$$

When the winding is exicted by a current

$$i_a = I_a \cos\omega_e t \quad (4.17)$$

the mmf distribution is given by

$$\begin{aligned} F_{\text{agl}} &= F_{\max}\cos\left(\frac{\text{poles}}{2}\theta_a\right)\cos\omega_e t \\ &= F_{\max}\cos(\theta_{\text{ae}})\cos\omega_e t \end{aligned} \quad (4.18)$$

$$F_{\max} = \frac{4}{\pi}\left(\frac{k_w N_{\text{ph}}}{\text{poles}}\right) I_a \quad (4.19)$$

-
    - This mmf distribution remains fixed in space with an amplitude that varies sinusoidally in time at frequency ω_c , as shown in Fig. 4.19(a).

- The air-gap mmf of a single-phase winding exicted by a source of ac current can be resolved into rotating traveling waves.
    - By the identity $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \cos(\alpha + \beta)$

$$F_{\text{agl}} = F_{\max}\left[\frac{1}{2}\cos(\theta_{\text{ae}} - \omega_e t) + \frac{1}{2}\cos(\theta_{\text{ae}} + \omega_e t)\right] \quad (4.20)$$

$$F^{+}_{\text{agl}} = F_{\max}\cos(\theta_{\text{ae}} - \omega_e t) \quad (4.21)$$

$F_{\mathrm{agl}}^{-} = \frac{1}{2} F_{\max}\cos(\theta_{\mathrm{ae}} + \omega_e t)$ (4.22)

- F_{agl}travels in the $+\theta_a$direction and F_{agl}travels in the $-\theta_a$ direction.
- This decomposition is shown graphically in Fig. 4.19(b) and in a phasor representation in Fig. 4.19(c).

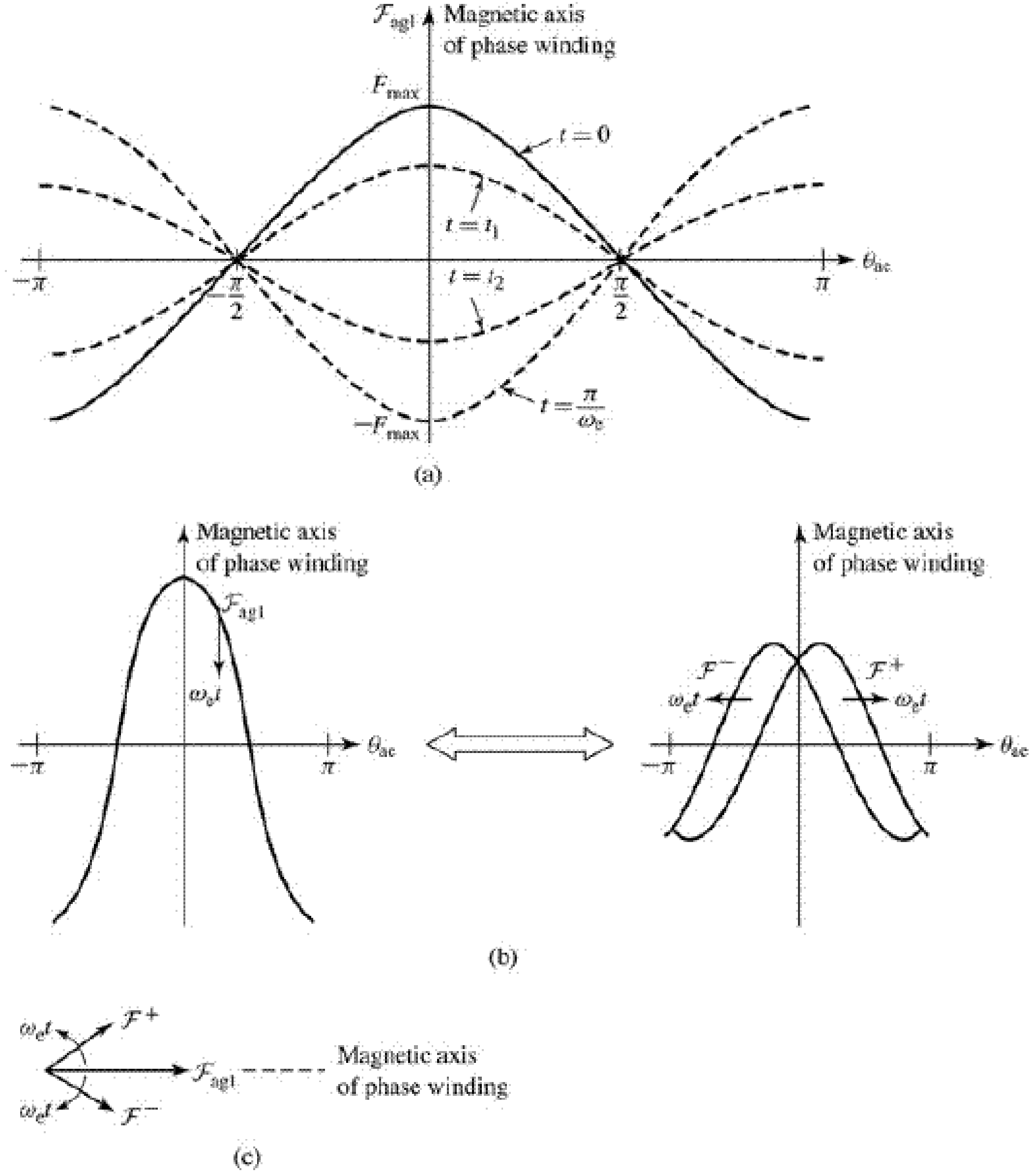


Figure 4.19 Single-phase-winding space-fundamental air-gap mmf: (a) mmf distribution of a

single-phase winding at various times; (b) total mmf F_{ag1}decomposed into two traveling waves F and F^{+} ; (c) phasor decomposition of F.

§4.5.2 MMF Wave of a Polyphase Winding

- We are to study the mmf distribution of three-phase windings such as those found on the stator of three-phase induction and synchronous machines.

In a three-phase machine, the windings of the individual phases are displaced from each other by 120 electrical degrees in space around the air-gap circumference as shown in Fig.4.20 in which the concentrated full-pitch coils may be considered to represent distributed windings.

- Under balanced three-phase conditions, the excitation currents (Fig. 4.20) are

$i_a = I_m \cos \omega_e t$ (4.23)

$i_b = I_m \cos(\omega_e t - 120^o)$ (4.24)

$i_c = I_m \cos(\omega_e t + 120^o)$ (4.25)

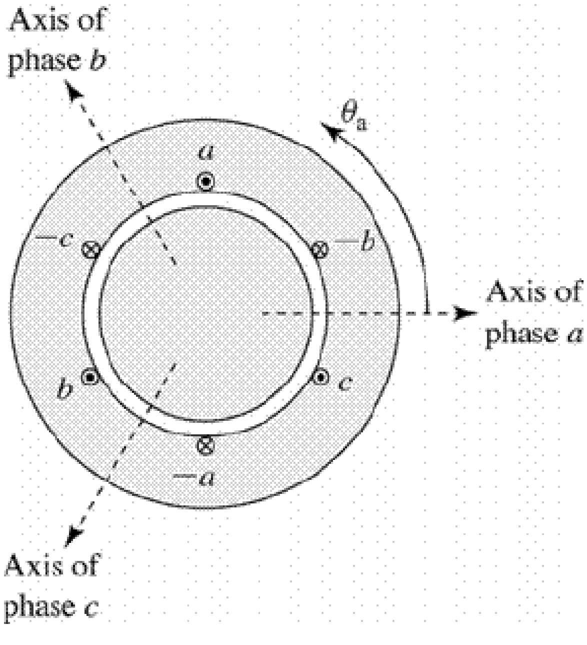


Figure 4.20 Simplified two-pole three-phase stator winding.

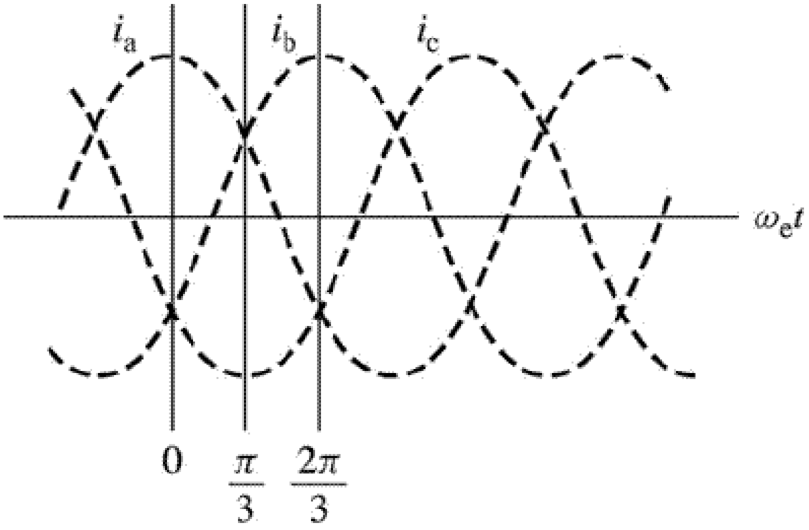


Figure 4.21 Instantaneous phase currents under balanced three-phase conditions.

- The mmf of phase a has been shown to be

$F_{a1} = F_{a1}^{+} + F_{a1}^{-}$ (4.26)

$F_{a1}^{+} = \frac{1}{2} F_{\mathrm{max}} \cos(\theta_{\mathrm{ae}} - \omega_e t)$ (4.27)

$F_{a1}^{-} = \frac{1}{2} F_{\mathrm{max}} \cos(\theta_{\mathrm{ae}} + \omega_e t)$ (4.28)

$F_{\mathrm{max}} = \frac{4}{\pi} \left(\frac{k_w N_{\mathrm{ph}}}{\mathrm{poles}}\right) I_m$ (4.29)

- Similarly, for phases b and c

$F_{b1} = F_{b1}^{+} + F_{b1}^{-}$ (4.30)

$F_{b1}^{+} = \frac{1}{2} F_{\mathrm{max}} \cos(\theta_{\mathrm{ae}} - \omega_e t)$ (4.31)

$F_{b1}^{-} = \frac{1}{2} F_{\mathrm{max}} \cos(\theta_{\mathrm{ae}} + \omega_e t + 120^{o})$(4.32)

$F_{c1} = F_{c1}^{+} + F_{c1}^{-}$ (4.33)

$F_{c1}^{+} = \frac{1}{2} F_{\mathrm{max}} \cos(\theta_{\mathrm{ae}} - \omega_e t)$ (4.34)

$F_{c1}^{-} = F_{\mathrm{max}} \cos(\theta_{\mathrm{ae}} + \omega_e t - 120^{o})$(4.35)

- The total mmf is the sum

$F(\theta_{\text{ae}},t) = F_{a1} + F_{b1} + F_{c1}$(4.36)

It can be performed in terms of the positive- and negative- traveling waves.

$$\begin{gathered} F^{-}(\theta_{\text{ae}},t) = F_{a1}^{-} + F_{b1}^{-} + F_{c1}^{-} \\ = \tfrac{1}{2}F_{\max}[\cos(\theta_{\text{ae}} + \omega_e t) + \cos(\theta_{\text{ae}} + \omega_e t - 120^o) + \cos(\theta_{\text{ae}} + \omega_e t + 120^o)] \\ = 0 \end{gathered}$$

$$\begin{aligned} F^{+}(\theta_{\text{ae}},t) &= F_{a1}^{+} + F_{b1}^{+} + F_{c1}^{+} \\ &= \tfrac{3}{2}F_{\max}\cos(\theta_{\text{ae}} - \omega_e t) \end{aligned} \quad (4.37)$$

- The result of displacing the three windings by 120^o in space phase and displacing the winding currents by 120^o in time phase is a single positive-traveling mmf wave

$$\begin{aligned} F(\theta_{\text{ae}},t) &= \tfrac{3}{2}F_{\max}\cos(\theta_{\text{ae}} - \omega_e t) \\ &\tfrac{3}{2}F_{\max}\cos((\tfrac{\text{poles}}{2})\theta_a - \omega_e t) \end{aligned} \quad (4.38)$$

- Under balanced three-phase conditions, the three-phase winding produces an air-gap mmf wave which rotates at synchronous angular velocity ω_s (rad/sec)

$\omega_s = (\frac{2}{\text{poles}})\omega_e$ (4.39)

ω_c: angular velocity of the applied electrical excitation (rad/sec)

- [missing_resource: graphics31.wmf]: synchronous speed

$f_e = \omega_e/(2\pi)$: applied electrical frequency

$n_s = (\frac{120}{\text{poles}})f_e r/\text{min}$ (4.40)

- A polyphase winding exicted by balanced polyphase currents produces a rotating mmf wave.

- It is the interaction of this magnetic flux wave with that of the rotor which produces torque.
- Constant torque is produced when rotor-produced magnetic flux rotates in synchronism with that of the stator.

§4.5.3 Graphical Analysis of Polyphase MMF

- For balanced three-phase currents, the production of a rotating mmf can also be shown graphically.

- Refer to Fig. 4.22.

- As time passes, the resultant mmf wave retains its sinusoidal form and amplitude but rotates progressively around the air gap.
- The net result is an mmf wave of constant amplitude rotating at uniform angular velocity.

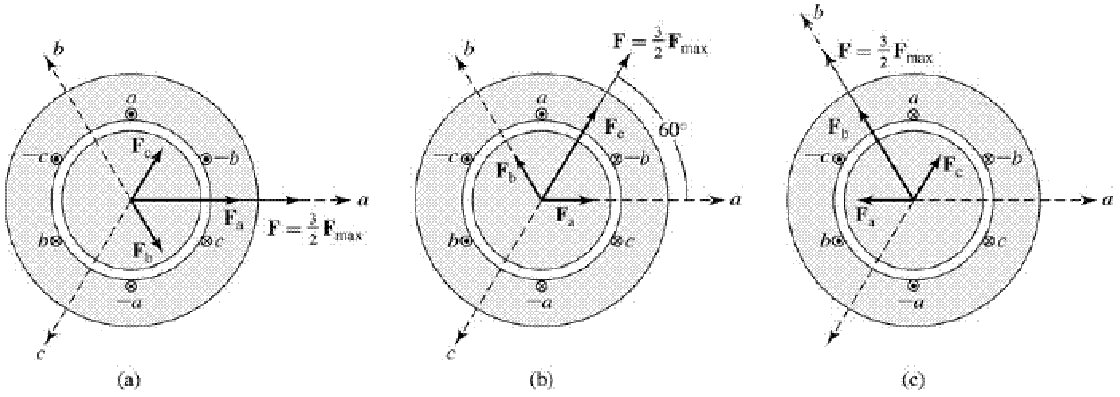


Figure 4.22 The production of a rotating magnetic field by means of three-phase currents.

§4.6 Generated Voltage

§4.6.1 AC Machines

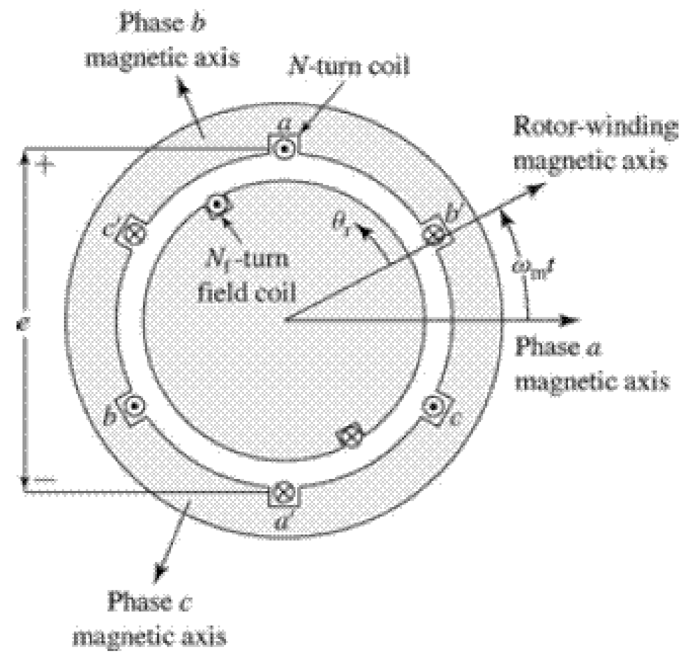


Figure 4.23 Cross-sectional view of an elementary three-phase ac machine.

$$B_{\text{peak}} = \frac{4\mu_o}{\pi g}\left(\frac{k_f N_f}{\text{poles}}\right) I_f \quad (4.41)$$

$$B = B_{\text{peak}} \cos\left(\frac{\text{poles}}{2}\theta_r\right) \quad (4.42)$$

$$\begin{aligned} \Phi &= l \int_{-\pi/\text{poles}}^{+\pi/\text{poles}} B_{\text{peak}} \cos\left(\frac{\text{poles}}{2}\theta_r\right) \mathrm{r d}\theta_r \\ &= \left(\frac{2}{\text{poles}}\right) 2\mathrm{B}_{\text{peak}} \mathrm{lr} \end{aligned} \quad (4.43)$$

$$\begin{aligned} \lambda_a &= k_w N_{\text{ph}} \Phi_p \cos\left(\left(\frac{\text{poles}}{2}\right)\omega_m t\right) \\ &= k_w N_{\text{ph}} \Phi_p \cos \omega_{\text{me}} t \end{aligned} \quad (4.44)$$

$$\omega_{\text{me}} = \left(\frac{\text{poles}}{2}\right)\omega_m \quad (4.45)$$

$$e_a = \frac{d\lambda_a}{\mathrm{dt}} = k_w N_{\text{ph}} \frac{d\Phi_p}{\mathrm{dt}} \cos \omega_{\text{me}} t - \omega_{\text{me}} k_w N_{\text{ph}} \Phi_p \sin \omega_{\text{me}} t \quad (4.46)$$

$$e_a = -\omega_{\text{me}} k_w N_{\text{ph}} \Phi_p \sin \omega_{\text{me}} t \quad (4.47)$$

$$E_{\max} = \omega_{\text{me}} k_w N_{\text{ph}} \Phi_p = 2\pi \mathrm{f}_{\text{me}} k_w N_{\text{ph}} \Phi_p \quad (4.48)$$

$$E_{\text{rms}} = \frac{2\pi}{\sqrt{2}} f_{\text{me}} k_w N_{\text{ph}} \Phi_p = \sqrt{2} f_{\text{me}} k_w N_{\text{ph}} \Phi_p \quad (4.49)$$

$$E_a = \frac{1}{\pi}\int_0^{\pi} \omega_{\mathrm{me}} N\Phi_p \sin(\omega_{\mathrm{me}}t)\, d(\omega_{\mathrm{me}}t) = \frac{2}{\pi}\omega_{\mathrm{me}} N\Phi_p \quad (4.50)$$

$$E_a = \left(\frac{\text{poles}}{\pi}\right) N\Phi_p\omega_m = \text{polesN}\Phi_p\left(\frac{n}{30}\right) \quad (4.51)$$

$$E_a = \left(\frac{\text{poles}}{2\pi}\right)\left(\frac{C_a}{m}\right)\Phi_p\omega_m = \left(\frac{\text{poles}}{60}\right)\left(\frac{C_a}{m}\right)\Phi_p n \quad (4.52)$$

§4.7 Torque in Nonsalient-pole Machines

- Consider the elementary smooth-air-gap machine of Fig.4.24 with one winding on the stator and one on the rotor and with θ_m being the mechanical angle between the axes of the two windings. These windings are distributed over a number of slots so that their mmf waves can be approximated by space sinusoids. In Fig.4.24a the coil sides s, -s and r, -r mark the positions of the centers of the belts of conductors comprising the distributed windings. An alternative way of drawing these windings is shown in Fig.4.24b, which also shows reference directions for voltages and currents. Here it is assumed that current in the arrow direction produces a magnetic field in the air gap in the arrow direction, so that a single arrow defines reference directions for both current and flux.

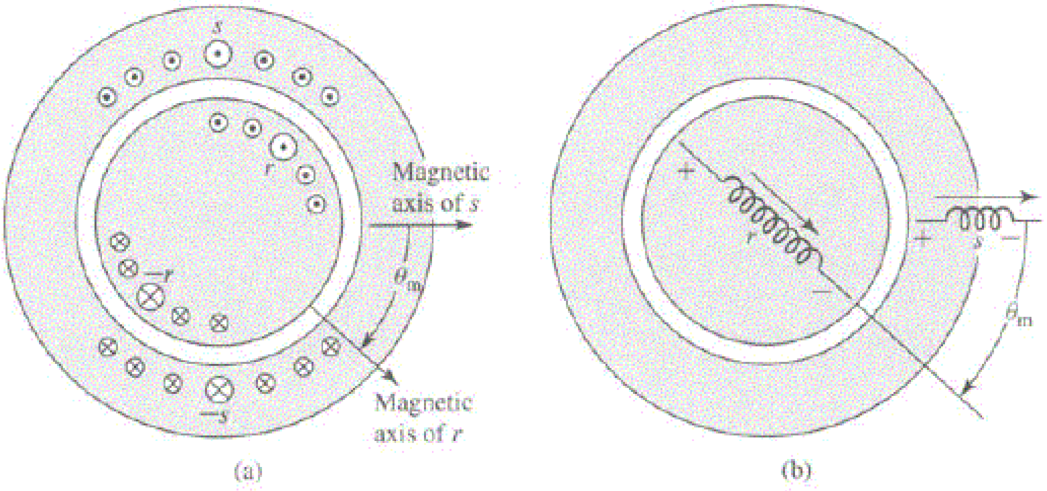


Figure 4.24 Elementary two-pole machine with smooth air gap: (a) winding distribution and (b) schematic representation.

- The stator and rotor are concentric cylinders, and slot openings are neglected. Consequently, our elementary model does not include the effects of salient poles, which are investigated in later chapters. We also assume that the reluctances of the stator and rotor iron are negligible. Finally, although Fig.4.34 shows a two-pole machine, we will write the derivations that follow for the general case of a multipole machine, replacing θ_m by the electrical rotor angle.

$\theta_{\mathrm{me}} = \left[\frac{\mathrm{poles}}{2}\right]\theta_m$ (4.53)

- Based upon these assumptions, the stator and rotor self-inductances L_{ss} and L_{rr} can be seen to be constant, but the stator-to-rotor mutual inductance depends on the electrical angle θ_{me} between the magnetic axes of the stator and rotor windings. The mutual inductance is at its positive maximum when θ_{me}=0 or 2 π, is zero when $\theta_{\mathrm{me}} = \pm\pi/2$, and is at its negative maximum when $\theta_{\mathrm{me}} = \pm\pi$. On the assumption of sinusoidal mmf waves and a uniform air gap, the space distribution of the air-gap flux wave is sinusoidal, and the mutual inductance will be of the form

$L_{\mathrm{sr}}(\theta_{\mathrm{me}}) = L_{\mathrm{sr}}\cos(\theta_{\mathrm{me}})$ (4.54)

where the script letter E denotes an inductance which is a function of the electrical angle θ_{me}. The italic capital letter L denotes a constant value. Thus Lsr is the magnitude of the mutual inductance; its value when the magnetic axes of the stator and rotor are aligned (θ_{me}= 0). In terms of the inductances, the stator and rotor flux linkages) $\lambda_s$and $\lambda_r$are

$\lambda_s = L_{\mathrm{ss}} + L_{\mathrm{sr}}(\theta_{\mathrm{me}})i_r = L_{\mathrm{ss}}i_s + L_{\mathrm{sr}}\cos(\theta_{\mathrm{me}})i_r$ (4.55)

$\lambda_r = L_{\mathrm{sr}}(\theta_{\mathrm{me}})i_s + L_{\mathrm{rr}}i_r = L_{\mathrm{sr}}\cos(\theta_{\mathrm{me}})i_s + L_{\mathrm{rr}}i_r$ (4.56)

In matrix notation

$\begin{bmatrix}\lambda_s \\ \lambda_r\end{bmatrix} = \begin{bmatrix} L_{\mathrm{ss}} & L_{\mathrm{sr}}(\theta_{\mathrm{me}}) \\ L_{\mathrm{sr}}(\theta_{\mathrm{me}}) & L_{\mathrm{rr}}\end{bmatrix}\begin{bmatrix} i_s \\ i_r\end{bmatrix}$ (4.57)

The terminal voltages $v_s$and $v_r$are

$v_s = R_s i_s + \frac{d\lambda_s}{\mathrm{dt}}$ (4.58)

$v_r = R_r i_r + \frac{d\lambda_r}{\mathrm{dt}}$ (4.59) where $R_s$and R_r are the resistances of the stator and rotor windings respectively.

- When the rotor is revolving, θ_{me} must be treated as a variable. Differentiation of Eqs.4.56 and 4.57 and substitution of the results in Eqs.4.59 and 4.60 then give

$$v_s = R_s i_s + L_{\text{ss}} \frac{\text{di}_s}{\text{dt}} + L_{\text{sr}} \cos(\theta_{\text{me}}) \frac{\text{di}_r}{\text{dt}} - L_{\text{sr}} i_r \sin(\theta_{\text{me}}) \frac{d\theta_{\text{me}}}{\text{dt}} \ (4.60)$$

$$v_r = R_r i_r + L_{\text{rr}} \frac{\text{di}_r}{\text{dt}} + L_{\text{sr}} \cos(\theta_{\text{me}}) \frac{\text{di}_r}{\text{dt}} - L_{\text{sr}} i_s \sin(\theta_{\text{me}}) \frac{d\theta_{\text{me}}}{\text{dt}} \ (4.61)$$

$$\frac{d\theta_{\text{me}}}{\text{dt}} = \omega_{\text{me}} = \left[\frac{\text{poles}}{2}\right]\omega_m \ (4.62)$$

is the instantaneous speed in electrical radians per second. In a two-pole machine, θme and ωme are equal to the instantaneous shaft angle θm and the shaft speed ωm respectively. In a multipole machine, they are related by Eqs.4.54 and 4.46. The second and third terms on the fight-hand sides of Eqs.4.61 and 4.62 are L(di/dt) induced voltages like those induced in stationary coupled circuits such as the windings of transformers. The fourth terms are caused by mechanical motion and are proportional to the instantaneous speed. These are the speed voltage terms which correspond to the interchange of power between the electric and mechanical systems.

- The electromechanical torque can be found from the coenergy.

$$\begin{aligned} W_{\text{fld}}^{'} &= \frac{1}{2} L_{\text{ss}} i_s^2 + \frac{1}{2} L_{\text{rr}} i_r^2 + L_{\text{sr}} i_s i_r \cos\theta_{\text{me}} \\ &= \frac{1}{2} L_{\text{ss}} i_s^2 + \frac{1}{2} L_{\text{rr}} i_r^2 + L_{\text{sr}} i_s i_r \cos\left[\left(\frac{\text{poles}}{2}\right)\theta_m\right] \end{aligned} \ (4.63)$$

Note that the coenergy of Eq.4.63 has been expressed specifically in terms of the shaft angle θ_m because the torque expression requires that the torque be obtained from the derivative of the coenergy with respect to the spatial angle θ_m and not with respect to the electrical angle θ_{me}. Thus,

$$\begin{aligned} T = \frac{\partial W_{\text{fld}}^{'}(i_s, i_r, \theta_m)}{\partial \theta_m} \Big|_{i_s, i_r} &= -\left[\frac{\text{poles}}{2}\right] L_{\text{sr}} i_s i_r \sin\left[\frac{\text{poles}}{2}\theta_m\right] \\ &= -\left[\frac{\text{poles}}{2}\right] L_{\text{sr}} i_s i_r \sin\theta_{\text{me}} \end{aligned} \ (4.64)$$

where T is the electromechanical torque acting to accelerate the rotor (i.e., a positive torque acts to increase θ_m). The negative sign in Eq.4.64 means that the electromechanical torque acts in the direction to bring the magnetic fields of the stator and rotor into alignment.

4.7.2 Magnetic Field Viewpoint

- In the discussion of Section 4.7.1 the characteristics of a rotating machine as viewed from its electric and mechanical terminals have been expressed in terms of its winding inductances. This viewpoint gives little insight into the physical phenomena which occur within the machine. In this section, we will explore an alternative formulation in terms of the interacting magnetic fields.

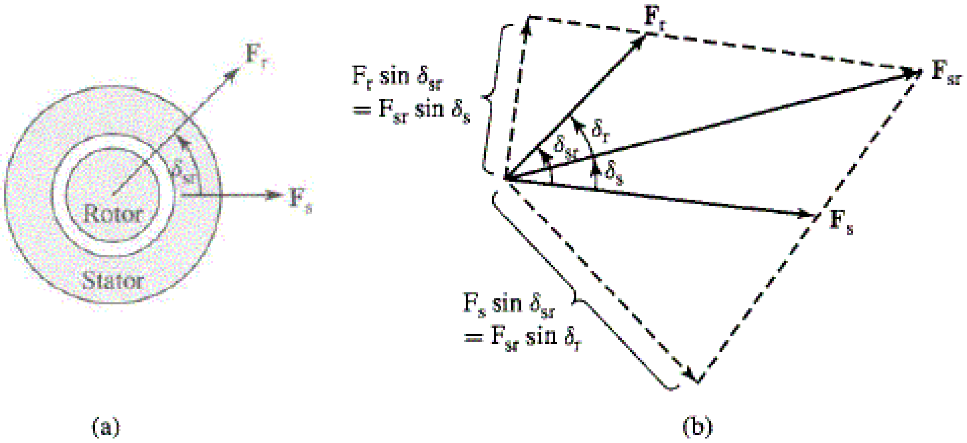


Figure 4.25 Simplified two-pole machine: (a) elementary model and

(b) vector diagram of mmf waves. Torque is produced by the tendency of the rotor and stator magnetic fields to align. Note that these figures are drawn with δ_{sr} positive, i.e., with the rotor mmf wave
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leading that of the stator F_s.

- As we have seen, currents in the machine windings create magnetic flux in the air gap between the stator and rotor, the flux paths being completed through the stator and rotor iron. This condition corresponds to the appearance of magnetic poles on both the stator and the rotor, centered on their respective magnetic axes, as shown in Fig.4.35a for a two-pole machine with a smooth air gap. Torque is produced by the tendency of the two component magnetic fields to line up their magnetic axes.

- We shall derive an expression for the magnetic coenergy stored in the air gap in terms of the stator and rotor mmfs and the angle δ_{sr} between their magnetic axes. The torque can then be found from the partial derivative of the coenergy with respect to angle δ_{sr}.
- The line integral of Hag across the gap then is simply $H_{\text{ag}}g$ and equals the resultant air-gap mmf F_{sr} produced by the stator and rotor windings; thus

$H_{\text{ag}}g = F_{\text{sr}}$ (4.65)

- The mmf waves of the stator and rotor are spatial sine waves with δ_{sr} being the phase angle between their magnetic axes in electrical degrees. They can be represented by the space vectors F_s and F_r drawn along the magnetic axes of the stator- and rotor mmf waves respectively. The resultant mmf F_{sr} acting across the air gap, also a sine wave, is their vector sum.

$F_{\text{sr}}^2 = F_s^2 + F_r^2 + 2\text{F}_s F_r \cos\delta_{\text{sr}}$ (4.66)

The resultant radial H_{ag}field is a sinusoidal space wave whose peak value $H_{\text{ag,peak}}$ is, from Eq.4.65,

$(H_{\text{ag}})_{\text{peak}} = \frac{F_{\text{sr}}}{g}$ (4.67)

Average coenergy density = $\frac{\mu_o}{2}\left[\frac{(H_{\text{ag}})^2_{\text{peak}}}{2}\right] = \frac{\mu_o}{4}\left[\frac{F_{\text{sr}}}{g}\right]^2$ (4.68)

$$\begin{aligned} W'_{\text{fld}} &= (\text{average coenergy density})(\text{volume of air gap}) \\ &= \frac{\mu_o}{4}\left[\frac{F_{\text{sr}}}{g}\right]^2 \pi D\text{lg} = \frac{\mu_o \pi \text{Dl}}{4\text{g}} F_{\text{sr}}^2 \end{aligned} \quad (4.69)$$

$W'_{\text{fld}} = \frac{\mu_o \pi \text{Dl}}{4\text{g}}\left(F_s^2 + F_r^2 + 2\text{F}_s F_r \cos\delta_{\text{sr}}\right)$ (4.70)

For a two-pole machine

$T = \frac{\partial W'_{\text{fld}}}{\partial \delta_{\text{sr}}}\ |_{F_s, F_r} = -\left[\frac{\mu_o \pi \text{Dl}}{2\text{g}}\right] F_s F_r \sin\delta_{\text{sr}}$ (4.71)

The general expression for the torque for a multipole machine is

$$T=-\left[\frac{\text{poles}}{2}\right]\left[\frac{\mu_o\pi \text{Dl}}{2\text{g}}\right]F_sF_r\sin\delta_{\text{sr}}\ (4.72)$$

- In this equation, δ_{sr} is the electrical space-phase angle between the rotor and stator mmf waves and the torque T acts in the direction to accelerate the rotor. Thus when δ_{sr} is positive, the torque is negative and the machine is operating as a generator.
- Similarly, a negative value of δ_{sr} corresponds to positive torque and, correspondingly, motor action. The torque, acting to accelerate the rotor, can then be expressed in terms of the resultant mmf wave F_{sr}; thus

$$T=-\left[\frac{\text{poles}}{2}\right]\left[\frac{\mu_o\pi \text{Dl}}{2\text{g}}\right]F_sF_{\text{sr}}\sin\delta_s\ (4.73)$$

$$T=-\left[\frac{\text{poles}}{2}\right]\left[\frac{\mu_o\pi \text{Dl}}{2\text{g}}\right]F_rF_{\text{sr}}\sin\delta_r\ (4.74)$$

$$T=\left[\frac{\text{poles}}{2}\right]\left[\frac{\pi \text{Dl}}{2}\right]B_{\text{sr}}F_r\sin\delta_r\ (4.75)$$

- One of the inherent limitations in the design of electromagnetic apparatus is the saturation flux density of magnetic materials. Because of saturation in the armature teeth the peak value B_{sr} of the resultant flux-density wave in the air gap is limited to about 1.5 to 2.0T. The maximum permissible value of the winding current, and hence the corresponding mmf wave, is limited by the temperature rise of the winding and other design requirements. Because the resultant flux density and mmf appear explicitly in Eq.4.75, this equation is in a convenient form for design purposes and can be used to estimate the maximum torque which can be obtained from a machineof a given size.

PROBLEMS - chapter 4

PROBLEMS

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

4.1 The rotor of a six-pole synchronous generator is rotating at a mechanical speed of 1200 r/min.

a. Express this mechanical speed in radians per second.

b. What is the frequency of the generated voltage in hertz and in radians per second?

c. What mechanical speed in revolutions per minute would be required to generate voltage at a frequency of 50 Hz?

4.2 The voltage generated in one phase of an unloaded three-phase synchronous generator is of the form v(t) = $V_0 \cos \omega t$. Write expressions for the voltage in the remaining two phases.

4.3 A three-phase motor is used to drive a pump. It is observed (by the use of a stroboscope) that the motor speed decreases from 898 r/min when the pump is unloaded to 830 r/min as the pump is loaded.

a. Is this a synchronous or an induction motor?

b. Estimate the frequency of the applied armature voltage in hertz.

c. How many poles does this motor have?

4.4 A three-phase Y-connected ac machine is initially operating under balanced three-phase conditions when one of the phase windings becomes open-circuited. Because there is no neutral connection on the winding, this requires that the currents in the remaining two windings become equal and opposite. Under this condition, calculate the relative magnitudes of the resultant positive- and negative-traveling mmf waves.

4.5 What is the effect on the rotating mmf and flux waves of a three-phase winding produced by balanced-three-phase currents if two of the phase connections are interchanges?

4.6 In a balanced two-phase machine, the two windings are displaced 90 electrical degrees in space, and the currents in the two windings are phase-displaced 90 electrical degrees in time. For such a machine, carry out the process leading to an equation for the rotating mmf wave corresponding to Eq.4.39 (which is derived for a three-phase machine).

4.7 This problem investigates the advantages of short-pitching the stator coils of an ac machine. Figure 4.1a shows a single full-pitch coil in a two-pole machine. Figure 4.1b shows a fractional-pitch coil for which the coil sides are β radians apart, rather than π radians (180^0) as is the case for the full-pitch coil.

For an air-gap radial flux distribution of the form

$$B_r = \sum_{\text{n odd}} B_n \cos n\theta$$

where n = 1 corresponds to the fundamental space harmonic, n = 3 to the third space harmonic, and so on, the flux linkage of each coil is the integral of B_r over the surface spanned by that coil. Thus for the nth space harmonic, the ratio of the maximum fractional-pitch coil flux linkage to that of the full-pitch coil is

$$\frac{\int_{-\beta/2}^{\beta/2} B_n \cos n\theta d\theta}{\int_{-\pi/2}^{\pi/2} B_n \cos n\theta d\theta} = \frac{\int_{-\beta/2}^{\beta/2} \cos n\theta d\theta}{\int_{-\pi/2}^{\pi/2} \cos n\theta d\theta} = \mid \sin(n\beta/2) \mid$$

It is common, for example, to fractional-pitch the coils of an ac machine by 30 electrical degrees ($\beta = \frac{5\pi}{6} = 150^0$). For n =1, 3, 5 calculate the fractional reduction in flux linkage due to short pitching.

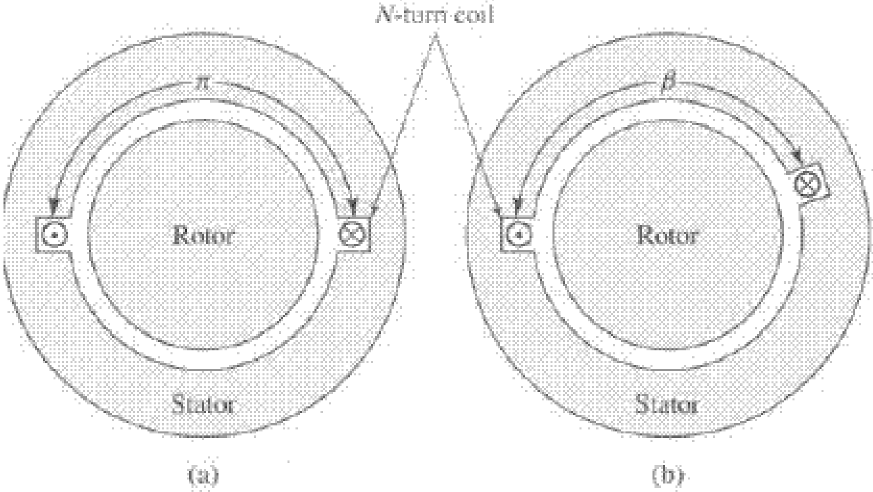


Figure 4.1 (a) full-pitch coil and (b) fractional-pitch coil.

4.8 A six-pole, 60-Hz synchronous machine has a rotor winding with a total of 138 series turns and a winding factor k_r = 0.935. The rotor length is 1.97m, the rotor radius is 58cm, and the air-gap length = 3.15 cm.

a. What is the rated operating speed in r/min?

b. Calculate the rotor-winding current required to achieve a peak fundamental air-gap flux density of 1.23 T.

c. Calculate the corresponding flux per pole.

4.9 Assume that a phase winding of the synchronous machine of Problem 4.8 consists of one full-pitch, 11-turn coil per pole pair, with the coils connected in series to form the phase winding. If the machine is operating at rated speed and under the operating conditions of Problem 4.8, calculate the rms generated voltage per phase.

4.10The synchronous machine of Problem 4.8 has a three-phase winding with 45 series turns per phase and a winding factor k_w = 0.928. For the flux condition and rated speed of Problem 4.8, calculate the rms-generated voltage per phase.

4.11The three-phase synchronous machine of Problem 4.8 is to be moved to an application which requires that its operating frequency be reduced from

60 to 50 Hz. This application requires that, for the operating condition considered in Problem 4.8, the rms generated voltage equal 13.0 kV line-to-line. As a result, the machine armature must be rewound with a different number of turns. Assuming a winding factor of k_w = 0.928, calculate the required number of series turns per phase.

4.12 Figure 4.2 shows a two-pole rotor revolving inside a smooth stator which carries a coil of 110 turns. The rotor produces a sinusoidal space distribution of flux at the stator surface; the peak value of the flux-density wave being 0.85 T when the current in the rotor is 15 A. The magnetic circuit is linear. The inside diameter of the stator is 11 cm, and its axial length is 0.17 m. The rotor is driven at a speed of 50 r/sec.

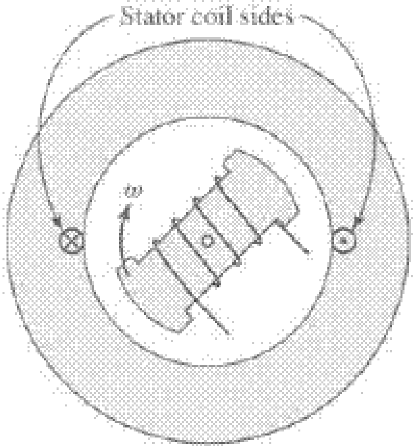


Figure 4.2 Elementary generator.

a. The rotor is excited by a current of 15 A. Taking zero time as the instant when the axis of the rotor is vertical, find the expression for the instantaneous voltage generated in the open-circuited stator coil.

b. The rotor is now excited by a 50-Hz sinusoidal alternating currrent whose peak value is 15 A. Consequently, the rotor current reverses every half revolution; it is timed to be at its maximum just as the axis of the rotor is vertical (i.e., just as it becomes aligned with that of the stator coil). Taking zero time as the instant when the axis of the rotor is vertical, find the expression for the instantaneous voltage generated in the open-circuited stator coil. This scheme is sometimes suggested as a dc generator without a commutator; the thought being that if alternative half cycles of the alternating voltage generated in part (a) are reversed by reversal of the polarity of the field (rotor) winding, then a pulsating direct voltage will be generated in the stator. Discuss whether or not this scheme will work.

4.13 A three-phase two-pole winding is excited by balanced three-phase 60-Hz currents as described by Eqs. 4.23 to 4.25. Although the winding distribution has been designed to minimize harmonics, there remains some third and fifth spatial harmonics. Thus the phase-a mmf can be written as

$$F_a = i_a(A_1 \cos\theta_a + A_3 \cos 3\theta_a + A_5 \cos 5\theta_a)$$

Similar expressions can be written for phases b (replace θ_a by $\theta_a - 120^0$) and c (replace θ_a by $\theta_a + 120^0$). Calculate the total three-phase mmf. What is the angular velocity and rotational direction of each component of the mmf?

4.14 The nameplate of a dc generator indicates that it will produce an output voltage of 24 V dc when operated at a speed of 1200 r/min. By what factor must the number of armature turns be changed such that, for the same field-flux per pole, the generator will produce an output voltage of 18 V dc at a speed of 1400 r/min?

4.15 The armature of a two-pole dc generator has a total of 320 series turns. When operated at a speed of 1800 r/min, the open-circuit generated voltage is 240 V. Calculate φ_P, the air-gap flux per pole.

4.16 The design of a four-pole, three-phase, 230-V, 60-Hz induction motor is to be based on a stator core of length 21 cm and inner diameter 9.52 cm. The stator winding distribution which has been selected has a winding factor k_w = 0.925. The armature is to be Y-connected, and thus the rated phase voltage will be 230/ $\sqrt{3}$ V.

a. The designer must pick the number of armature turns so that the flux density in the machine is large enough to make efficient use of the magnetic material without being so large as to result in excessive saturation. To achieve this objective, the machine is to be designed with a peak fundamental air-gap flux density of 1.25 T. Calculate the required number of series turns per phase.

b. For an air-gap length of 0.3 mm, calculate the self-inductance of an armature phase based upon the result of part (a) and using the inductance formulas of Appendix B. Neglect the reluctance of the rotor and stator iron and the armature leakage inductance.

4.17 A two-pole, 60-Hz, three-phase, laboratory-size synchronous generator has a rotor radius of 5.71 cm, a rotor length of 18.0 cm, and an air-gap length of 0.25 mm. The rotor field winding consists of 264 turns with a winding factor of k_r = 0.95. The Y-connected armature winding consists of 45 turns per phase with a winding factor k_w = 0.93.

a. Calculate the flux per pole and peak fundamental air-gap flux density which will result in an open-circuit, 60-Hz armature voltage of 120 V rms/phase (line-to-neutral).

b. Calculate the dc field current required to achieve the operating condition of part (a).

c. Calculate the peak value of the field-winding to armature-phase-winding mutual inductance.

4.18 Write a MATLAB script which calculates the required total series field- and armature-winding turns for a three-phase, Y-connected synchronous motor given the following information:

Rotor radius, R (meters)

Air-gap length, g (meters)

Electrical frequency, f_e

Field-winding factor, k_f

Rotor length, l (meters)

Number of poles, poles

Peak fundamental air-gap flux density, B_{peak}

Armature-winding factor, k_w

Rated rms open-circuit line-to-line terminal voltage, V_{rated}

Field-current at rated-open-circuit terminal voltage, I_f

4.19 A four-pole, 60-Hz synchronous generator has a rotor length of 5.2 m, diameter of 1.24 m, and air-gap length of 5.9 cm. The rotor winding consists of a series connection of 63 turns per pole with a winding factor of k_r = 0.91. The peak value of the fundamental air-gap flux density is limited to 1.1 T and the rotor winding current to 2700 A. Calculate the maximum torque (N.m) and power output (MW) which can be supplied by this machine.

4.20 Thermal considerations limit the field-current of the laboratory-size synchronous generator of Problem 4.17 to a maximum value of 2.4 A. If the peak fundamental air-gap flux density is limited to a maximum of 1.3 T, calculate the maximum torque (N.m) and power (kW) which can be produced by this generator.

4.21 Figure 4.3 shows in cross section a machine having a rotor winding f and two identical stator windings a and b whose axes are in quadrature. The self-inductance of each stator winding is L_{aa} and of the rotor is L_{ff}. The air gap is uniform. The mutual inductance between a stator winding depends on the angular position of the rotor and may be assumed to be of the form

$M_{\text{af}} = M\cos\theta_0 \; M_{\text{bf}} = M\sin\theta_0$

where M is the maximum value of the mutual inductance. The resistance of each stator winding is R_a.

a. Derive a general expression for the torque T in terms of the angle θ_0, the inductance parameters, and the instantaneous currents i_a, I_b, and i_f. Does this expression apply at standstill? When the rotor is revolving?

b. Suppose the rotor is stationary and constant direct currents I_a = I_0, $I_b = I_0$, and $I_f = 2\text{I}_0$ are supplied to the windings in the directions indicated by the dots and crosses in Fig. 4.45. If the rotor is allowed to move, will it rotate continuously or will it tend to come to rest? If the latter, at what value of θ_0 ?

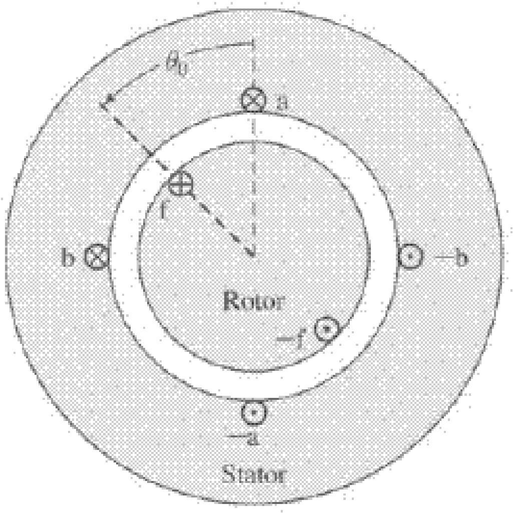


Figure 4.3 Elementary cylindrical-rotor, two-phase synchronous machine.

c. The rotor winding is now excited by a constant direct current If while the stator windings carry balanced two-phase currents

$$i_a = \sqrt{2}I_a\text{cos}\omega t i_b = \sqrt{2}I_a\text{sin}\omega t$$

The rotor is revolving at synchronous speed so that its instantaneous angular position is given by $\theta_0 = \omega t - \delta$, where δ is a phase angle describing the position of the rotor at t = 0. The machine is an elementary two-phase synchronous machine. Derive an expression for the torque under these conditions.

d. Under the conditions of part (c), derive an expression for the instantaneous terminal voltages of stator phases a and b.

4.22 Consider the two-phase synchronous machine of Problem 4.21. Derive an expression for the torque acting on the rotor if the rotor is rotating at constant angular velocity, such that $\theta_0 = \omega t + \delta$, and the phase currents become unbalanced such that

$$i_a = \sqrt{2}I_a\text{cos}\omega t i_b = \sqrt{2}(I_a + I^{'})\text{sin}\omega t$$

What are the instantaneous and time-averaged torque under this condition?

4.23 Figure 4.4 shows in schematic cross section a salient-pole synchronous machine having two identical stator windings a and b on a laminated steel core. The salient-pole rotor is made of steel and carries a field winding f connected to slip rings.

Because of the nonuniform air gap, the self- and mutual inductances are functions of the angular position θ_0 of the rotor. Their variation with θ0 can be approximated as:

$L_{\mathrm{aa}} = L_0 + L_2 \mathrm{cos} 2\theta_0$ $L_{\mathrm{bb}} = L_0 - L_2 \mathrm{cos} 2\theta_0$ $M_{\mathrm{ab}} = L_2 \mathrm{sin} \theta_0$

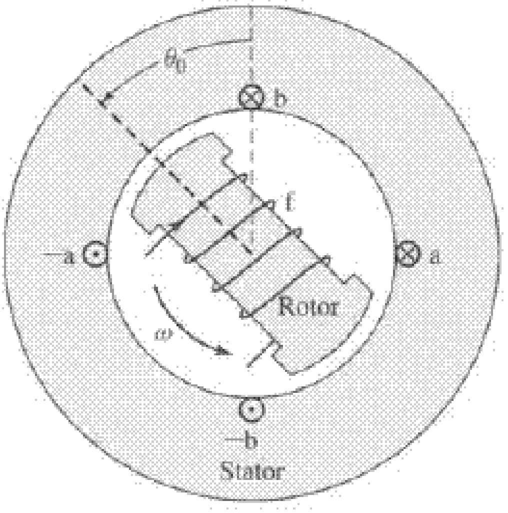


Figure 4.4 Schematic two-phase, salient-pole synchronous machine.

where L_0 and L_2 are positive constants. The mutual inductance between the rotor and the stator windings are functions of θ_0

$M_{\mathrm{af}} = M \mathrm{cos} \theta_0$ $M_{\mathrm{bf}} = M \mathrm{sin} \theta_0$

where M is also a positive constant. The self-inductance of the field winding, L_{ff}, is constant, independent of θ_0.

Consider the operating condition in which the field winding is excited by direct current I_f and the stator windings are connected to a balanced two-phase voltage source of frequency ω. With the rotor revolving at synchronous speed, its angular position will be given by $\theta_0 = \omega t$.

Under this operating condition, the stator currents will be of the form

$i_a = \sqrt{2} I_a \cos(\omega t + \delta) i_b = \sqrt{2} I_a \sin(\omega t + \delta)$

a. Derive an expression for the electromagnetic torque acting on the rotor.

b. Can the machine be operated as a motor and/or a generator? Explain.

c. Will the machine continue to run if the field current If is reduced to zero? Support you answer with an expression for the torque and an explanation as to why such operation is or is not possible.

4.24 A three-phase linear ac motor has an armature winding of wavelength 25 cm. A three-phase balanced set of currents at a frequency of 100 Hz is applied to the armature.

a. Calculate the linear velocity of the armature mmf wave.

b. For the case of a synchronous rotor, calculate the linear velocity of the rotor.

c. For the case of an induction motor operating at a slip of 0.045, calculate the linear velocity of the rotor.

4.25 The linear-motor armature of Problem 4.24 has a total active length of 7 wavelengths, with a total of 280 turns per phase with a winding factor k_w = 0.91. For an air-gap length of 0.93 cm, calculate the rms magnitude of the balanced three-phase currents which must be supplied to the armature to achieve a peak space-fundamental air-gap flux density of 1.45 T.

4.26 A two-phase linear permanent-magnet synchronous motor has an air-gap of length 1.0 mm, a wavelength of 12 cm, and a pole width of 4 cm. The rotor is 5 wavelengths in length. The permanent magnets on the rotor are arranged to produce an air-gap magnetic flux distribution that is uniform over the width of a pole but which varies sinusoidally in space in the direction of rotor travel. The peak density of this air-gap flux is 0.97 T.

a. Calculate the net flux per pole.

b. Each armature phase consists of 10 turns per pole, with all the poles connected in series. Assuming that the armature winding extends many wavelengths past either end of the rotor, calculate the peak flux linkages of the armature winding.

c. If the rotor is traveling at a speed of 6.3 m/sec, calculate the rms voltage induced in the armature winding.

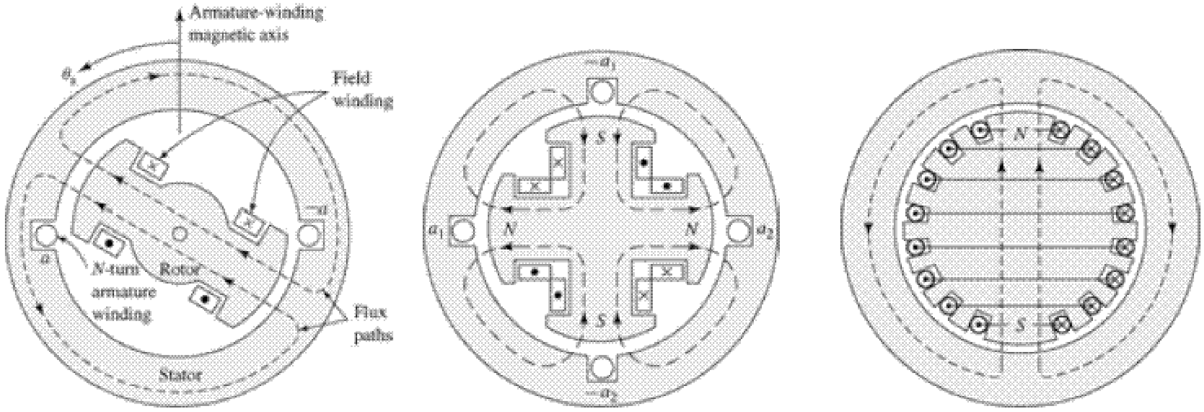
Chapter 5: Synchronous Machines

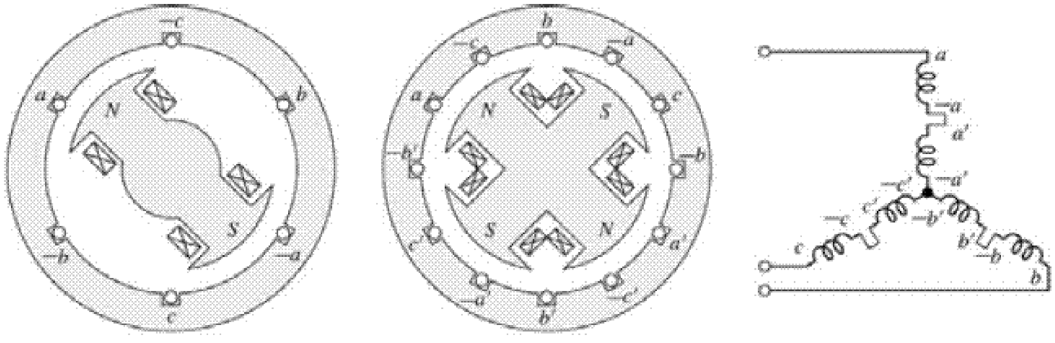
Chapter 5: Synchronous Machines

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- Main features of synchronous machines:

- A synchronous machine is an ac machine whose speed under steady-state conditions is proportional to the frequency of the current in its armature.
- The rotor, along with the magnetic field created by the dc field current on the rotor, rotates at the same speed as, or in synchronism with, the rotating magnetic field produced by the armature currents, and a steady torque results.





(a) (b) (c)

Figure 5.1 Schematic views of three-phase generators: (a) two-pole, (b) four-pole, and

(c) Y connection of the windings.

§5.1 Introduction to Polyphase Synchronous Machines

- Synchronous machines:

- Armature winding: on the stator, alternating current.
- Field winding: on the rotor, dc power supplied by the excitation system.
    - Cylindrical rotor: for two- and four-pole turbine generators.
    - Salient-pole rotor: for multipolar, slow-speed, hydroelectric generators and for most synchronous motors.

- Acting as a voltage source:
    - Frequency determined by the speed of its mechanical drive (or prime mover).
    - The amplitude of the generated voltage is proportional to the frequency and the field current.

$$\begin{aligned}\lambda_a &= k_w N_{\mathrm{ph}} \mathit{\Phi}_p \cos\left(\left(\frac{\text{poles}}{2}\right)\omega_m t\right) \\ &= k_w N_{\mathrm{ph}} \mathit{\Phi}_p \cos \omega_{\mathrm{me}} t\end{aligned} \quad (5.1)$$

$$\omega_{\mathrm{me}} = \left(\frac{\text{poles}}{2}\right)\omega_m \quad (5.2)$$

$$e_a = \frac{d\lambda_a}{\mathrm{dt}} = k_w N_{\mathrm{ph}} \frac{d\mathit{\Phi}_p}{\mathrm{dt}} \cos \omega_{\mathrm{me}} t - \omega_{\mathrm{me}} k_w N_{\mathrm{ph}} \mathit{\Phi}_p \sin \omega_{\mathrm{me}} t \quad (5.3)$$

$$e_a = -\omega_{\mathrm{me}} k_w N_{\mathrm{ph}} \mathit{\Phi}_p \sin \omega_{\mathrm{me}} t \quad (5.4)$$

$$E_{\max} = \omega_{\mathrm{me}} k_w N_{\mathrm{ph}} \mathit{\Phi}_p = 2\pi \mathrm{f}_{\mathrm{me}} k_w N_{\mathrm{ph}} \mathit{\Phi}_p \quad (5.5)$$

$$E_{\mathrm{rms}} = \frac{2\pi}{\sqrt{2}} f_{\mathrm{me}} k_w N_{\mathrm{ph}} \mathit{\Phi}_p = \sqrt{2}\pi f_{\mathrm{me}} k_w N_{\mathrm{ph}} \mathit{\Phi}_p \quad (5.6)$$

- Synchronous generators can be readily operated in parallel: interconnected power systems.
- When a synchronous generator is connected to a large interconnected system containing many other synchronous generators, the voltage and frequency at its armature terminals are substantially fixed by the system.
  - It is often useful, when studying the behavior of an individual generator or group of generators, to represent the remainder of the system as a constant-frequency, constant-voltage source, commonly referred to as an infinite bus.
  - Analysis of a synchronous machine connected to an infinite bus.
- Torque equation:

$T == \frac{\pi}{2}\left(\frac{\text{poles}}{2}\right)^2 \varPhi_R F_f \sin\delta_{\text{RF}}$ (5.7)

where

$\varPhi_R$ =resultant air-gap flux per pole

F_f =mmf of the dc field winding

δ_{RF} =electric phase angle between magnetic axes of $\varPhi_R$and F_f

- The minus sign indicates that the electromechanical torque acts in the direction to bring the interacting fields into alignment.
- In a generator, the prime-mover torque acts in the direction of rotation of the rotor, and the electromechanical torque opposes rotation. The rotor mmf wave leads the resultant air-gap flux.
- In a motor, the electromechanical torque is in the direction of rotation, in opposition to the retarding torque of the mechanical load on the shaft.
- Torque-angle curve: Fig. 5.2.

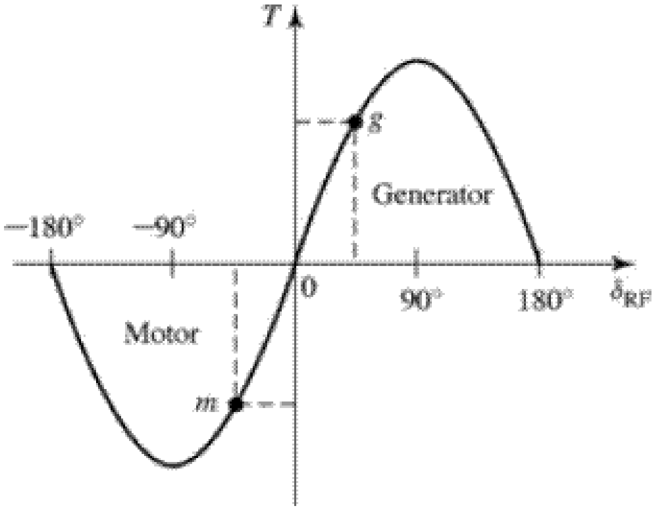


Figure 5.2 Torque-angle characteristics.

- An increase in prime-mover torque will result in a corresponding increase in the torque angle.
- $T = T_{\max}$: pull-out torque at $\delta = 90$.Any further increase in prime-mover torque cannot be balanced by a corresponding increase in synchronous electromechanical torque, with the result that synchronism will no longer be maintained and the rotor will speed up. ⇒ loss of synchronism, pulling out of step.

§5.2 Synchronous-Machine Inductances; Equivalent Circuits

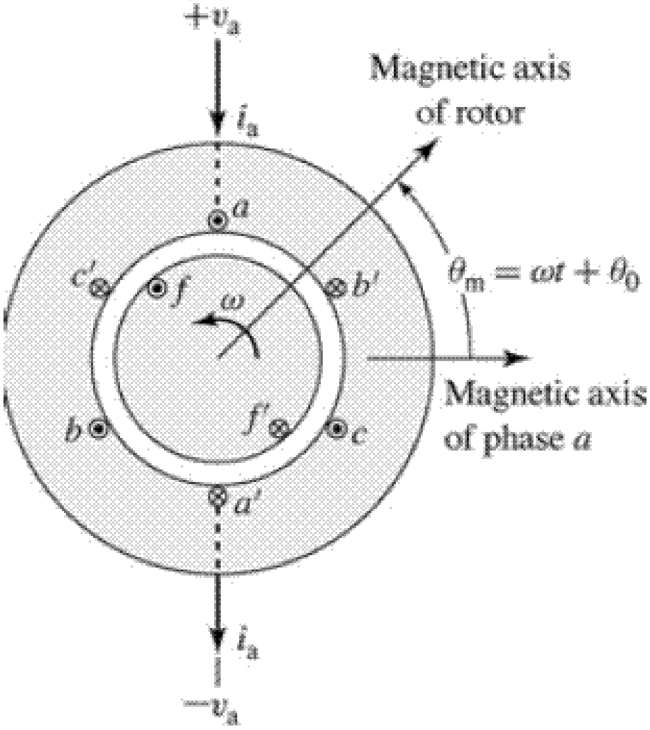


Figure 5.3 Schematic diagram of a two-pole,

three-phase cylindrical-rotor synchronous machine.

- A cross-sectional sketch of a three-phase cylindrical-rotor synchronous machine is shown schematically in Fig.5.3. The figure shows a two-pole machine; alternatively, this can be considered as two poles of a multipole machine. The three-phase armature winding on the stator is of the same type used in the discussion of rotating magnetic fields in Section 4.5. Coils $aa^{'}$, $bb^{'}$ and $cc^{'}$ I represent distributed windings producing sinusoidal mmf and flux-density waves in the air gap. The reference directions for the currents are shown by dots and crosses. The field winding $ff^{'}$on the rotor also represents a distributed winding which produces a sinusoidal mmf and flux-density wave centered on its magnetic axis and rotating with the rotor.

- When the flux linkages with armature phases a, b, c and field winding f are expressed in terms of the inductances and currents as follows,

$$\lambda_a = L_{\text{aa}} i_a + L_{\text{ab}} i_b + L_{\text{ac}} i_c + L_{\text{af}} i_f \quad (5.8)$$

$\lambda_b = L_{\text{ba}} i_a + L_{\text{bb}} i_b + L_{\text{bc}} i_c + L_{\text{bf}} i_f$ (5.9)

$\lambda_c = L_{\text{ca}} i_a + L_{\text{cb}} i_b + L_{\text{cc}} i_c + L_{\text{cf}} i_f$ (5.10)

$\lambda_f = L_{\text{fa}} i_a + L_{\text{fb}} i_b + L_{\text{fc}} i_c + L_{\text{ff}} i_f$ (5.11)

the induced voltages can be found from Faraday's law. Here, two like subscripts denote a self-inductance, and two unlike subscripts denote a mutual inductance between the two windings. The script is used to indicate that, in general, both the self- and mutual inductances of a three-phase machine may vary with rotor angle.

§5.2.1 Rotor Self-Inductance

- With a cylindrical stator, the self-inductance of the field winding is independent of the rotor position 0m when the harmonic effects of stator slot openings are neglected.

$L_{\text{ff}} = L_{\text{ff}} = L_{\text{ff0}} + L_{f1}$ (5.12)

where the italic L is used for an inductance which is independent of θ_m. The component L_{ff0}corresponds to that portion of L_{ff} due to the space-fundamental component of air-gap flux

§5.2.2 Stator-to-Rotor Mutual Inductances

- The stator-to-rotor mutual inductances vary periodically with θ_{me}, the electrical angle between the magnetic axes of the field winding and the armature phase a as shown in Fig.5.2 and as defined by Eq.4.54. With the space-mmf and air-gap flux distribution assumed sinusoidal, the mutual inductance between the field winding f and phase a varies as cosθ_{me}; thus

$L_{\text{af}} = L_{\text{fa}} = L_{\text{af}} \cos\theta_{\text{me}}$ (5.13)

$\theta_{\text{me}} = \left[\frac{\text{poles}}{2}\right]\theta_m = \omega_e t + \delta_{e0}$ (5.14)

$L_{\text{af}} = L_{\text{fa}} = L_{\text{af}} \cos(\omega_e t + \delta_{e0})$ (5.15)

§5.2.3 Stator Inductances; Synchronous Inductance

- With a cylindrical rotor, the air-gap geometry is independent of θ_m if the effects of rotor slots are neglected. The stator self-inductances then are constant; thus

$L_{\mathrm{aa}} = L_{\mathrm{bb}} = L_{\mathrm{cc}} = L_{\mathrm{aa}} = L_{\mathrm{aa0}} + L_{a1}$ (5.16)

§5.2.4 Equivalent Circuit

- Equivalent circuit for the synchronous machine:

- Single-phase, line-to-neutral equivalent circuits for a three-phase machine operating under balanced, three-phase conditions.

$L_s =$effective inductance seen by phase a under steady-state, balanced three-phase

machine operating conditions.

$X_s = \omega_e L_s$: synchronous reactance

$R_a =$armature winding resistance

$e_{\mathrm{af}} =$voltage induced by the field winding flux (generated voltage, internal voltage)

$I_a =$ armature current

$v_a =$ terminal voltage

Motor reference direction:

$\hat{V}_a = R_a \hat{I}_a + \mathrm{jX}_s \hat{I}_a + \hat{E}_{\mathrm{af}}$ (5.17)

Generator reference direction:

$\hat{V}_a = -R_a \hat{I}_a - \mathrm{jX}_s \hat{I}_a + \hat{E}_{\mathrm{af}}$ (5.18)

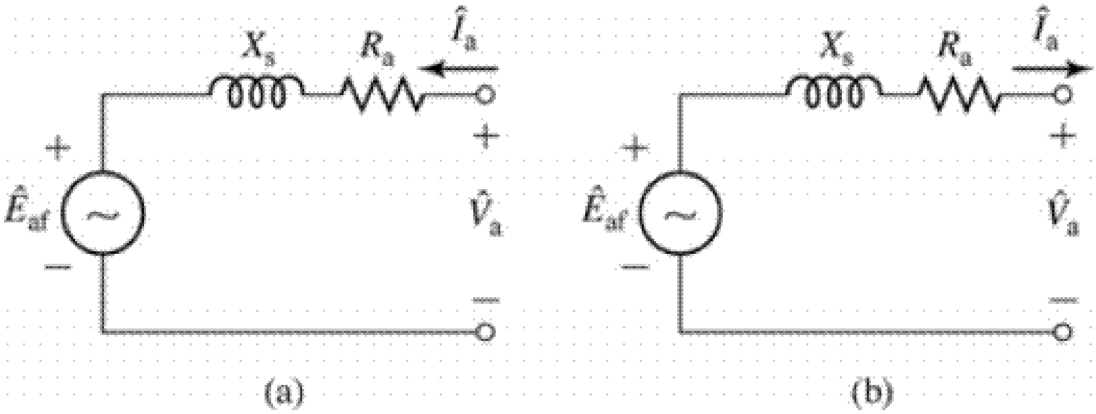


Figure 5.4 Synchronous-machine equivalent circuits:

(a) motor reference direction and (b) generator reference direction.

$X_s = X_{\text{al}} + X_\phi$

X_{al} =armature leakage reactance

X_ϕ=magnetizing reactance of the armature winding

$\hat{E}_R$= air-gap voltage or the voltage behind leakage reactance

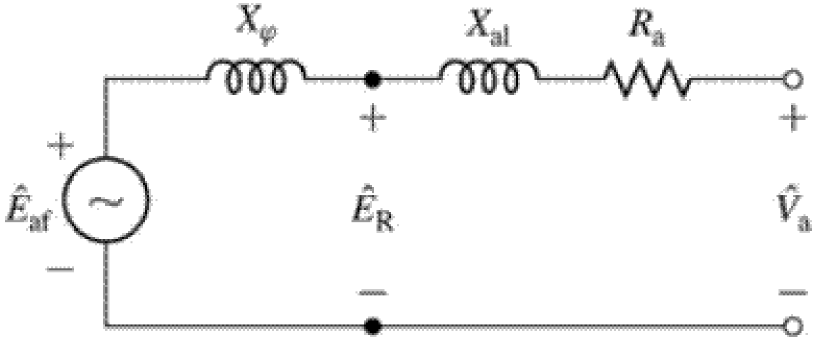


Figure 5.5 Synchronous-machine equivalent circuit showing air-gap and

leakage components of synchronous reactance and air-gap voltage.

§5.4 Steady-State Power-Angle Characteristics

- The maximum power a synchronous machine can deliver is determined by the maximum torque that can be applied without loss of synchronism with the external system to which it is connected.

- Both the external system and the machine itself can be represented as an impedance in series with a voltage source.

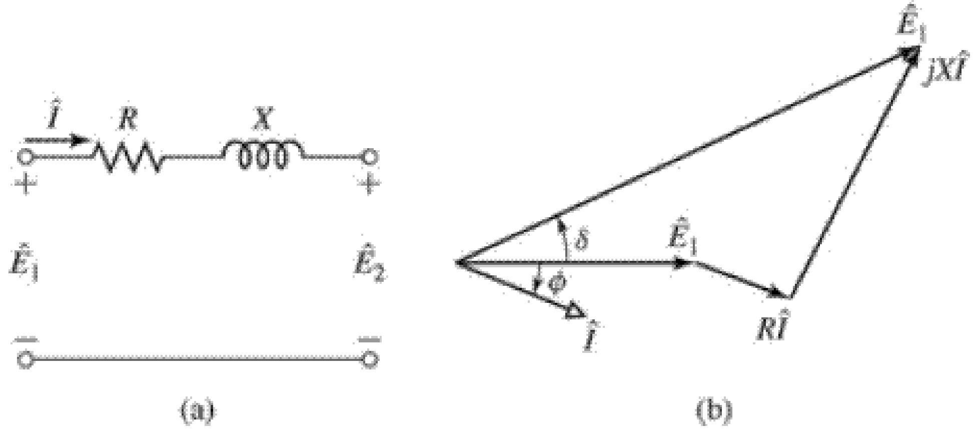


Figure 5.6 (a) Impedance interconnecting two voltages; (b) phasor diagram.

$$P_2 = E_2 I \cos\varphi \quad (5.19)$$

$$\hat{I} = \frac{\hat{E}_1 - \hat{E}_2}{Z} \quad (5.20)$$

$$\hat{E}_1 = E_1 e^{j\delta} \quad (5.21)$$

$$\hat{E}_2 = E_2 \quad (5.22)$$

$$Z = R + \mathrm{jX} = | Z | \, e^{j\varphi_z} (5.23)$$

$$\hat{I} = \mathrm{I}e^{j\varphi} = \frac{E_1 e^{j\delta} - E_2}{|Z| e^{j\varphi_z}} = \frac{E_1}{|Z|} e^{j(\delta - \varphi_z)} - \frac{E_2}{|Z|} e^{-j\varphi_z} (5.24)$$

$$I\cos\varphi = \frac{E_1}{|Z|} \cos(\delta - \varphi_z) - \frac{E_2}{|Z|} \cos(-\varphi_z) (5.25)$$

$$P_2 = \frac{E_1 E_2}{|Z|} \cos(\delta - \varphi_z) - \frac{E_2^2 R}{|Z|^2} \quad (5.26)$$

$$P_2 = \frac{E_1 E_2}{|Z|} \sin(\delta + \alpha_z) - \frac{E_2^2 R}{|Z|^2} \quad (5.27)$$

Where

$$\alpha_z = 90^o - \varphi_z = \tan^{-1}(\frac{R}{X}) \text{ (5.28)}$$

$$P_1 = \frac{E_1E_2}{|Z|}\sin(\delta - \varphi_z) - \frac{E_1^2R}{|Z|^2} \text{ (5.29)}$$

Frequently, $R << \mid Z \mid, \mid Z \mid \approx X \text{and} \alpha_z \approx 0$,

$$P_1 = P_2 = \frac{E_1E_2}{X}\sin\delta \text{ (5.30)}$$

Equation (5.30) is commonly referred to as the power-angle characteristic for a synchronous machine.

- The angle δ is known as the power angle.
- Note that E_1 and E_2 are the line-to-neutral voltages.
- For three-phase systems, a factor “3” shall be placed in front of the equation.
- The maximum power transfer is

$$P_{1,\text{max}} = P_{2,\text{max}} = \frac{E_1E_2}{X} \text{ (5.31)}$$

occurring when $\delta = \pm 90^o$.

- If $\delta > 0$, $\hat{E}_1$ leads and power flows from source to $\hat{E}_2$
- When $\delta < 0$, $\hat{E}_1$ lags $\hat{E}_2$ and power flows from source $\hat{E}_1$ to $\hat{E}_2$
- Consider Fig. 5.7 in which a synchronous machine with generated voltage Êaf and synchronous X_s is connected to a system whose Thevenin equivalent is a voltage source V_{EQ} in series with a reactive impedance jX_{EQ}. The power-angle characteristic can be written

$$P = \frac{E_{\text{af}}V_{\text{EQ}}}{X_s + X_{\text{EQ}}}\sin\delta \text{ (5.32)}$$

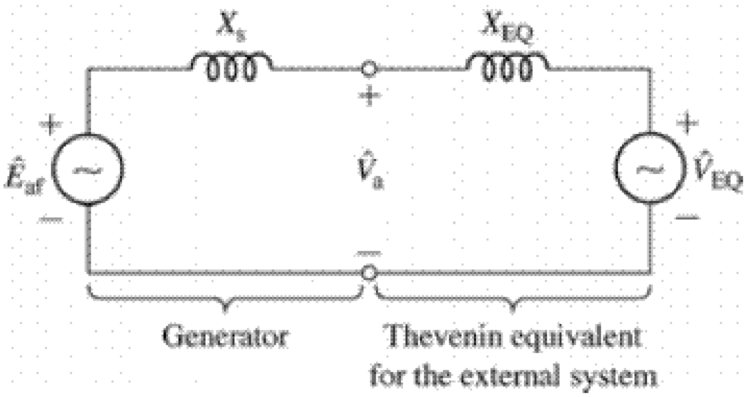


Figure 5.7 Equivalent-circuit representation of

a synchronous machine connected to an external system.

- Note that $P \propto E_1 E_2$,$P \propto X$,$P_{\max} \propto E_1 E_2$, and $P_{\max} \propto X$.
- In general, stability considerations dictate that a synchronous machine achieve steady-state operation for a power angle considerably less than 90^o.

§5.3 Open- and Short-Circuit Characteristics

§5.3.1 Open-Circuit Saturation Characteristic and No-Load Rotational Losses

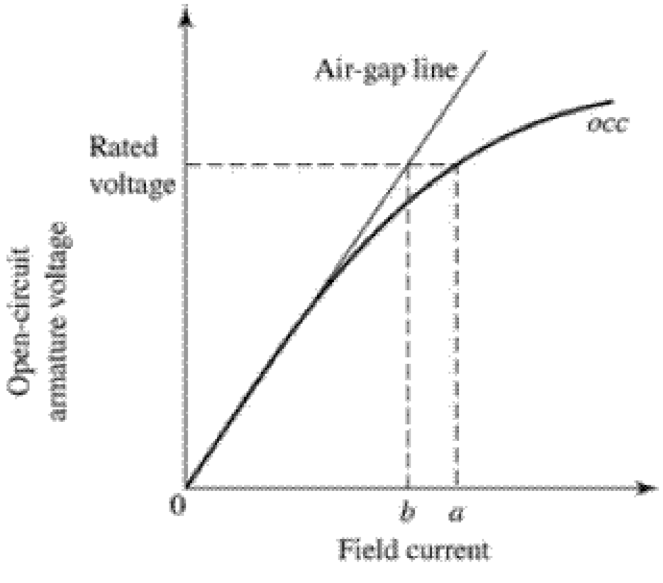


Figure 5.8 Open-circuit characteristic of a synchronous machine.

§5.3.2 Short-Circuit Characteristic and Load Loss

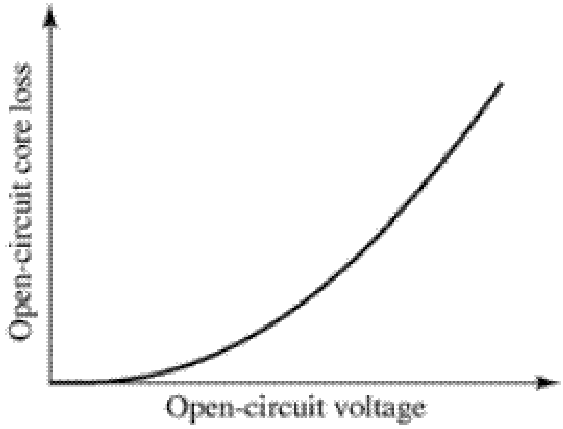


Figure 5.9 Typical form of an open-circuit core-loss curve.

$\hat{E}_{\text{af}} = \hat{I}_a(R_a + \text{j}\text{X}_s)$ (5.33)

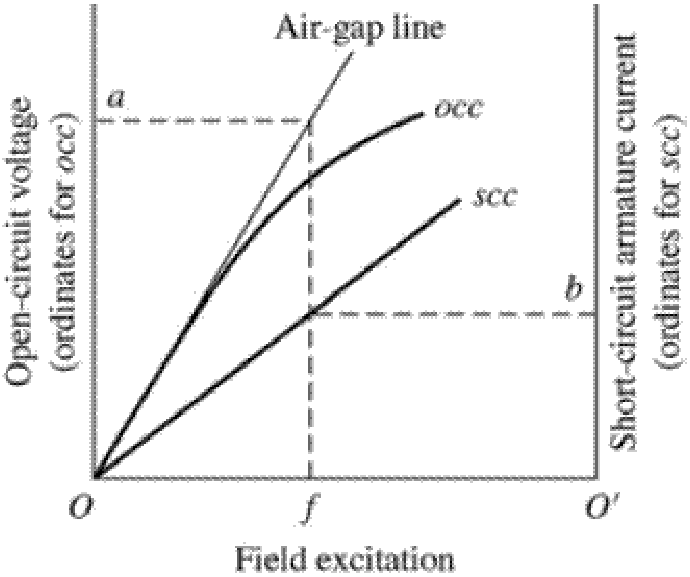


Figure 5.10 Open- and short-circuit characteristics of a synchronous machine.

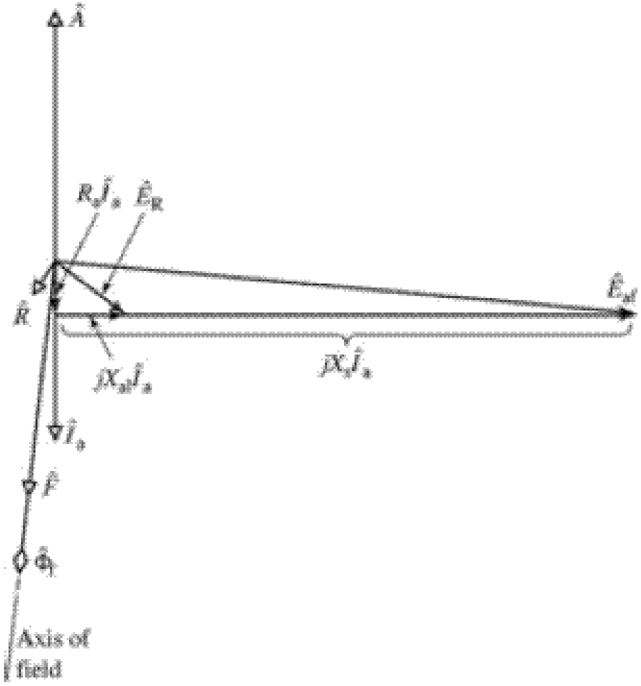


Figure 5.11 Phasor diagram for short-circuit conditions.

$$\hat{E}_R = \hat{I}_a(R_a + \mathrm{jX_{al}}) \quad (5.34)$$

$$X_{s,u} = \frac{V_{a,\mathrm{ag}}}{I_{a,\mathrm{ac}}} \quad (5.35)$$

$$X_s = \frac{V_{a,\mathrm{rated}}}{I_a^{'}} \quad (5.36)$$

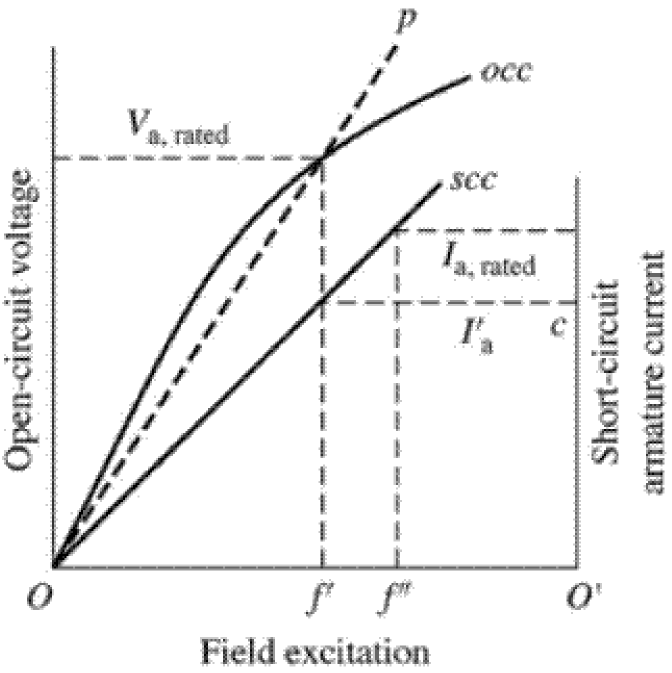


Figure 5.12 Open- and short-circuit characteristics showing equivalent magnetization line for saturated operating conditions.

$$\text{SCR} = \frac{Of'}{Of''} \quad (5.37)$$

$$\text{SCR} = \frac{\text{AFNL}}{\text{AFSC}} \quad (5.38)$$

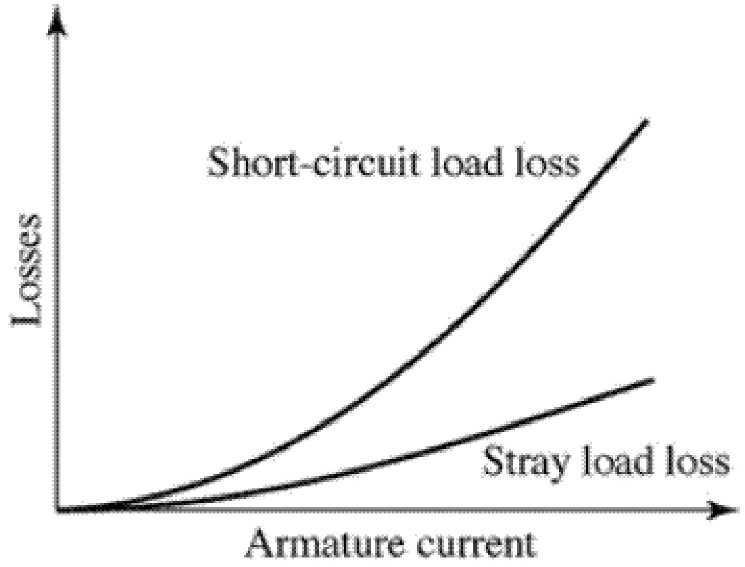


Figure 5.13 Typical form of short-circuit load loss and stray load-loss curves.

$$\frac{R_T}{R_T} = \frac{234.5+T}{234.5+t} \quad (5.39)$$

$$R_{a,\text{eff}} = \frac{\text{short}-\text{circuit load loss}}{(\text{short}-\text{circuit armature current})^2} \quad (5.40)$$

§5.5 Steady-State Operating Characteristics

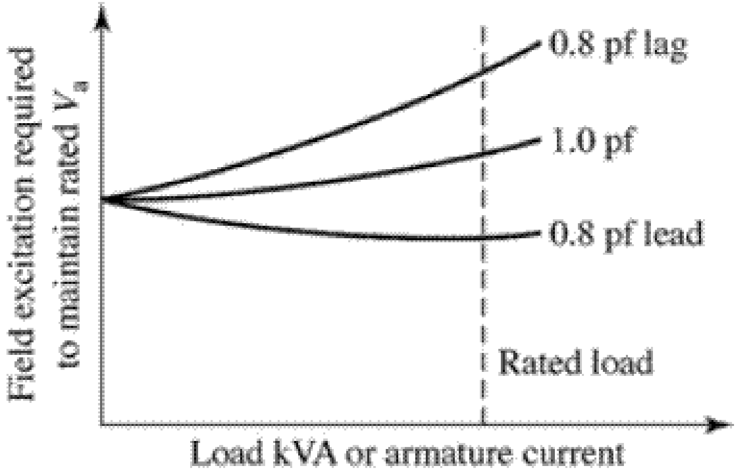


Figure 5.14 Characteristic form of synchronous-generator compounding curves.

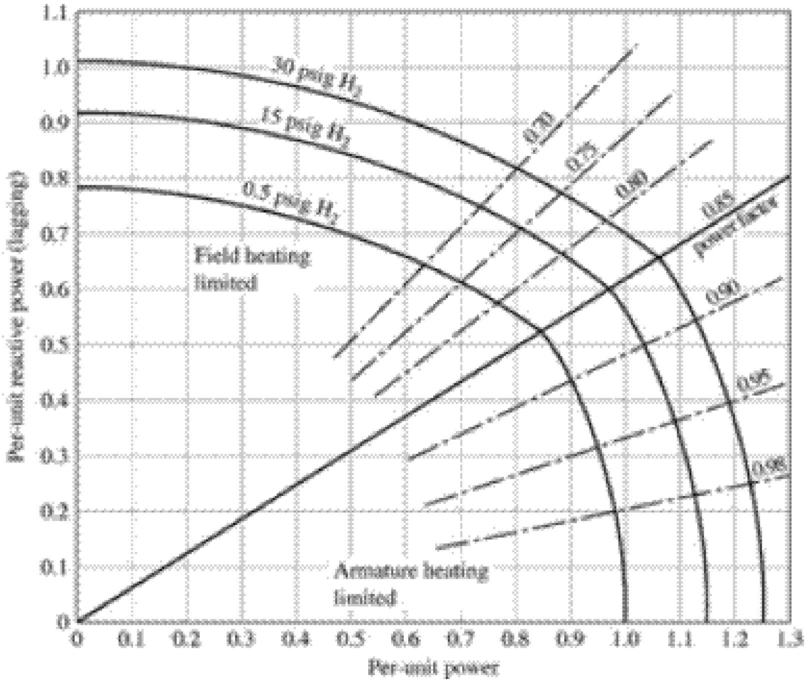


Figure 5.15 Capability curves of an 0.85 power factor, 0.80 short-circuit ratio,

hydrogen-cooled turbine generator. Base MVA is rated MVA at 0.5 psig hydrogen.

$$\text{Apprent power} = \sqrt{P^2 + Q^2} = V_a I_a$$

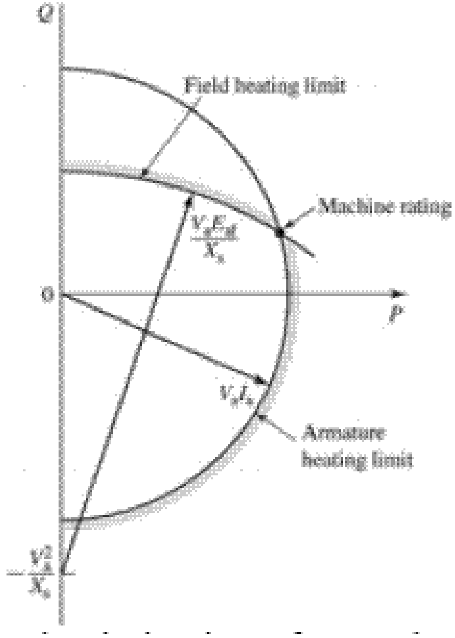


Figure 5.16 Construction used for the derivation of a synchronous generator capability curve.

$P - \text{jQ} = \hat{V}_a + \text{jX}_s \hat{I}_a$ (5.41)

$\hat{E}_{\text{af}} = \hat{V}_a + \text{jX}_s \hat{I}_a$ (5.42)

$P^2 + (Q + \frac{V_a^2}{X_s})^2 = (\frac{V_a E_{\text{af}}}{X_s})^2$ (5.43)

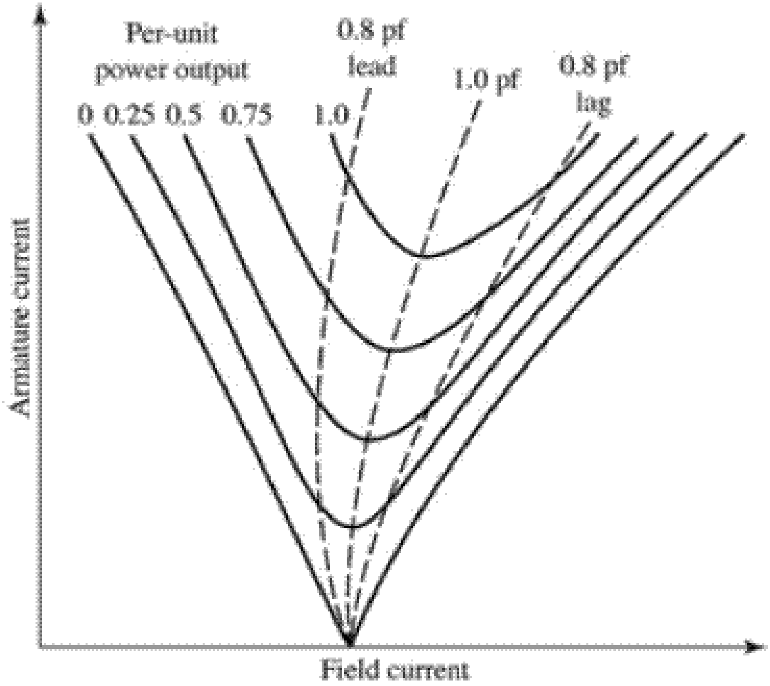


Figure 5.17 Typical form of synchronous-generator V curves.

§5.6 Effects of Salient Poles; Introduction to Direct-And Quadrature-Axis Theory

§5.6.1 Flux and MMF Waves

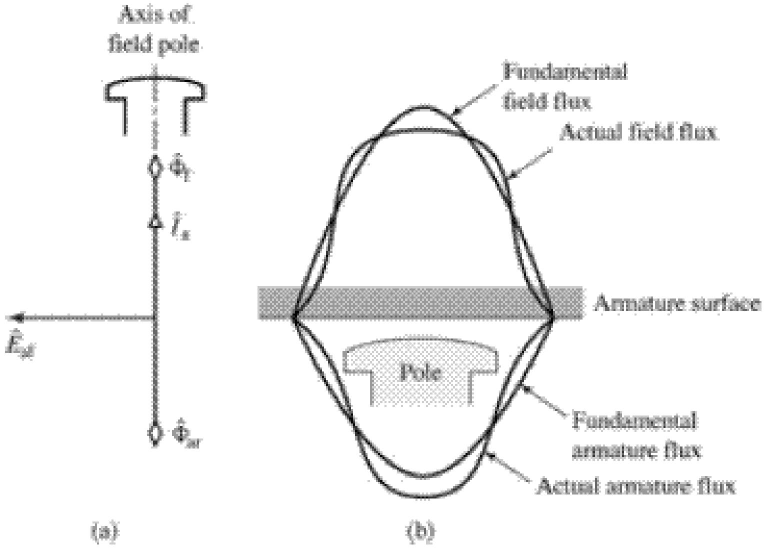


Figure 5.18 Direct-axis air-gap fluxes in a salient-pole synchronous machine.

$E_{3,a} = \sqrt{2}V_3\cos(3\omega_e t + \varphi_3)$ (5.44)

$E_{3,b} = \sqrt{2}V_3\cos(3(\omega_e - 120^o) + \varphi_3) = \sqrt{2}V_3\cos(3\omega_e t + \varphi_3)$ (5.45)

$E_{3,c} = \sqrt{2}V_3\cos(3(\omega_e t - 120^o) + \varphi_3) = \sqrt{2}V_3\cos(3\omega_e t + \varphi_3)$ (5.45)

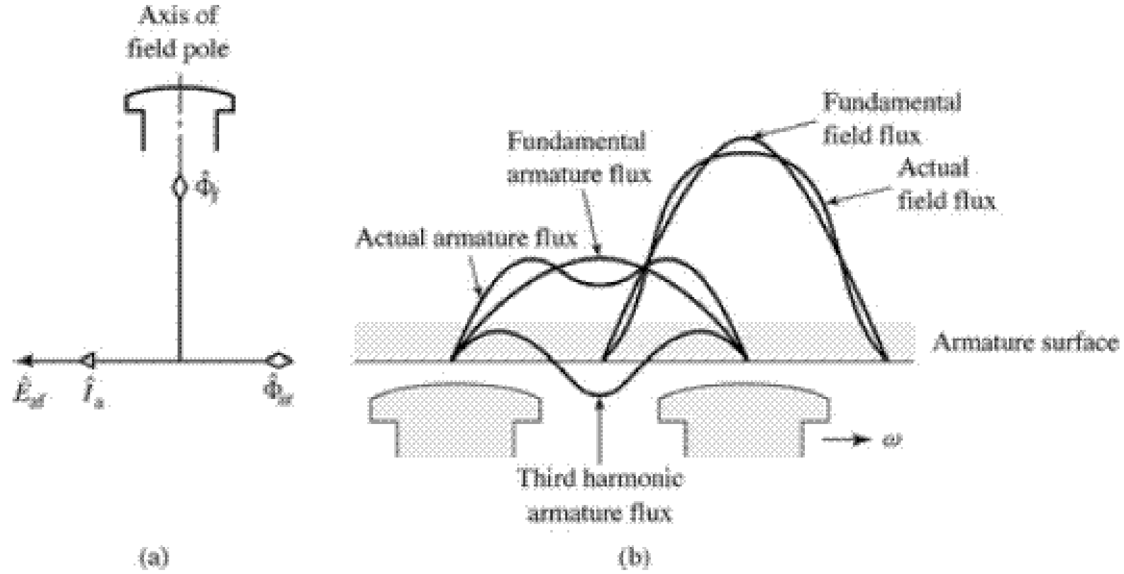


Figure 5.19 Quadrature-axis air-gap fluxes in a salient-pole synchronous machine.

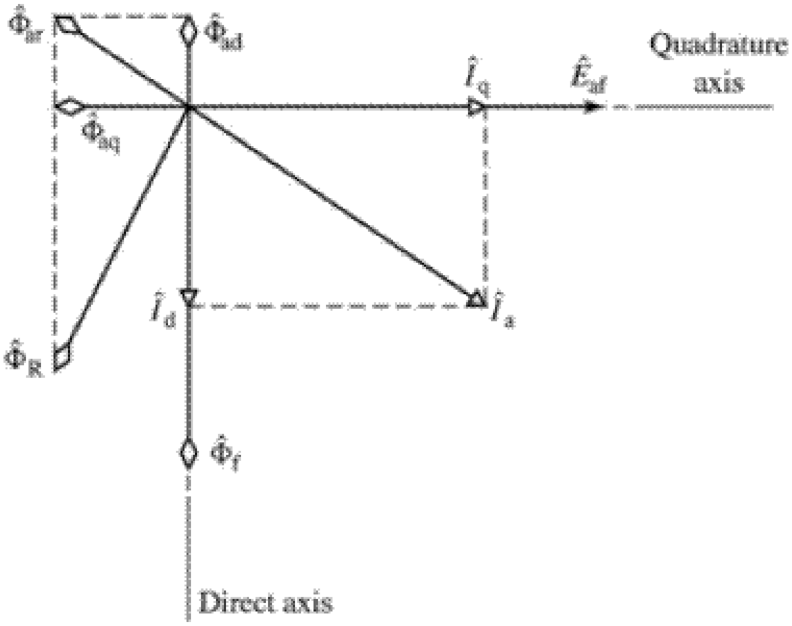


Figure 5.20 Phasor diagram of a salient-pole synchronous generator.

§5.6.2 Phasor Diagrams for Salient-Pole Machines

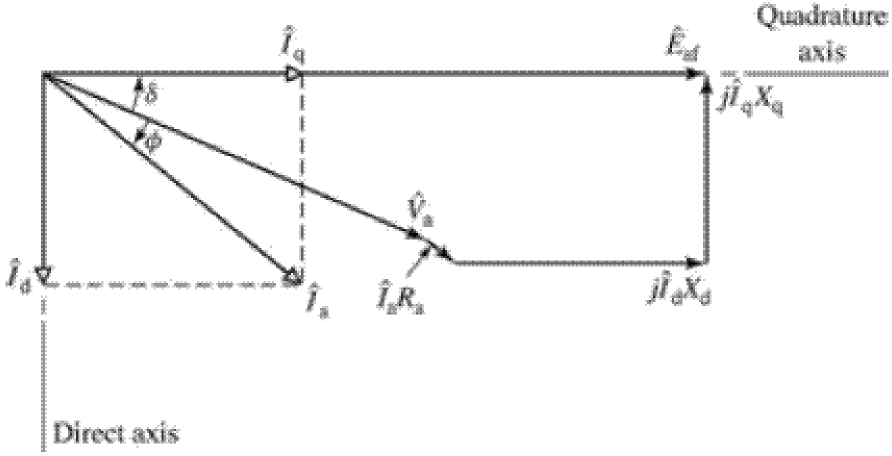


Figure 5.21 Phasor diagram for a synchronous generator showing the relationship between the voltages and the currents.

$X_d = X_{\mathrm{al}} + X_{\phi d}$ (5.46)

$X_q = X_{\mathrm{al}} + X_{\phi q}$ (5.47)

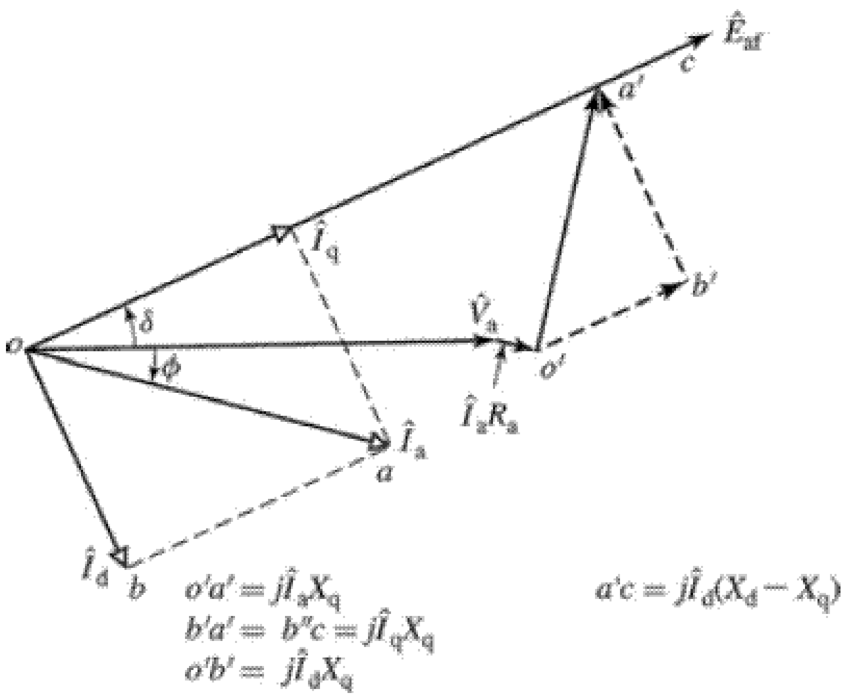


Figure 5.22 Relationships between component voltages in a phasor diagram.

$\frac{o'a'}{\mathrm{oa}} = \frac{b'a'}{\mathrm{ba}}$ (5.48)

$o'a' = (\frac{b'a'}{\mathrm{ba}})\mathrm{oa} = \frac{|\hat{I}_q|X_q}{|\hat{I}_q|} \mid \hat{I}_a \mid = X_q \mid \hat{I}_a \mid$ (5.49)

$\hat{E}_{\mathrm{af}} = \hat{V}_a + R_a\hat{I}_a + \mathrm{jX}_d\hat{I}_d + \mathrm{jX}_q\hat{I}_q$ (5.50)

5.7 Power-Angle Characteristics Of Salient-Pole Machines

- For the purposes of this discussion, it is sufficient to limit our discussion to the simple system shown in the schematic diagram of Fig.5.23a, consisting of a salient pole synchronous machine SM connected to an infinite bus of voltage $\hat{V}_{\mathrm{EQ}}$through a series impedance of reactance X_{EQ}. Resistance will be neglected because it is usually small. Consider that the synchronous machine is acting as a generator. The phasor diagram is shown by the solid-line phasors in Fig.5.23b. The dashed phasors show the external reactance drop resolved into components due to $\hat{I}_d$and $\hat{I}_q$. The effect of the external impedance is merely to add its reactance to the reactances of the machine; the total values of the reactance between the excitation voltage $\hat{E}_{\mathrm{af}}$and the bus voltage $\hat{V}_{\mathrm{EQ}}$ is therefore

$$X_{\mathrm{dT}} = X_d + X_{\mathrm{EQ}} \ (5.50)$$

$$X_{\mathrm{qT}} = X_q + X_{\mathrm{EQ}} \ (5.51)$$

- If the bus voltage $\hat{V}_{\mathrm{EQ}}$ is resolved into components its direct-axis component $V_d = V_{\mathrm{EQ}}\mathrm{sin}\delta$ and quadrature-axis component $V_q = V_{\mathrm{EQ}}\mathrm{cos}\delta$ in phase with $\hat{I}_d$ and $\hat{I}_q$, respectively, the power P delivered to the bus per phase (or in per unit) is

$$P = I_d V_d + I_q V_q = I_d V_{\mathrm{EQ}}\mathrm{sin}\delta + I_q V_{\mathrm{EQ}}\mathrm{cos}\delta (5.52)$$

$$I_d = \frac{E_{\mathrm{af}} - V_{\mathrm{EQ}}\mathrm{cos}\delta}{X_{\mathrm{dT}}} (5.53)$$

$$I_q = \frac{V_{\mathrm{EQ}}\mathrm{sin}\delta}{X_{\mathrm{qT}}} (5.54)$$

$$P = \frac{E_{\mathrm{af}} V_{\mathrm{EQ}}}{X_{\mathrm{dT}}}\mathrm{sin}\delta + \frac{V_{\mathrm{EQ}}^2 (X_{\mathrm{dT}} - X_{\mathrm{qT}})}{2\mathrm{X}_{\mathrm{dT}} X_{\mathrm{qT}}}\mathrm{sin}2\delta (5.55)$$

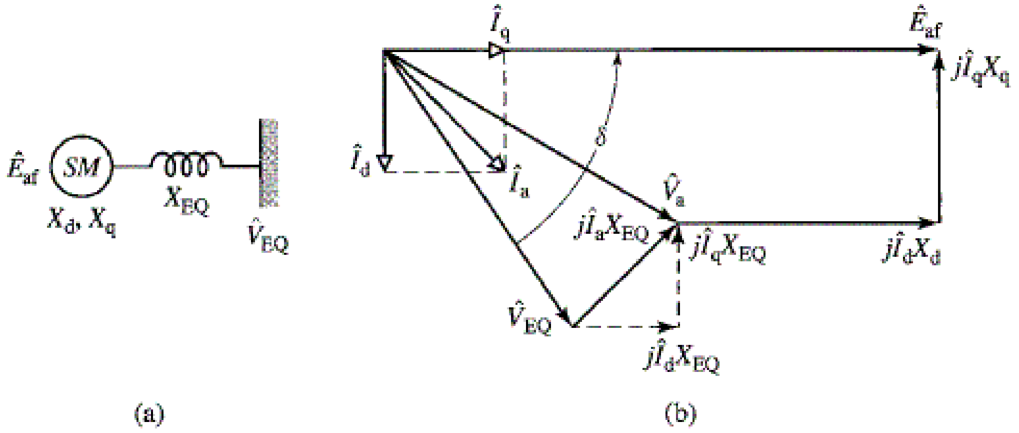


Figure 5.23 Salient-pole synchronous machine and series impedance: (a) single-line diagram and (b) phasor diagram.

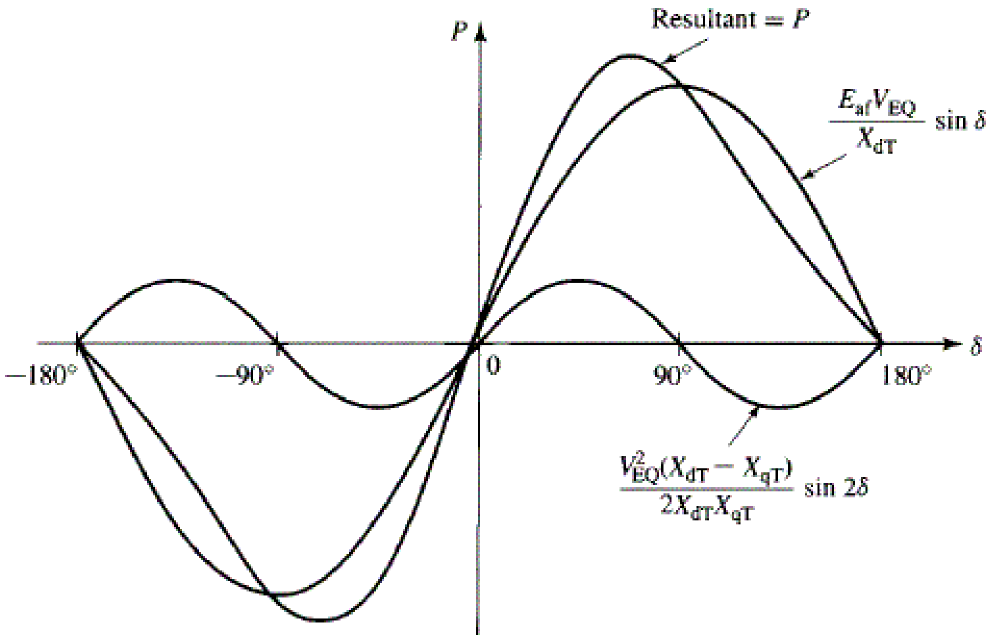


Figure 5.24 Power-angle characteristic of a salient-pole synchronous machine showing the fundamental component due to field excitation and the second-harmonic component due to reluctance torque.

- The general form of this power-angle characteristic is shown in Fig.5.24. The first term is the same as the expression obtained for a cylindrical-rotor machine. The second term includes the effect of salient poles. It represents the fact that the airgap flux wave creates torque, tending to align the field poles in the position of minimum reluctance. This term is the power corresponding to the reluctance torque and is of the same general nature as the reluctance torque. Note that the reluctance torque is independent of field excitation. Also note that, if $X_{\mathrm{dT}} = X_{\mathrm{qT}}$as in a uniform-air-gap machine, there is no preferential direction of magnetization, the reluctance torque is zero and Eq.5.55 reduces to the power-angle equation for a cylindrical-rotor machine.

5.8 Permanent-Magnet Ac Motors

- Permanent-magnet ac motors are polyphase synchronous motors with permanentmagnet rotors. Thus they are similar to the synchronous machines discussed up to this point in this chapter with the exception that the field windings are replaced by permanent magnets.

- Figure 5.25 is a schematic diagram of a three-phase permanent-magnet ac machine. Comparison of this figure with Fig.5.1 emphasizes the similarities between the permanent-magnet ac machine and the conventional synchronous machine. In fact, the permanent-magnet ac machine can be readily analyzed with the techniques of this chapter simply by assuming that the machine is excited by a field current of constant value, making sure to calculate the various machine inductances based on the effective permeability of the permanent-magnet rotor.

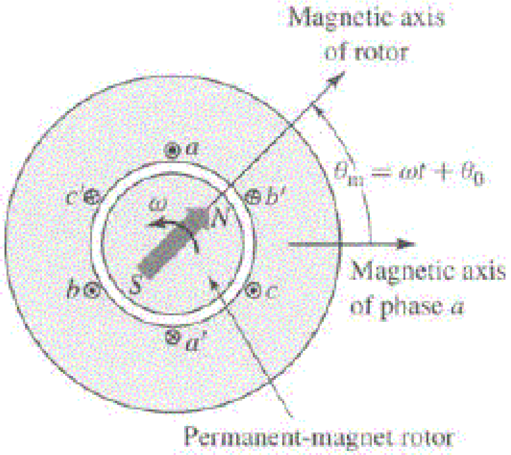


Figure 5.25 Schematic diagram of a three-phase permanent-magnet ac machine. The arrow indicates the direction of rotor magnetization.

- Figure 5.26 shows a cutaway view of a typical permanent-magnet ac motor. This figure also shows a speed and position sensor mounted on the rotor shaft. This sensor is used for control of the motor. A number of techniques may be used for shaft-position sensing, including Hall-effect devices, light-emitting diodes and phototransistors in combination with a pulsed wheel, and inductance pickups.

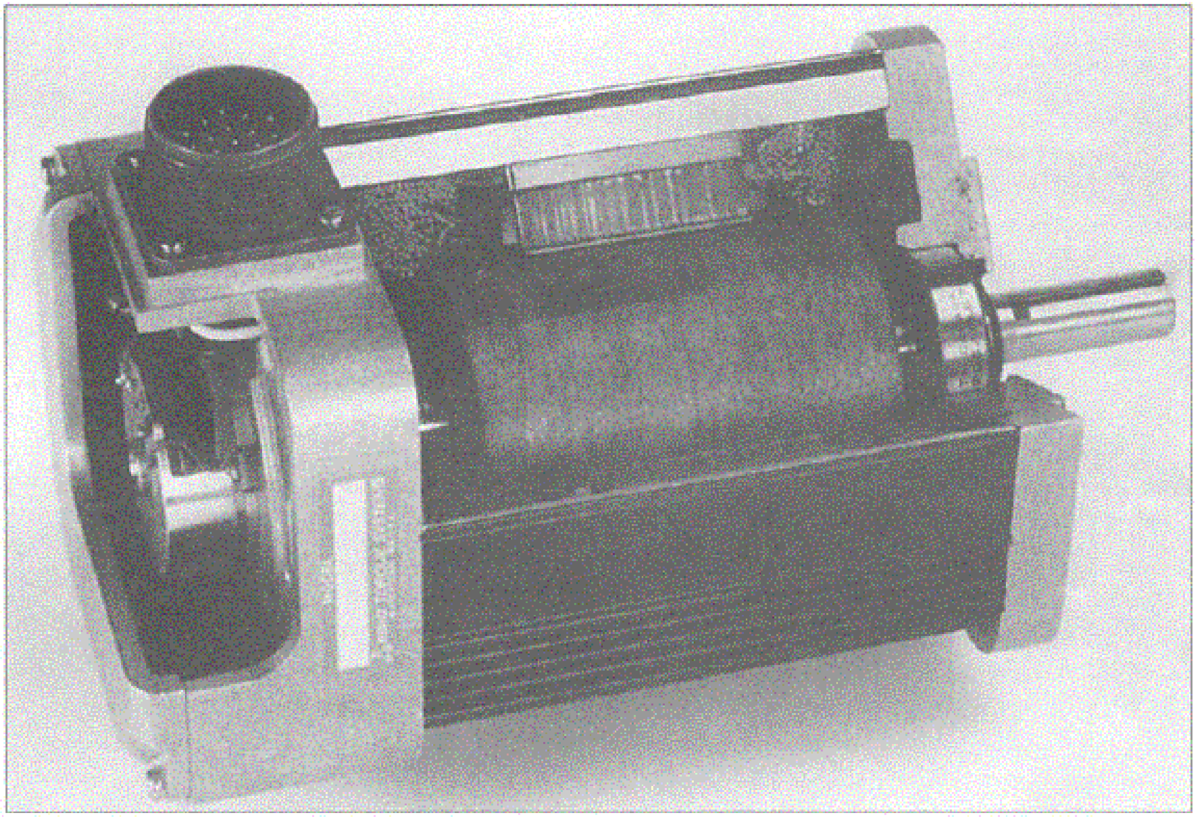


Figure 5.26 Cutaway view of a permanent-magnet ac motor. Also shown is the shaft speed and position sensor used to control the motor. (EG&G Torque Systems.)

- Permanent-magnet ac motors are typically operated from variable-frequency motor drives. Under conditions of constant-frequency, sinusiodal polyphase excitation, a permanent-magnet ac motor behaves similarly to a conventional ac synchronous machine with constant field excitation.
- An alternate viewpoint of a permanent-magnet ac motor is that it is a form of permanent-magnet stepping motor with a nonsalient stator. Under this viewpoint, the only difference between the two is that there will be little, if any, saliency (cogging) torque in the permanent-magnet ac motor. In the simplest operation, the phases can be simply excited with stepped waveforms so as to cause the rotor to step sequentially from one equilibrium position to the next. Alternatively, using rotor-position feedback from a shaft-position sensor, the motor

phase windings can be continuously excited in such a fashion as to control the torque and speed of the motor.

- As with the stepping motor, the frequency of the excitation determines the motor speed, and the angular position between the rotor magnetic axis and a given phase and the level of excitation in that phase determines the torque which will be produced.
- Permanent-magnet ac motors are frequently referred to as brushless motors or brushless dc motors. This terminology comes about both because of the similarity, when combined with a variable-frequency, variable-voltage drive system, of their speed-torque characteristics to those of dc motors and because of the fact that one can view these motors as inside-out dc motors, with their field winding on the rotor and with their armature electronically commutated

PROBLEMS - chapter 5

PROBLEMS

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

5.1 The full-load torque angle of a synchronous motor at rated voltage and frequency is 35 electrical degrees. Neglect the effects of armature resistance and leakage reactance. If the field current is held constant, how would the full-load torque angle be affected by the following changes in operating condition?

a. Frequency reduced 10 percent, load torque and applied voltage constant.

b. Frequency reduced 10 percent, load power and applied voltage constant.

c. Both frequency and applied voltage reduced 10 percent, load torque constant.

d. Both frequency and applied voltage reduced 10 percent, load power constant.

5.2 Design calculations show the following parameters for a three-phase, cylindrical-rotor synchronous generator:

Phase-a self-inductance L_{aa} = 4.83 mH

Armature leakage inductance L_{a1} = 0.33 mH

Calculate the phase-phase mutual inductance and the machine synchronous inductance.

5.3 The open-circuit terminal voltage of a three-phase, 60-Hz synchronous generator is found to be 15.4 kV rms line-to-line when the field current is 420 A.

a. Calculate the stator-to-rotor mutual inductance L_{af}.

b. Calculate the open-circuit terminal voltage if the field current is held constant while the generator speed is reduced so that the frequency of the generated voltage is 50 Hz.

5.4 A 460-V, 50-kW, 60-Hz, three-phase synchronous motor has a synchronous reactance of $X_S = 4.15\Omega$ and an armature-to-field mutual inductance, L_{af} = 83 mH. The motor is operating at rated terminal voltage and an input power of 40 kW. Calculate the magnitude and phase angle of the line-toneutral generated voltage $\hat{E}_{\mathrm{af}}$ and the field current I_f if the motor is operating at (a) 0.85 power factor lagging, (b) unity power factor, and (c) 0.85 power factor leading.

5.5 The motor of Problem 5.4 is supplied from a 460-V, three-phase source through a feeder whose impedance is Z_f = 0.084 + j0.82 Ω. Assuming the system (as measured at the source) to be operating at an input power of 40 kW, calculate the magnitude and phase angle of the line-to-neutral generated voltage $\hat{E}_{\mathrm{af}}$and the field current I_f for power factors of (a) 0.85 lagging, (b) unity, and (c) 0.85 leading.

5.6 A 50-Hz, two-pole, 750 kVA, 2300 V, three-phase synchronous machine has a synchronous reactance of 7.75 Ω and achieves rated open-circuit terminal voltage at a field current of 120 A.

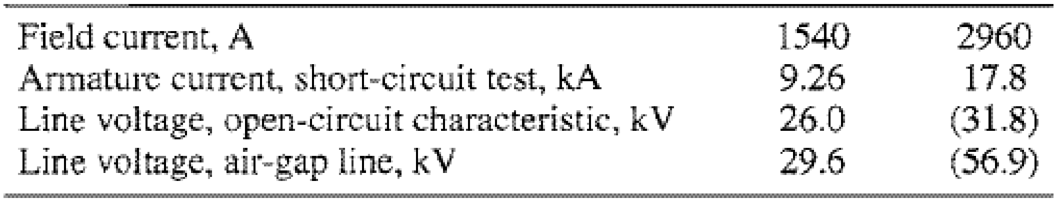
a. Calculate the armature-to-field mutual inductance.

b. The machine is to be operated as a motor supplying a 600 kW load at its rated terminal voltage. Calculate the internal voltage E_{af} and the corresponding field current if the motor is operating at unity power factor.

c. For a constant load power of 600 kW, write a MATLAB script to plot the terminal current as a function of field current. For your plot, let the field current vary between a minimum value corresponding to a machine loading of 750 kVA at leading power factor and a maximum value corresponding to a machine loading of 750 kVA at lagging power factor. What value of field current produces the minimum terminal current? Why?

5.7 The manufacturer's data sheet for a 26-kV, 750-MVA, 60-Hz, three-phase synchronous generator indicates that it has a synchronous reactance X_S = 2.04 and a leakage reactance X_{a1} = 0.18, both in per unit on the generator base. Calculate (a) the synchronous inductance in mH, (b) the armature leakage inductance in mH, and (c) the armature phase inductance L_{aa} in mH and per unit.

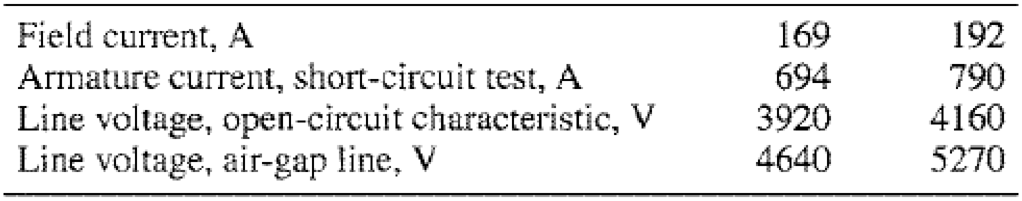
5.8 The following readings are taken from the results of an open- and a shortcircuit test on an 800-MVA, three-phase, Y-connected, 26-kV, two-pole, 60-Hz turbine generator driven at synchronous speed:



Field current, A	1540	2960
Armature current, short-circuit test, kA	9.26	17.8
Line voltage, open-circuit characteristic, kV	26.0	(31.8)
Line voltage, air-gap line, kV	29.6	(56.9)

The number in parentheses are extrapolations based upon the measured data. Find (a) the short-circuit ratio, (b) the unsaturated value of the synchronous reactance in ohms per phase and per unit, and (c) the saturated synchronous reactance in per unit and in ohms per phase.

5.9 The following readings are taken from the results of an open- and a short-circuit test on a 5000-kW, 4160-V, three-phase, four-pole, 1800-rpm synchronous motor driven at rated speed:



Field current, A	169	192
Armature current, short-circuit test, A	694	790
Line voltage, open-circuit characteristic, V	3920	4160
Line voltage, air-gap line, V	4640	5270

The armature resistance is 11 m Ω/phase. The armature leakage reactance is estimated to be 0.12 per unit on the motor rating as base. Find (a) the short-circuit ratio, (b) the unsaturated value of the synchronous reactance in ohms per phase and per unit, and (c) the saturated synchronous reactance in per unit and in ohms per phase.

5.10 Write a MATLAB script which automates the calculations of Problems 5.8 and 5.9. The following minimum set of data is required:

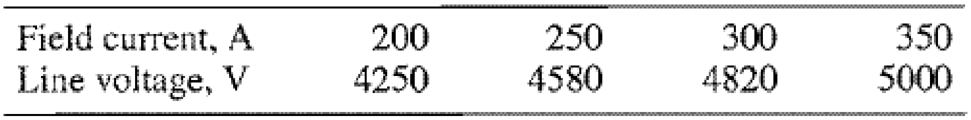
- AFNL: The field current required to achieve rated open-circuit terminal voltage.
- The corresponding terminal voltage on the air gap line.
- AFSC: The field current required to achieve rated short-circuit current on the short-circuit characteristic.

Your script should calculate (a) the short-circuit ratio, (b) the unsaturated value of the synchronous reactance in ohms per phase and per unit, and (c) the saturated synchronous reactance in per unit and in ohms per phase.

5.11 Consider the motor of Problem 5.9.

a. Compute the field current required when the motor is operating at rated voltage, 4200 kW input power at 0.87 power factor leading. Account for saturation under load by the method described in the paragraph relating to Eq. $X_s = \frac{V_{a,\text{rated}}}{I_a'}$.

b. In addition to the data given in Problem 5.9, additional points on the open-circuit characteristic are given below:



Field current, A	200	250	300	350
Line voltage, V	4250	4580	4820	5000

If the circuit breaker supplying the motor of part (a) is tripped, leaving the motor suddenly open-circuited, estimate the value of the motor terminal voltage following the trip (before the motor begins to slow down and before any protection circuitry reduces the field current).

5.12 Using MATLAB, plot the field current required to achieve unity-power-factor operation for the motor of Problem 5.9 as the motor load varies from zero to full load. Assume the motor to be operating at rated terminal voltage.

5.13 Loss data for the motor of Problem 5.9 are as follows:

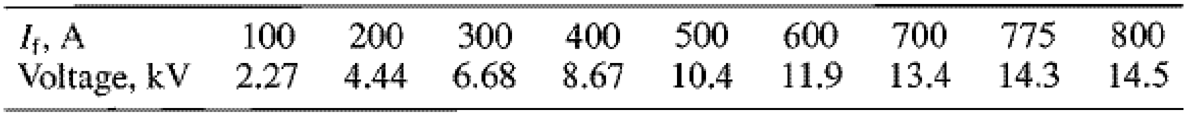
Open-circuit core loss at 4160 V = 37 kW

Friction and windage loss = 46 kW

Field-winding resistance at $75^0 C$ = 0.279 Ω

Compute the output power and efficiency when'the motor is operating at rated input power, unity power factor, and rated voltage. Assume the field-winding to be operating at a temperature of $125^0 C$.

5.14 The following data are obtained from tests on a 145-MVA, 13.8-kV, threephase, 60-Hz, 72-pole hydroelectric generator. Open-circuit characteristic:



I_f, A	100	200	300	400	500	600	700	775	800
Voltage, kV	2.27	4.44	6.68	8.67	10.4	11.9	13.4	14.3	14.5

Short-circuit test:

I_f = 710 A, I_a = 6070 A

a. Draw (or plot using MATLAB) the open-circuit saturation curve, the air-gap line, and the short-circuit characteristic.

b. Find AFNL and AFSC. (Note that if you use MATLAB for part (a), you can use the MATLAB function 'polyfit' to fit a second-order polynomial to the open-circuit saturation curve. You can then use this fit to find AFNL.)

c. Find (i) the short-circuit ratio, (ii) the unsaturated value of the synchronous reactance in ohms per phase and per unit and (iii) the saturated synchronous reactance in per unit and in ohms per phase.

5.15 What is the maximum per-unit reactive power that can be supplied by a synchronous machine operating at its rated terminal voltage whose synchronous reactance is 1.6 per unit and whose maximum field current is limited to 2.4 times that required to achieve rated terminal voltage under open-circuit conditions?

5.16 A 25-MVA, 11.5 kV synchronous machine is operating as a synchronous condenser, as discussed in Appendix D (section D.4.1). The generator short-circuit ratio is 1.68 and the field current at rated voltage, no load is 420 A. Assume the generator to be connected directly to an 11.5 kV source.

a. What is the saturated synchronous reactance of the generator in per unit and in ohms per phase?

The generator field current is adjusted to 150 A.

b. Draw a phasor diagram, indicating the terminal voltage, internal voltage, and armature current.

c. Calculate the armature current magnitude (per unit and amperes) and its relative phase angle with respect to the terminal voltage.

d. Under these conditions, does the synchronous condenser appear inductive or capacitive to the 11.5 kV system?

e. Repeat parts (b) through (d) for a field current of 700 A.

5.17 The synchronous condenser of Problem 5.16 is connected to a 11.5 kV system through a feeder whose series reactance is 0.12 per unit on the machine base. Using MATLAB, plot the voltage (kV) at the synchronous-condenser terminals as the synchronous-condenser field current is varied between 150 A and 700 A.

5.18 A synchronous machine with a synchronous reactance of 1.28 per unit is operating as a generator at a real power loading of 0.6 per unit connected to a system with a series reactance of 0.07 per unit. An increase in its field current is observed to cause a decrease in armature current.

a. Before the increase, was the generator supplying or absorbing reactive power from the power system?

b. As a result of this increase in excitation, did the generator terminal voltage increase or decrease?

c. Repeat parts (a) and (b) if the synchronous machine is operating as a motor.

5.19 Superconducting synchronous machines are designed with superconducting fields windings which can support large current densities and create large magnetic flux densities. Since typical operating magnetic flux densities exceed the saturation flux densities of iron, these machines are typically designed without iron in the magnetic circuit; as a result, these machines exhibit no saturation effects and have low synchronous reactances.

Consider a two-pole, 60-Hz, 13.8-kV, 10-MVA superconducting generator which achieves rated open-circuit armature voltage at a field current of 842 A. It achieves rated armature current into a three-phase terminal short circuit for a field current of 226 A.

a. Calculate the per-unit synchronous reactance. Consider the situation in which this generator is connected to a 13.8 kV distribution feeder of negligible impedance and operating at an output power of 8.75 MW at 0.9 pf lagging. Calculate:

b. the field current in amperes, the reactive-power output in MVA, and the rotor angle for this operating condition.

c. the resultant rotor angle and reactive-power output in MVA if the field current is reduced to 842 A while the shaft-power supplied by the prime mover to the generator remains constant.

5.20For a synchronous machine with constant synchronous reactance X_S operating at a constant terminal voltage V_t and a constant excitation voltage E_{af}, show that the locus of the tip of the armature-current phasor is a circle. On a phasor diagram with terminal voltage shown as the reference phasor, indicate the position of the center of this circle and its radius. Express the coordinates of the center and the radius of the circle in terms of $V_t, E_{\mathrm{af}}, X_S$.

5.21 A four-pole, 60-Hz, 24-kV, 650-MVA synchronous generator with a synchronous reactance of 1.82 per unit is operating on a power system which can be represented by a 24-kV infinite bus in series with a reactive impedance of j0.21 Ω. The generator is equipped with a voltage regulator that adjusts the field excitation such that the generator terminal voltage remains at 24 kV independent of the generator loading.

a. The generator output power is adjusted to 375 MW.

(i) Draw a phasor diagram for this operating condition.

(ii) Find the magnitude (in kA) and phase angle (with respect to the generator terminal voltage) of the terminal current.

(iii) Determine the generator terminal power factor.

(iv) Find the magnitude (in per unit and kV) of the generator excitation voltage E_{af}.

b. Repeat part (a) if the generator output power is increased to 600 MW.

5.22 The generator of Problem 5.21 achieves rated open-circuit armature voltage at a field current of 850 A. It is operating on the system of Problem 5.21 with its voltage regulator set to maintain the terminal voltage at 0.99 per unit (23.8 kV).

a. Use MATLAB to plot the generator field current (in A) as a function of load (in MW) as the load on the generator output power is varied from zero to full load.

b. Plot the corresponding reactive output power in MVAR as a function of output load.

c. Repeat the plots of parts (a) and (b) if the voltage regulator is set to regulate the terminal voltage to 1.01 per unit (24.2 kV).

5.23 The 145 MW hydroelectric generator of Problem 5.14 is operating on a 13.8-kV power system. Under normal operating procedures, the generator is operated under automatic voltage regulation set to maintain its terminal voltage at 13.8 kV. In this problem you will investigate the possible consequences should the operator forget to switch over to the automatic voltage regulator and instead leave the field excitation constant at AFNL, the value corresponding to rated open-circuit voltage. For the purposes of this problem, neglect the effects of saliency and assume that the generator can be represented by the saturated synchronous reactance found in Problem 5.14.

a. If the power system is represented simply by a 13.8 kV infinite (ignoring the effects of any equivalent impedance), can the generator be loaded to full load? If so, what is the power angle δ corresponding to full load? If not, what is the maximum load that can be achieved?

b. Repeat part (a) with the power system now represented by a 13.8 kV infinite bus in series with a reactive impedance of j0.14 Ω.

5.24 Repeat Problem 5.23 assuming that the saturated direct-axis synchronous inductance X_d is equal to that found in Problem 5.14 and that the saturated quadrature-axis synchronous reactance X_q is equal to 75 percent of this value. Compare your answers to those found in Problem 5.23.

5.25 Write a MATLAB script to plot a set of per-unit power-angle curves for a salient-pole synchronous generator connected to an infinite bus (V_{bus} = 1.0 per unit). The generator reactances are X_d = 1.27 per unit and X_q = 0.95 per unit. Assuming E_{af} = 1.0 per unit, plot the following curves:

a. Generator connected directly to the infinite bus.

b. Generator connected to the infinite bus through a reactance X_{bus} = 0.1 per unit.

c. Generator connected directly to the infinite bus. Neglect saliency effects, setting $X_d = X_q$.

d. Generator connected to the infinite bus through a reactance X_{bus} = 0.1 per unit. Neglect saliency effects, setting $X_q = X_d$.

5.26 Draw the steady-state, direct- and quadrature-axis phasor diagram for a salient-pole synchronous motor with reactances X_d and X_q and armature resistance R_a. From this phasor diagram, show that the torque angle δ between the generated voltage $\hat{E}_{\mathrm{af}}$ (which lies along the quadrature axis) and the terminal voltage $\hat{V}_t$ is given by

$$\tan\delta = \frac{I_a X_q \cos\varphi + I_a R_a \sin\varphi}{V_t + I_a X_q \sin\varphi - I_a R_a \cos\varphi}$$

Here φ is the phase angle of the armature current $\hat{I}_a$ and V_t, considered to be negative when $\hat{I}_a$ lags $\hat{V}_t$.

5.27 Repeat Problem 5.26 for synchronous generator operation, in which case the equation for δ becomes

$$\tan\delta = \frac{I_a X_q \cos\varphi + I_a R_a \sin\varphi}{V_t - I_a X_q \sin\varphi + I_a R_a \cos\varphi}$$

5.28 What maximum percentage of its rated output power will a salient-pole motor deliver without loss of synchronism when operating at its rated terminal voltage with zero field excitation (***SORRY, THIS MEDIA TYPE IS NOT SUPPORTED.*** = 0) if X_d = 0.90 per unit and X_q = 0.65 per unit? Compute the per-unit armature current and reactive power for this operating condition.

5.29 If the synchronous motor of Problem 5.28 is now operated as a synchronous generator connected to an infinite bus of rated voltage, find the minimum per-unit field excitation (where 1.0 per unit is the field current required to achieve rated open-circuit voltage) for which the generator will remain synchronized at (a) half load and (b) full load.

5.30 A salient-pole synchronous generator with saturated synchronous reactances ***SORRY, THIS MEDIA TYPE IS NOT SUPPORTED.*** =1.57 per unit and X_q = 1.34 per unit is connected to an infinite bus of rated voltage through an external impedance X_{bus} = 0.11 per unit. The generator is supplying its rated MVA at 0.95 power factor lagging, as measured at the generator terminals.

a. Draw a phasor diagram, indicating the infinite-bus voltage, the armature current, the generator terminal voltage, the excitation voltage, and the rotor angle (measured with respect to the infinite bus).

b. Calculate the per-unit terminal and excitation voltages, and the rotor angle in degrees.

5.31 A salient-pole synchronous generator with saturated synchronous reactances X_d = 0.78 per unit and X_q = 0.63 per unit is connected to a rated-voltage infinite bus through an external impedance X_{bus} = 0.09 per unit.

a. Assuming the generator to be supplying only reactive power

(i) Find minimum and maximum per-unit field excitations (where 1.0 per unit is the field current required to achieve rated open-circuit voltage) such that the generator does not exceed its rated terminal current.

(ii) Using MATLAB, plot the armature current as a function of field excitation as the field excitation is varied between the limits determined in part (i).

b. Now assuming the generator to be supplying 0.25 per unit rated real power, on the same axes add a plot of the per-unit armature current as a function of field excitation as the field current is varied in the range for which the per-unit armature current is less than 1.0 per unit.

c. Repeat part (b) for generator output powers of 0.50 and 0.75 per unit. The final result will be a plot of V-curves for this generator in this configuration.

5.32 A two-phase permanent-magnet ac motor has a rated speed of 3000 r/min and a six-pole rotor. Calculate the frequency (in Hz) of the armature voltage required to operate at this speed.

5.33 A 5-kW, three-phase, permanent-magnet synchronous generator produces an open-circuit voltage of 208 V line-to-line, 60-Hz, when driven at a speed of 1800 r/min. When operating at rated speed and supplying a resistive load, its terminal voltage is observed to be 192 V line-to-line for a power output of 4.5 kW.

a. Calculate the generator phase current under this operating condition.

b. Assuming the generator armature resistance to be negligible, calculate the generator 60-Hz synchronous reactance.

c. Calculate the generator terminal voltage which will result if the motor generator load is increased to 5 kW (again purely resistive) while the speed is maintained at 1800 r/min.

5.34 Small single-phase permanent-magnet ac generators are frequently used to generate the power for lights on bicycles. For this application, these generators are typically designed with a significant amount of leakage inductance in their armature winding. A simple model for these generators is an ac voltage source $e_a(t) = \omega K_a \cos \omega t$ in series with the armature leakage inductance La and the armature resistance R_a. Here ω is the electrical frequency of the generated voltage which is determined by the speed of the generator as it rubs against the bicycle wheel.

Assuming that the generator is running a light bulb which can be modeled as a resistance R_b, write an expression for the minimum frequency $\omega_{\min}$ which must be achieved in order to insure that the light operates at constant brightness, independent of the speed of the bicycle.

Chapter 6: Polyphase Induction Machines

Chapter 6: Polyphase Induction Machines

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- Study on the behavior of polyphase induction machines:

The analysis begins with the development of single-phase equivalent circuits.

- The general form is suggested by the similarity of an induction machine to a transformer.
- The equivalent circuits can be used to study the electromechanical characteristics of an induction machine as well as the loading presented by the machine on its supply source.

§6.1 Introduction to Polyphase Induction Machines

- An induction machine is one in which alternating current is supplied to the stator directly and to the rotor by induction or transformer action from the stator.

- The stator winding is excited from a balanced polyphase source and produces a magnetic field in the air gap rotating at synchronous speed.
- The rotor winding may one of two types.

- A wound rotor is built with a polyphase winding similar to, and wound with the same number of poles as, the stator. The rotor terminals are available external to the motor.
- A squirrel-cage rotor has a winding consisting of conductor bars embedded in slots in the rotor iron and short-circuited at each end buy conducting end rings. It is the most commonly used type of motor in sizes ranging from fractional horsepower on up.

- The difference between synchronous speed and the rotor speed is commonly referred to as the slip of the rotor. The fractional slip s is

$$s = \frac{n_s - n}{n_s} \text{ (6.1)}$$

- The slip is often expressed in percent.
- n : rotor speed in rpm

$$n = (1 - s)n_s \text{ (6.2)}$$

- ω_m: mechanical angular velocity

$$\omega_m = (1 - s)\omega_s \text{ (6.3)}$$

- f_r: the frequency of induced voltages, the slip frequency

$$f_r = \mathrm{sf}_e \text{ (6.4)}$$

–A wound-rotor induction machine can be used as a frequency changer.

The rotor currents produce an air-gap flux wave that rotates at synchronous speed and in synchronism with that produced by the stator currents.

- With the rotor revolving in the same direction of rotation as the stator field, the rotor currents produce a rotating flux wave rotating at sn_s with respect to the rotor in the forward direction.
- With respect to the stator, the speed of the flux wave produced by the rotor currents (with frequency sf_c) equals

$$\mathrm{sn}_s + n = \mathrm{sn}_s + n_s(1 - s) = n_s \text{ (6.5)}$$

- Because the stator and rotor fields each rotate synchronously, they are stationary with respect to each other and produce a steady torque, thus maintaining rotation of the rotor. Such torque is called an asynchronous torque.
- Equation (4.81) $T = -\frac{\pi}{2}\ \frac{\text{poles}}{2}^{2} \mathit{\Phi}_{\text{sr}} F_r \sin\delta_r$can be expressed in the form

$T=-\mathrm{KI}_r \sin \delta_r$ (6.6)

I_r: the rotor current

δ_r : the angle by which the rotor mmf wave leads the resultant air-gap mmf wave

- Fig. 6.1 shows a typical polyphase squirrel-cage induction motor torque-speed curve. The factors influencing the shape of this curve can be appreciated in terms of the torque equation.

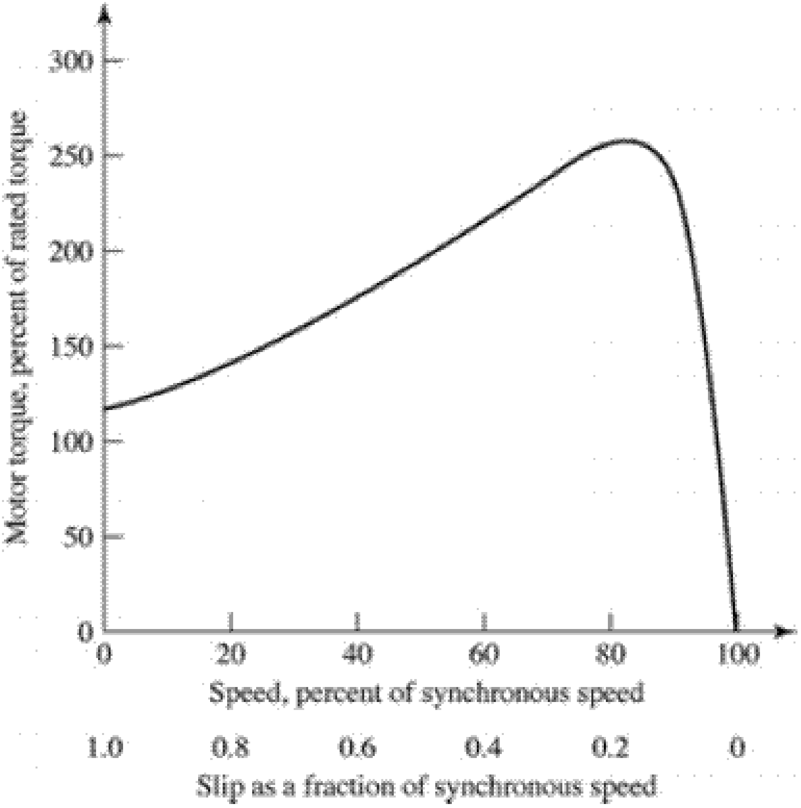


Figure 6.1 Typical induction-motor torque-speed

curve for constant-voltage, constant-frequency operation

- Under normal running conditions the slip is small: 2 to 10 percent at full load.
- The maximum torque is referred to as the breakdown torque.

- The slip at which the peak torque occurs is proportional to the rotor resistance.

§6.2 Currents and Fluxes in Polyphase Induction Machines

§6.3 Induction-Motor Equivalent Circuit

- Only machines with symmetric polyphase windings exited by balanced polyphase voltages are considered. It is helpful to think of three-phase machines as being Y-connected.

- Stator equivalent circuit:

$\hat{V}_1 = \hat{E}_2 + \hat{I}_1(R_1 + \mathrm{jX}_1)$ (6.7)

$\hat{V}_1 =$ Stator line-to-neutral terminal voltage

$\hat{E}_2 =$Counter emf (line-to-neutral) generated by the resultant air-gap flux

$\hat{I}_1 =$Stator current

$R_1 =$ Stator effective resistance

$X_1 =$Stator leakage reactance

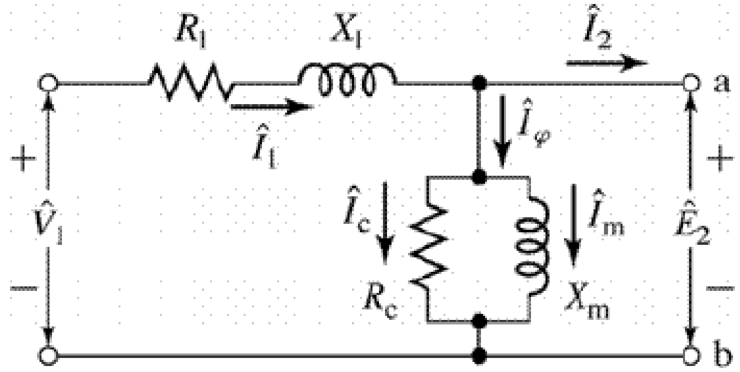


Figure 6.2 Stator equivalent circuit for a polyphase induction motor.

- Rotor equivalent circuit:

$Z_2 = \frac{\hat{E}_2}{\hat{I}_2}$ (6.8)

$Z_{2\mathrm{s}} = \frac{\hat{E}_{2\mathrm{s}}}{\hat{I}_{2\mathrm{s}}} N_{\mathrm{eff}}^2 (\frac{E_{\mathrm{rotor}}}{I_{\mathrm{rotor}}}) = N_{\mathrm{eff}}^2 Z_{\mathrm{rotor}}$ (6.9)

: the slip-frequency leakage impedance of the equivalent rotor

Z_{rotor}: the slip-frequency leakage impedance

$Z_{2\mathrm{s}} = \frac{\hat{E}_{2\mathrm{s}}}{\hat{I}_{2\mathrm{s}}} = R_2 + \mathrm{jsX}_2$ (6.10)

$R_2 =$Referred rotor resistance

$\mathrm{sR}_2 =$Referred rotor leakage reactance at slip frequency

$X_2 =$Referred rotor leakage reactance at stator frequency f_c

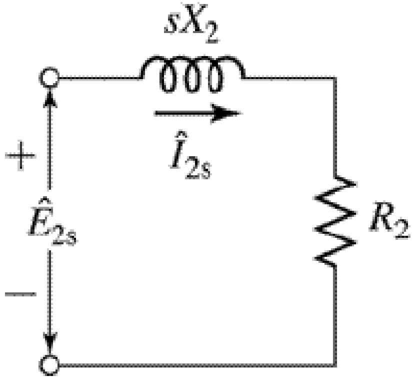


Figure 6.3 Rotor equivalent circuit for a polyphase induction motor at slip frequency.

$\hat{I}_{2\mathrm{s}} = \hat{I}_2$ (6.11)

$E_{2\mathrm{s}} = \mathrm{sE}_2$ (6.12)

$\hat{E}_{2\mathrm{s}} = s\hat{E}_2$ (6.13)

$\frac{\hat{E}_{2\mathrm{s}}}{\hat{I}_{2\mathrm{s}}} = \frac{s\hat{E}_2}{\hat{I}_2} = Z_{2\mathrm{s}} = R_2 + \mathrm{jsX}_2$ (6.14)

$Z_2 = \frac{\hat{E}_2}{\hat{I}_2} = \frac{R_2}{s} + \mathrm{jX}_2$ (6.15)

- Fig. 6.4 shows the single-phase equivalent circuit.

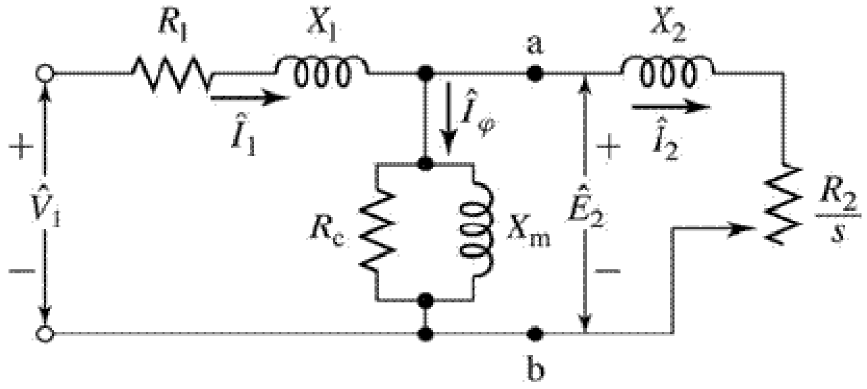


Figure 6.4 Single-phase equivalent circuit for a polyphase induction motor.

§6.4 Analysis of the Equivalent Circuit

- The single-phase equivalent circuit can be used to determine a wide variety of steady-state performance characteristics of polyphase induction machines.

- P_{gap}: the total power transferred across the air gap from the stator

P_{rotor}: the total rotor ohmic loss

$P_{\mathrm{gap}} = n_{\mathrm{ph}} I_2^2 (\frac{R_2}{s})$ (6.16)

$P_{\mathrm{rotor}} = n_{\mathrm{ph}} I_{2\mathrm{s}}^{s} R_2$ (6.17)

$P_{\mathrm{rotor}} = n_{\mathrm{ph}} I_2^2 R_2$ (6.18)

$P_{\mathrm{mech}} = P_{\mathrm{gap}} - P_{\mathrm{rotor}} = n_{\mathrm{ph}} I_2^2 (\frac{R_2}{s}) - n_{\mathrm{ph}} I_2^2 R_2$ (6.19)

$P_{\mathrm{mech}} = n_{\mathrm{ph}} I_2^2 R_2 (\frac{1-s}{s})$ (6.20)

$P_{\mathrm{mech}} = (1 - s) P_{\mathrm{gap}}$ (6.21)

- Of the total power delivered across the air gap to the rotor, the fraction 1   s is converted to mechanical power and the fraction s is dissipated as ohmic loss in the rotor conductors.
- When power aspects are to be emphasized, the equivalent circuit can be redrawn in the manner of Fig. 6.5.

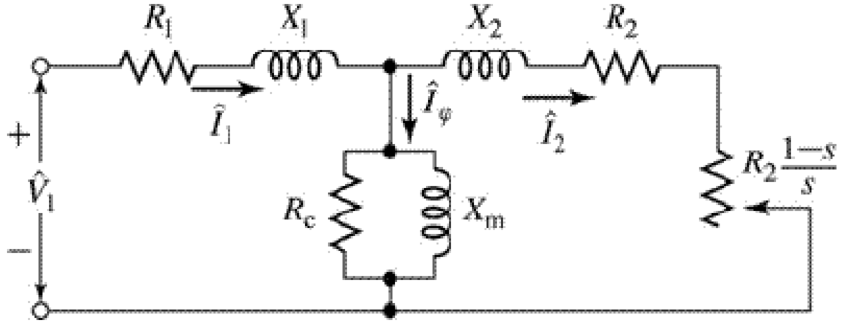


Figure 6.5 Alternative form of equivalent circuit.

Consider the electromechanical torque T_{mech} .

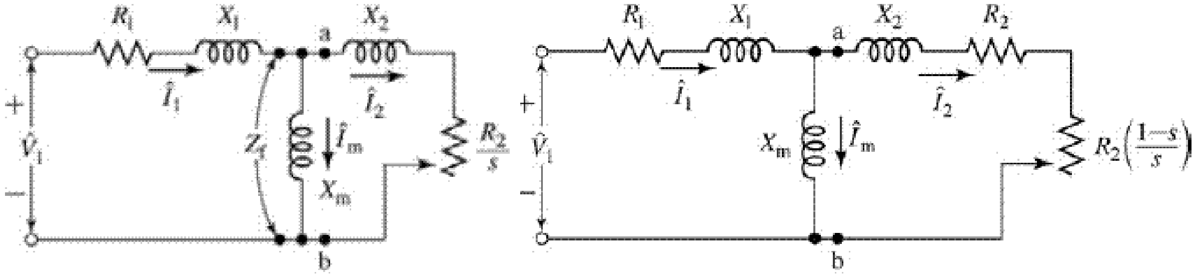
$$P_{\text{mech}} = \omega_m T_{\text{mech}} = (1-s)\omega_s T_{\text{mech}} \text{ (6.22)}$$

$$T_{\text{mech}} = \frac{P_{\text{mech}}}{\omega_m} = \frac{P_{\text{gap}}}{\omega_s} = \frac{n_{\text{ph}} I_2^2 (R_2/s)}{\omega_s} \text{ (6.23)}$$

$$\omega_s = \frac{4\pi f_e}{\text{poles}} = \left[\frac{2}{\text{poles}}\right]\omega_e \text{ (6.24)}$$

$$P_{\text{shaft}} = P_{\text{mech}} - P_{\text{rot}} \text{ (6.25)}$$

$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_m} = T_{\text{mech}} - T_{\text{rot}} \text{ (6.26)}$$



(a) (b)

Figure 6.6 Equivalent circuits with the core-loss resistance $R_c$neglected .

§6.5 Torque and Power by Use of Thevenin’s Theorem

- Considerable simplification will be obtained from application of Thevenin’s network theorem to the induction-motor equivalent circuit.

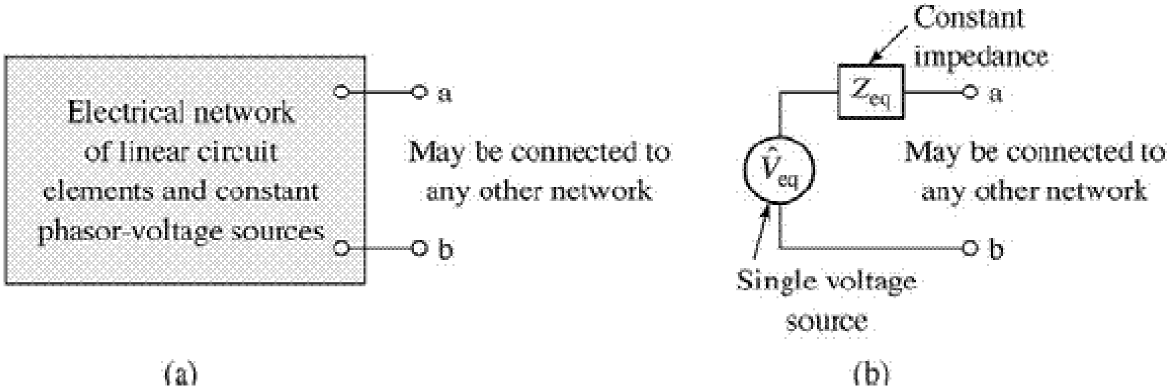

Electrical network of linear circuit elements and constant phasor-voltage sources
a
b
May be connected to any other network
(a)
Constant impedance
Z_{eq}
a
$\hat{V}_{eq}$
May be connected to any other network
b
Single voltage source
(b)

Figure 6.7 (a) General linear network and

(b) its equivalent at terminals ab by Thevenin’s theorem.

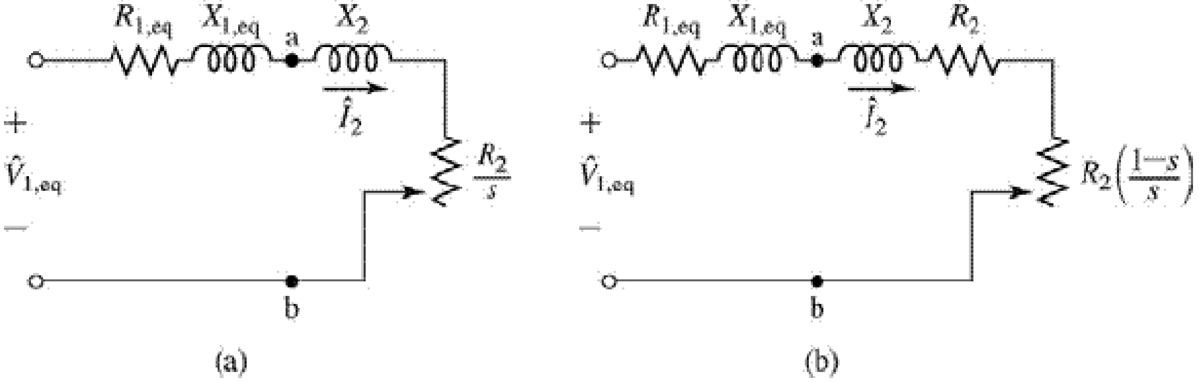


Figure 6.8 Induction-motor equivalent circuits simplified by Thevenin’s theorem.

$$\hat{V}_{1,\text{eq}} = \hat{V}_1 \left[\frac{\mathrm{jX}_m}{R_1+j(X_1+X_m)}\right] \text{ (6.27)}$$

$$Z_{1,\text{eq}} = R_{1,\text{eq}} + \mathrm{jX}_{1,\text{eq}} = (R_1 + \mathrm{jX}_1)\text{in parallel}\mathrm{jX}_m$$

$$Z_{1,\text{eq}} = \hat{V}_1 \frac{\mathrm{jX}_m(R_1+\mathrm{jX}_1)}{R_1+j(X_1+X_m)} \text{ (6.28)}$$

$\hat{I}_2 = \frac{\hat{V}_{1,\text{eq}}}{Z_{1,\text{eq}}+\text{jX}_2+R_2/s}$ (6.29)

$T_{\text{mech}} = \frac{1}{\omega_s}\left[\frac{n_{\text{ph}}V_{1,\text{eq}}^2(R_2/s)}{(R_{1,\text{eq}}+(R_2/s))^2+(X_{1,\text{eq}}+X_2)^2}\right]$ (6.30)

- The general shape of the torque-speed or torque-slip curve with motor connected to a constant-voltage, constant-frequency source is shown in Figs. 6.9 and 6.10.

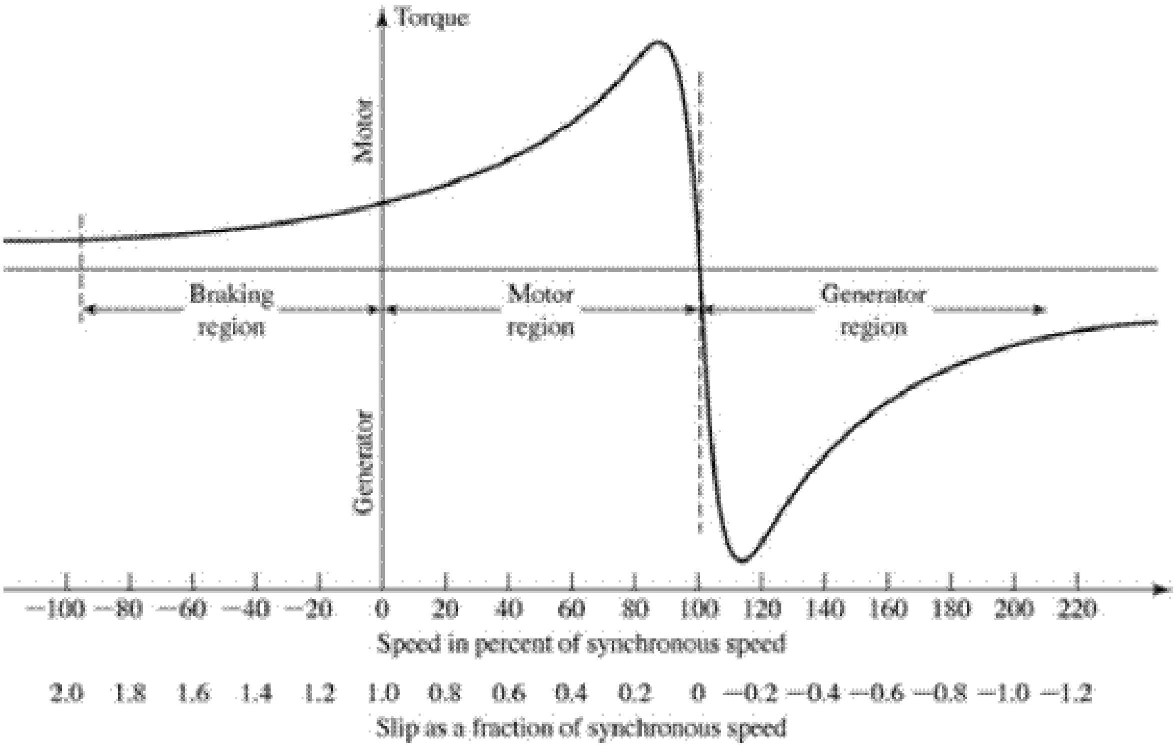


Figure 6.9 Induction-machine torque-slip curve showing braking, motor, and generator regions.

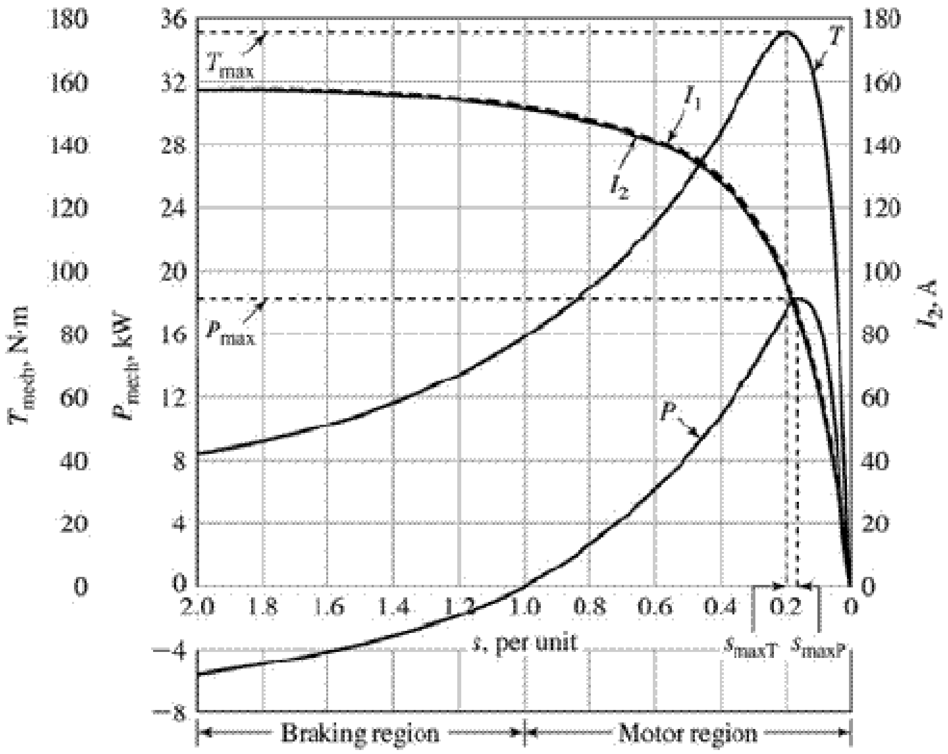


Figure 6.10 Computed torque, power, and current curves for the 7.5-kW motor in Exps 6.2 and 6.3.

- Maximum electromechanical torque will occur at a value of slip $s_{\max \mathrm{T}}$ for which

$$\frac{R_2}{s_{\max T}} = \sqrt{R_{1,\text{eq}}^2 + (X_{1,\text{eq}} + X_2)^2} \text{ (6.31)}$$

$$s_{\max T} = \frac{R_2}{\sqrt{R_{1,\text{eq}}^2 + (X_{1,\text{eq}} + X_2)^2}} \text{ (6.32)}$$

$$T_{\max} = \frac{1}{\omega_s}\left[\frac{0.5\mathrm{n}_{\mathrm{ph}} V_{1,\text{eq}}^2}{R_{1,\text{eq}} + \sqrt{R_{1,\text{eq}}^2 + (X_{1,\text{eq}} + X_2)^2}}\right] \text{ (6.33)}$$

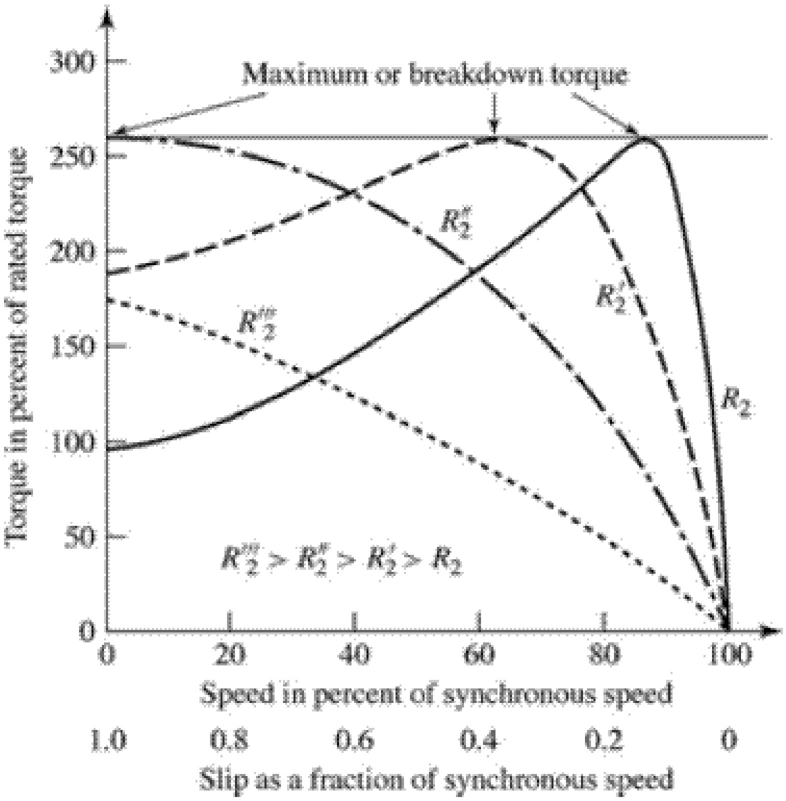


Figure 6.11 Induction-motor torque-slip curves showing effect of changing rotor-circuit resistance.

§6.5Parameter Determination from No-Load and Blocked-Rotor Tests

- The equivalent-circuit parameters needed for computing the performance of a poly-phase induction motor under load can be obtained from the results of a no-load test, a blocked-rotor test, and measurement of the dc resistances of the stator windings.

§6.6.1 No-Load Test

- Like the open-circuit test on a transformer, the no-load test on an induction motor gives information with respect to exciting current and no-load losses.

§6.6.2 Blocked-Rotor Test

- Like the short-circuit test on a transformer, the blocked-rotor test on an induction motor give information with respect to the leakage impedances.

PROBLEMS - chapter 6

PROBLEMS

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

6.1 The nameplate on a 460-V, 50-hp, 60-Hz, four-pole induction motor indicates that its speed at rated load is 1755 r/min. Assume the motor to be operating at rated load.

a. What is the slip of the rotor?

b. What is the frequency of the rotor currents?

c. What is the angular velocity of the stator-produced air-gap flux wave with respect to the stator? With respect to the rotor?

d. What is the angular velocity of the rotor-produced air-gap flux wave with respect to the stator? With respect to the rotor?

6.2 Stray leakage fields will induce rotor-frequency voltages in a pickup coil mounted along the shaft of an induction motor. Measurement of the frequency of these induced voltages can be used to determine the rotor speed.

a. What is the rotor speed in r/min of a 50-Hz, six-pole induction motor if the frequency of the induced voltage is 0.89 Hz?

b. Calculate the frequency of the induced voltage corresponding to a four-pole, 60-Hz induction motor operating at a speed of 1740 r/min. What is the corresponding slip?

6.3A three-phase induction motor runs at almost 1198 r/min at no load and 1112 r/min at full load when supplied from a 60-Hz, three-phase source.

a. How many poles does this motor have?

b. What is the slip in percent at full load?

c. What is the corresponding frequency of the rotor currents?

d. What is the corresponding speed of the rotor field with respect to the rotor? With respect to the stator?

6.4 Linear induction motors have been proposed for a variety of applications including high-speed ground transportation. A linear motor based on the induction-motor principle consists of a car riding on a track. The track is a developed squirrel-cage winding, and the car, which is 4.5 m long and 1.25 m wide, has a developed three-phase, 12-pole-pair armature winding. Power at 75 Hz is fed to the car from arms extending through slots to rails below ground level.

a. What is the synchronous speed in km/hr?

b. Will the car reach this speed? Explain your answer.

c. What is the slip if the car is traveling 95 km/hr? What is the frequency of the track currents under this condition?

d. If the control system controls the magnitude and frequency of the car currents to maintain constant slip, what is the frequency of the armaturewinding currents when the car is traveling 75 km/hr? What is the frequency of the track currents under this condition?

6.5 A three-phase, variable-speed induction motor is operated from a variablefrequency, variable-voltage source which is controlled to maintain constant peak air-gap flux density as the frequency of the applied voltage is varied. The motor is to be operated at constant slip frequency while the motor speed is varied between one half rated speed and rated speed.

a. Describe the variation of magnitude and frequency of the applied voltage with speed.

b. Describe how the magnitude and frequency of the rotor currents will vary as the motor speed is varied.

c. How will the motor torque vary with speed?

6.6 Describe the effect on the torque-speed characteristic of an induction motor produced by (a) halving the applied voltage and (b) halving both the applied voltage and the frequency. Sketch the resultant torque-speed curves relative to that of rated-voltage and rated-frequency. Neglect the effects of stator resistance and leakage reactance.

6.7 Figure 6.1 shows a system consisting of a three-phase wound-rotor induction machine whose shaft is rigidly coupled to the shaft of a three-phase synchronous motor. The terminals of the three-phase rotor winding of the induction machine are brought out to slip rings as shown. With the system supplied from a three-phase, 60-Hz source, the induction machine is driven by the synchronous motor at the proper speed and in the proper direction of rotation so that three-phase, 120-Hz voltages appear at the slip rings. The induction motor has four-pole stator winding.

a. How many poles are on the rotor winding of the induction motor?

b. If the stator field in the induction machine rotates in a clockwise direction, what is the rotation direction of its rotor?

c. What is the rotor speed in r/min?

d. How many poles are there on the synchronous motor?

e. It is proposed that this system can produce dc voltage by reversing two of the phase leads to the induction motor stator. Is this proposal valid?

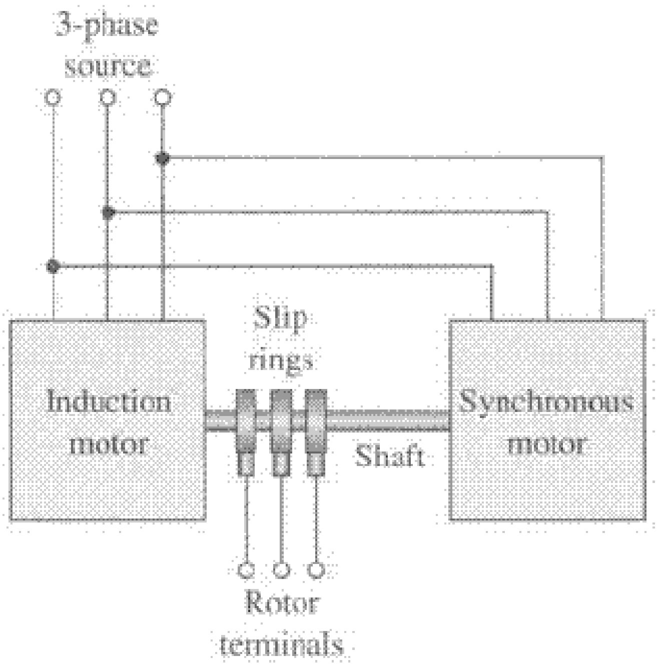


Figure 6.1 Interconnected induction and synchronous machines.

6.8 A system such at that shown in Fig. 6.1 is used to convert balanced 50-Hz voltages to other frequencies. The synchronous motor has four poles and drives the interconnected shaft in the clockwise direction. The induction machine has six poles and its stator windings are connected to the source in such a fashion as to produce a counterclockwise rotating field (in the direction opposite to the rotation of the synchronous motor). The machine has a wound rotor whose terminals are brought out through slip rings.

a. At what speed does the motor run?

b. What is the frequency of the voltages produced at the slip rings of the induction motor?

c. What will be the frequency of the voltages produced at the slip rings of the induction motor if two leads of the induction-motor stator are interchanged, reversing the direction of rotation of the resultant rotating field?

6.9 A three-phase, eight-pole, 60-Hz, 4160-V, 1000-kW squirrel-cage induction motor has the following equivalent-circuit parameters in ohms per phase Y referred to the stator:

R_1 = 0.220 R_2 = 0.207 X_1 = 1.95 X_2 = 2.42 X_m = 45.7

Determine the changes in these constants which will result from the following proposed design modifications. Consider each modification separately.

a. Replace the stator winding with an otherwise identical winding with a wire size whose cross-sectional area is increased by 4 percent.

b. Decrease the inner diameter of the stator laminations such that the air gap is decreased by 15 percent.

c. Replace the aluminum rotor bars (conductivity 3.5×10^7 mhos/m) with copper bars (conductivity 5.8×10^7 mhos/m).

d. Reconnect the stator winding, originally connected in Y for 4160-V operation, in Δ for 2.4 kV operation.

6.10 A three-phase, Y-connected, 460-V (line-line), 25-kW, 60-Hz, four-pole induction motor has the following equivalent-circuit parameters in ohms per phase referred to the stator:

R_1=0.103 R_2=0.225 X_1=1.10 X_2=1.13 X_m=59.4

The total friction and windage losses may be assumed constant at 265 W, and the core loss may be assumed to be equal to 220 W. With the motor connected directly to a 460-V source, compute the speed, output shaft torque and power, input power and power factor and efficiency for slips of 1, 2 and 3 percent. You may choose either to represent the core loss by a resistance connected directly across the motor terminals or by resistance R_C connected in parallel with the magnetizing reactance X_m.

6.11 Consider the induction motor of Problem 6.10.

a. Find the motor speed in r/min corresponding to the rated shaft output power of 25 kW. (Hint: This can be easily done by writing a MATLAB script which searches over the motor slip.)

b. Similarly, find the speed in r/min at which the motor will operate with no external shaft load (assuming the motor load at that speed to consist only of the friction and windage losses).

c. Write a MATLAB script to plot motor efficiency versus output power as the motor output power varies from zero to full load.

d. Make a second plot of motor efficiency versus output power as the motor output power varies from roughly 5 kW to full load.

6.12 Write a MATLAB script to analyze the performance of a three-phase induction motor operating at its rated frequency and voltage. The inputs should be the rated motor voltage, power and frequency, the number of poles, the equivalent-circuit parameters, and the rotational loss. Given a specific speed, the program should calculate the motor output power, the input power and power factor and the motor efficiency. Exercise your program on a 500-kW, 4160 V, three-phase, 60-Hz, four-pole induction motor operating at 1725 r/min whose rated speed rotational loss is 3.5 kW and whose equivalent-circuit parameters are:

R_1 =0.521 R_2=1.32 X_1 =4.98 X_2=5.32 X_m=136

6.13 A 15-kW, 230-V, three-phase, Y-connected, 60-Hz, four-pole squirrel-cage induction motor develops full-load internal torque at a slip of 3.5 percent when operated at rated voltage and frequency. For the purposes of this problem, rotational and core losses can be neglected. The following motor parameters, in ohms per phase, have been obtained:

R_1= 0.21 $X_1 = X_2$ = 0.26 X_m = 10.1

Determine the maximum internal torque at rated voltage and frequency, the slip at maximum torque, and the internal starting torque at rated voltage and frequency.

6.14 The induction motor of Problem 6.13 is supplied from a 230-V source through a feeder of impedance Z_f = 0.05 + j0.14 ohms. Find the motor slip and terminal voltage when it is supplying rated load.

6.15 A three-phase induction motor, operating at rated voltage and frequency, has a starting torque of 135 percent and a maximum torque of 220 percent, both with respect to its rated-load torque. Neglecting the effects of stator resistance and rotational losses and assuming constant rotor resistance, determine:

a. the slip at maximum torque.

b. the slip at rated load.

c. the rotor current at starting (as a percentage of rotor current at rated load).

6.16 When operated at rated voltage and frequency, a three-phase squirrel-cage induction motor (of the design classification known as a high-slip motor) delivers full load at a slip of 8.7 percent and develops a maximum torque of 230 percent of full load at a slip of 55 percent. Neglect core and rotational losses and assume that the rotor resistance and inductance remain constant, independent of slip. Determine the torque at starting, with rated voltage and frequency, in per unit based upon its full-load value.

6.17 A 500-kW, 2400-V, four-pole, 60-Hz induction machine has the following equivalent-circuit parameters in ohms per phase Y referred to the stator:

R_1 = 0.122 R_2 = 0.317 X_1 = 1.364 X_2 = 1.32 X_m = 45.8

It achieves rated shaft output at a slip of 3.35 percent with an efficiency of 94.0 percent. The machine is to be used as a generator, driven by a wind turbine. It will be connected to a distribution system which can be represented by a 2400-V infinite bus.

a. From the given data calculate the total rotational and core losses at rated load.

b. With the wind turbine driving the induction machine at a slip of -3.2 percent, calculate (i) the electric power output in kW, (ii) the efficiency (electric power output per shaft input power) in percent and (iii) the power factor measured at the machine terminals.

c. The actual distribution system to which the generator is connected has an effective impedance of 0.18 + j0.41 Ω/phase. For a slip of -3.2 percent, calculate the electric power as measured (i) at the infinite bus and (ii) at the machine terminals.

6.18 Write a MATLAB script to plot the efficiency as a function of electric power output for the induction generator of Problem 6.17 as the slip varies from -0.5 to -3.2 percent. Assume the generator to be operating into the system with the feeder impedance of part (c) of Problem 6.17.

6.19 For a 25-kW, 230-V, three-phase, 60-Hz squirrel-cage motor operating at rated voltage and frequency, the rotor I^2R loss at maximum torque is 9.0 times that at full-load torque, and the slip at full-load torque is 0.023. Stator resistance and rotational losses may be neglected and the rotor resistance and inductance assumed to be constant. Expressing torque in per unit of the full-load torque, find

a. the slip at maximum torque.

b. the maximum torque.

c. the starting torque.

6.20 A squirrel-cage induction motor runs at a full-load slip of 3.7 percent. The rotor current at starting is 6.0 times the rotor current at full load. The rotor resistance and inductance is independent of rotor frequency and rotational losses, stray-load losses and stator resistance may be neglected. Expressing torque in per unit of the full-load torque, compute

a. the starting torque.

b. the maximum torque and the slip at which the maximum torque occurs.

6.21 A A-connected, 25-kW, 230-V, three-phase, six-pole, 50-Hz squirrel-cage induction motor has the following equivalent-circuit parameters in ohms per phase Y:

R_1 = 0.045 R_2 = 0.054 X_1 = 0.29 X_2 = 0.28 X_m = 9.6

a. Calculate the starting current and torque for this motor connected directly to a 230-V source.

b. To limit the starting current, it is proposed to connect the stator winding in Y for starting and then to switch to the Δ connection for normal operation. (i) What are the equivalent-circuit parameters in ohms per phase for the Y connection? (ii) With the motor Y-connected and running directly off of a 230-V source, calculate the starting current and torque.

6.22 The following data apply to a 125-kW, 2300-V, three-phase, four pole, 60-Hz squirrel-cage induction motor:

Stator-resistance between phase terminals = 2.23 Ω

No-load test at rated frequency and voltage:

Line current = 7.7 A Three-phase power = 2870 W

Blocked-rotor test at 15 Hz:

Line voltage = 268 V Line current = 50.3 A

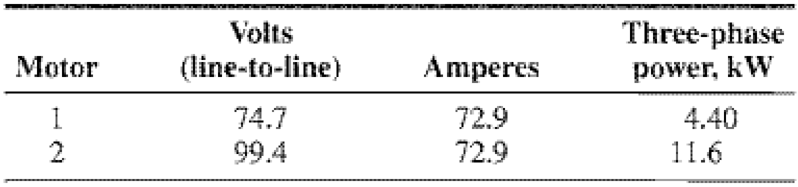
Three-phase power = 18.2 kW

a. Calculate the rotational losses.

b. Calculate the equivalent-circuit parameters in ohms. Assume that $X_1 = X_2$.

c. Compute the stator current, input power and power factor, output power and efficiency when this motor is operating at rated voltage and frequency at a slip of 2.95 percent.

6.23 Two 50-kW, 440-V, three-phase, six-pole, 60-Hz squirrel-cage induction motors have identical stators. The dc resistance measured between any pair of stator terminals is 0.204 Ω. Blocked-rotor tests at 60-Hz produce the following results:



Motor	Volts (line-to-line)	Amperes	Three-phase power, kW
1	74.7	72.9	4.40
2	99.4	72.9	11.6

Determine the ratio of the internal starting torque developed by motor 2 to that of motor 1 (a) for the same current and (b) for the same voltage. Make reasonable assumptions.

6.24 Write a MATLAB script to calculate the parameters of a three-phase induction motor from open-circuit and blocked-rotor tests.

Input:

Rated frequency

Open-circuit test: Voltage, current and power

Blocked-rotor test: Frequency, voltage, current and power

Stator-resistance measured phase to phase

Assumed ratio $\frac{X_1}{X_2}$

Output:

Rotational loss

Equivalent circuit parameters R_1, R_2, X_1, X_2, X_m

Exercise your program on a 2300-V, three-phase, 50-Hz, 250-kW

induction motor whose test results are:

Stator-resistance between phase terminals = 0.636 Ω

No-load test at rated frequency and voltage:

Line current = 20.2 A Three-phase power = 3.51 kW

Blocked-rotor test at 12.5 Hz:

Line voltage = 142 V Line current = 62.8 A

Three-phase power = 6.55 kW

You may assume that $X_1 = 0.4(X_1 + X_2)$.

6.25 A 230-V, three-phase, six-pole, 60-Hz squirrel-cage induction motor develops a maximum internal torquelof 288 percent at a slip of 15 percent when operated at rated voltage and frequency. If the effect of stator resistance is neglected, determine the maximum internal torque that this motor would develop if it were operated at 190 V and 50 Hz. Under these conditions, at what speed would the maximum torque be developed?

6.26 A 75-kW, 50-Hz, four-pole, 460-V three-phase, wound-rotor induction motor develops full-load torque at 1438 r/min with the rotor short-circuited. An external non-inductive resistance of 1.1 Ω is placed in series with each phase of the rotor, and the motor is observed to develop its rated torque at a speed of 1405 r/min. Calculate the rotor resistance per phase of the motor itself.

6.27 A 75-kW, 460-V, three-phase, four-pole, 60-Hz, wound-rotor induction motor develops a maximum internal torque of 225 percent at a slip of 16 percent when operated at rated voltage and frequency with its rotor short-circuited directly at the slip rings. Stator resistance and rotational losses may be neglected, and the rotor resistance and inductance may be assumed to be constant, independent of rotor frequency. Determine

a. the slip at full load in percent.

b. the rotor I^2R loss at full load in watts.

c. the starting torque at rated voltage and frequency in per unit and in N. m. If the rotor resistance is doubled (by inserting external series resistance at the slip rings) and the motor load is adjusted for such that the line current is equal to the value corresponding to rated load with no external resistance, determine

d. the corresponding slip in percent and

e. the torque in N. m.

6.28 Neglecting any effects of rotational and core losses, use MATLAB to plot the internal torque versus speed curve for the induction motor of Problem 6.10 for rated-voltage, rated-frequency operation. On the same plot, plot curves of internal torque versus speed for this motor assuming the rotor resistance increases by a factor of 2, 5 and 10.

6.29 A 100-kW, three-phase, 60-Hz, 460-V, six-pole wound-rotor induction motor develops its rated full-load output at a speed of 1158 r/min when operated at rated voltage and frequency with its slip rings short-circuited. The maximum torque it can develop at rated voltage and frequency is 310 percent of full-load torque. The resistance of the rotor winding is 0.17 Ω /phase Y. Neglect any effects of rotational and stray-load loss and stator resistance.

a. Compute the rotor I^2R loss at full load.

b. Compute the speed at maximum torque in r/min.

c. How much resistance must be inserted in series with the rotor windings to produce maximum starting torque?

With the rotor windings short-circuited, the motor is now run from a 50-Hz supply with the applied voltage adjusted so that the air-gap flux wave is essentially equal to that at rated 60-Hz operation.

d. Compute the 50-Hz applied voltage.

e. Compute the speed at which the motor will develop a torque equal to its rated 60Hz value with its slip-tings shorted.

6.30 A 460-V, three-phase, six-pole, 60-Hz, 150-kW, wound-rotor induction motor develops an internal torque of 190 percent with a line current of 200 percent (torque and current expressed as a percentage of their full-load values) at a slip of 5.6 percent when running at rated voltage and frequency with its rotor terminals short-circuited. The rotor resistance is measured to be 90 m Ω between each slip ring and may be assumed to remain constant. A balanced set of Y-connected resistors is to be connected to the slip rings in order to limit the rated-voltage starting current to 200 percent of its rated value. What resistance must be chosen for each leg of the Y connection? What will be the starting torque under these conditions?

6.31 The resistance measured between each pair of slip rings of a three-phase, 60-Hz, 250-kW, 16-pole, wound-rotor induction motor is 49 m Ω. With the slip tings short-circuited, the full-load slip is 0.041. For the purposes of this problem, it may be assumed that the slip-torque curve is a straight line from no load to full load. The motor drives a fan which requires 250 kW at the full-load speed of the motor. Assuming the torque to drive the fan varies as the square of the fan speed, what resistance should be connected in series with the rotor resistance to reduce the fan speed to 400 r/min?

Chapter 7: DC Machines

Chapter 7: DC Machines

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- Dc machines are characterized by their versatility.
- By means of various combinations of shunt-, series-, and separately-excited field windings they can be designed to display a wide variety of volt-ampere or speed-torque characteristics for both dynamic and steady-state operation.
- Because of the ease with which they can be controlled, systems of dc machines have been frequently used in applications requiring a wide range of motor speeds or precise control of motor output.

§7.1 Introduction

- The essential features of a dc machine are shown schematically in Fig.7.1
- Fig. 7.1(b) shows the circuit representation of the machine.
- The stator has salient poles and is excited by one or more field coils.
    - The air-gap flux distribution created by the field windings is symmetric about the center line of the field poles. This axis is called the field axis or direct axis.
- The ac voltage generated in each rotating armature coil is converted to dc in the external armature terminals by means of a rotating commutator and stationary brushes to which the armature leads are connected.
    - The commutator-brush combination forms a mechanical rectifier, resulting in a dc armature voltage as well as an armature-mmf wave which is fixed in space.

  - The brushes are located so that commutation occurs when the coil sides are in the neutral zone, midway between the field poles.
  - The axis of the armature-mmf wave is 90 electrical degrees from the axis of the field poles, i.e., in the quadrature axis.
  - The armature-mmf wave is along the brush axis.

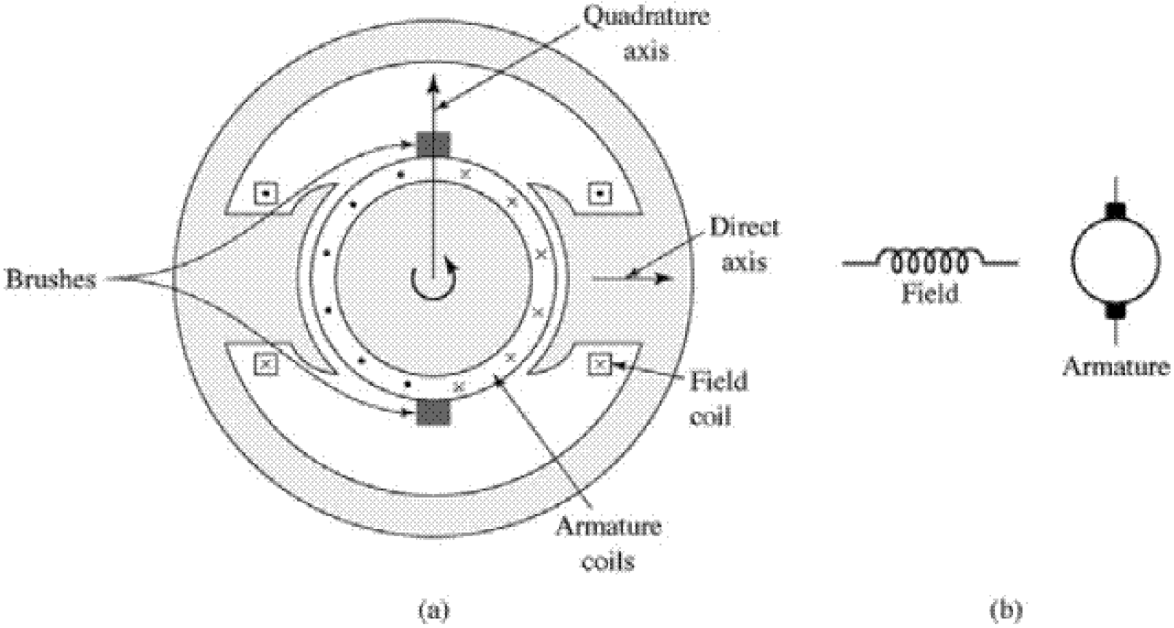


Figure 7.1 Schematic representations of a dc machine.

- Recall equation (7.1). Note that the torque is proportional to the product of the magnitudes of the interacting fields and to the sine of the electrical space angle between their magnetic axes.The negative sign indicates that the electromechanical torque acts in a direction to decrease the displacement angle between the fields.

$$T=-\frac{\pi}{2}\left[\frac{\text{poles}}{2}\right]^2 \Phi_{\mathrm{sr}} F_r \sin\delta_r \quad (7.1)$$

- For the dc machine, the electromagnetic torque T_{mech}can be expressed in terms of the interaction of the direct-axis air-gap per pole Φ_d and the space-fundamental component $F_{a1}$of the armature-mmf wave, in a form similar to (7.1). Note that $\sin\delta_r=1$.

$T_{\text{mech}} = \frac{\pi}{2}\left[\frac{\text{pole}}{2}\right]^2 \Phi_d F_{\text{al}}$ (7.2)

$T_{\text{mech}} = K_a \Phi_d i_a$ (7.3)

$K_a = \frac{\text{poles}.C_a}{2\pi\text{m}}$ (7.4)

K_a: a constant determined by the design of the winding, the winding constant

i_a =current in external armature circuit

C_a =total number of conductors in armature winding,

m= number of parallel paths through winding

- The rectified voltage e_a between brushes, known also as the speed voltage, is

$e_a = K_a \Phi_d \omega_m$ (7.5)

- The generated voltage as observed from the brushes is the sum of the rectified voltage of all the coils in series between brushes and is shown by the rippling line labeled e_a in Fig.7.2.
- With a dozen or so commutator segments per pole, the ripple becomes very small and the average generated voltage observed from the brushes equals the sum of the average values of the rectified coils voltages.

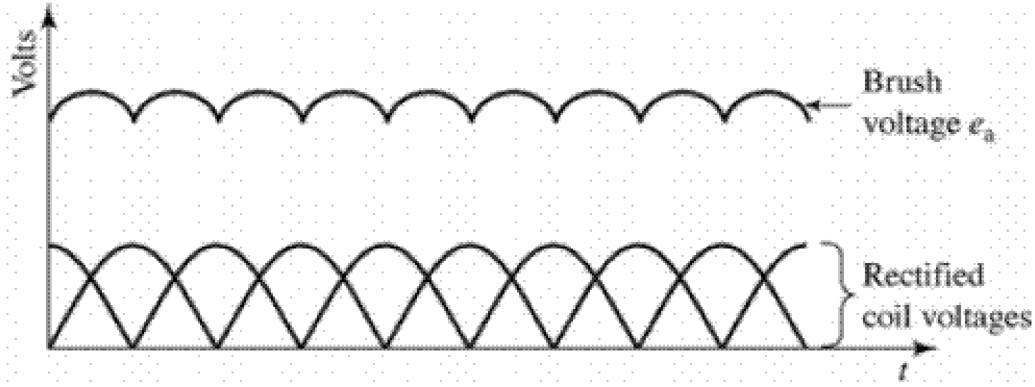


Figure 7.2 Rectified coil voltages and resultant voltage between brushes in a dc machine.

- Note that the electric power equals the mechanical power.

$e_a i_a = T_{\text{mech}} \omega_m$ (7.6)

- The flux-mmf characteristic is referred to as the magnetization curve.

- The direct-axis air-gap flux is produced by the combined mmf $\sum N_f i_f$ of the field winding.
- The form of a typical magnetization curve is shown in Fig.7.3(a).
- The dashed straight line through the origin coinciding with the straight portion of the magnetization curves is called the air-gap line.
- It is assumed that the armature mmf has no effect on the direct-axis flux because the axis of the armature-mmf wave is along the quadrature axis and hence perpendicular to the field axis. (This assumption needs reexamining!)
- Note the residual magnetism in the figure.The magnetic material of the field does not fully demagnetize when the net field mmf is reduced to zero.
- It is usually more convenient to express the magnetization curve in terms of the armature emf e_{a0}at a constant speed ω_{m0}as shown in Fig.7.3(b).

$\frac{e_a}{\omega_m} = K_a \Phi_d = \frac{e_{a0}}{\omega_{m0}}$ (7.7)

$e_a = (\frac{\omega_m}{\omega_{m0}}) e_{a0}$ (7.8)

$e_a = (\frac{n}{n_0}) e_{a0}$ (7.9)

- Fig. 7.3(c) shows the magnetization curve with only one field winding excited. This curve can easily be obtained by test methods.

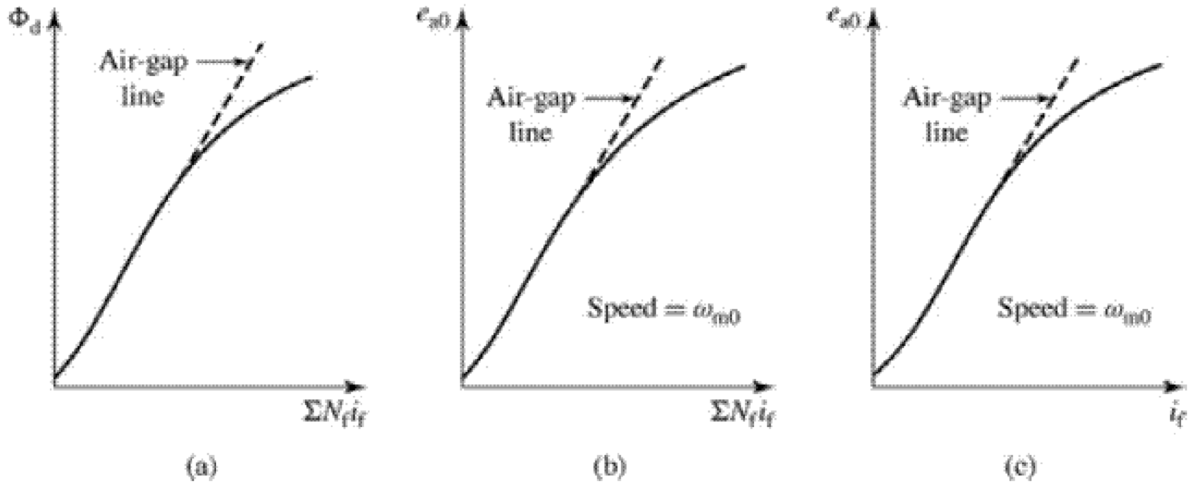


Figure 7.3 Typical form of magnetization curves of a dc machine.

- Various methods of excitation of the field windings are shown in Fig.7.4.

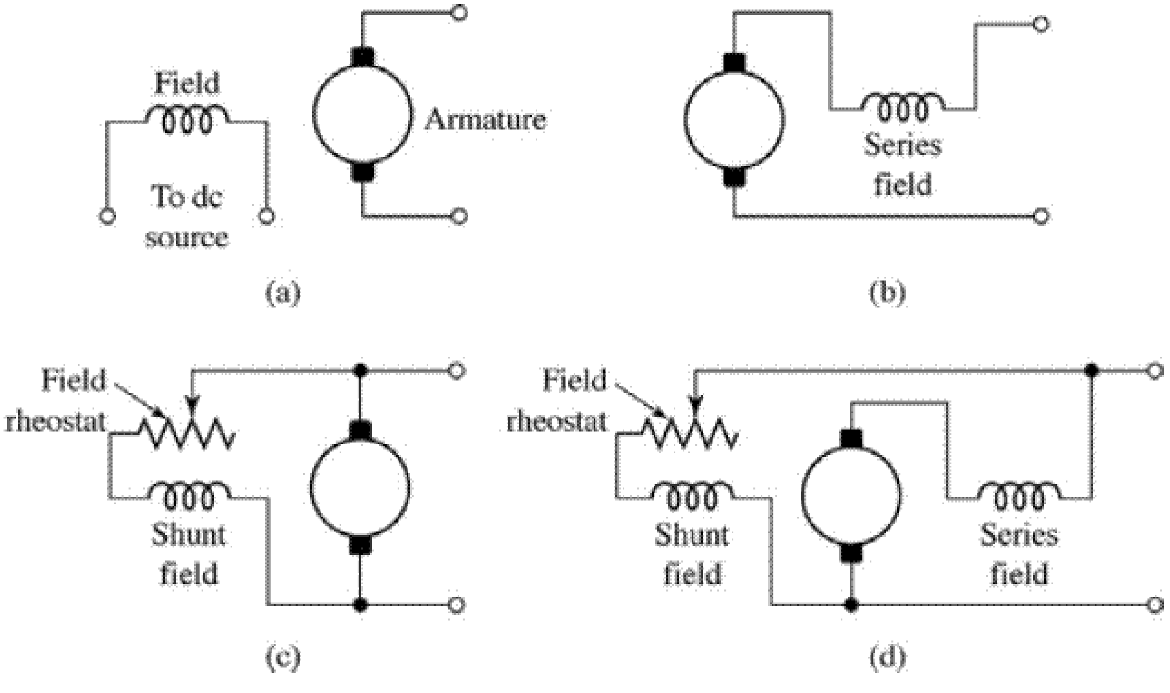


Figure 7.4 Field-circuit connections of dc machines:

(a) separate excitation, (b) series, (c) shunt, (d) compound.

Consider first dc generators.

- Separately-excited generators.
- Self-excited generators: series generators, shunt generators, compound generators.
    - With self-excited generators, residual magnetism must be present in the machine iron to get the self-excitation process started.
    - N.B.: long- and short-shunt, cumulatively and differentially compound.
- Typical steady-state volt-ampere characteristics are shown in Fig.7.5, constant-speed operation being assumed.
- The relation between the steady-state generated emf Ea and the armature terminal voltage V_a is

$V_a = E_a - I_a R_a$ (7.10)

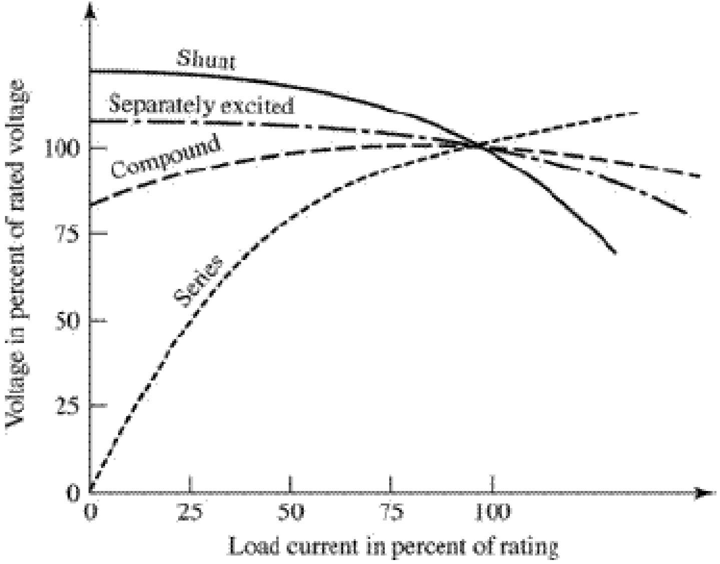


Figure 7.5 Volt-ampere characteristics of dc generators.

Any of the methods of excitation used for generators can also be used for motors.

- Typical steady-state dc-motor speed-torque characteristics are shown in Fig.7.6, in which it is assumed that the motor terminals are supplied from a constant-voltage source.

- In a motor the relation between the emf E_a generated in the armature and and the armature terminal voltage V_a is

$V_a = E_a + I_a R_a$ (7.11)

$I_a = \frac{V_a - E_a}{R_a}$ (7.12)

- The application advantages of dc machines lie in the variety of performance characteristics offered by the possibilities of shunt, series, and compound excitation.

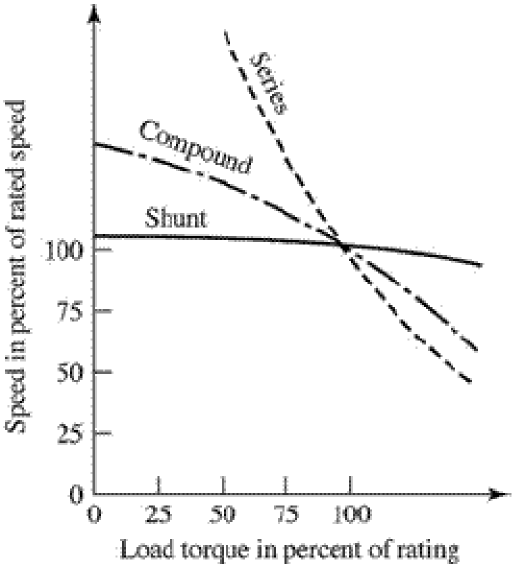


Figure 7.6 Speed-torque characteristics of dc motors.

§7.4 Analytical Fundamentals: Electric-Circuit Aspects

- Analysis of dc machines: electric-circuit and magnetic-circuit aspects
- Torque and power:

The electromagnetic torque T_{mech}

$T_{\text{mech}} = K_a \Phi_d I_a$ (7.13)

The generated voltage E_a

$E_a = K_a \Phi_d \omega_m$ (7.14)

$K_a = \frac{\text{poles}.C_a}{2\pi\text{m}}$ (7.15)

$E_a I_a$: electromagnetic power

$T_{\text{mech}} = \frac{E_a I_a}{\omega_m} = K_a \Phi_d I_a$ (7.16)

Note that the electromagnetic power differs from the mechanical power at the machine shaft by the rotational losses and differs from the electric power at the machine terminals by the shunt-field and armature I^2R losses.

- Voltage and current (Refer to Fig.7.7.):

V_a: the terminal voltage of the armature winding

V_t: the terminal voltage of the dc machine, including the voltage drop across the series connected field winding

$V_a = V_t$ if there is no series field winding

R_a: the resistance of armature, R_s: the resistance of the series field

$V_a = E_a \pm I_a R_a$ (7.17)

$V_t = E_a \pm I_a(R_a + R_s)$ (7.18)

$I_L = I_a \pm I_f$ (7.19)

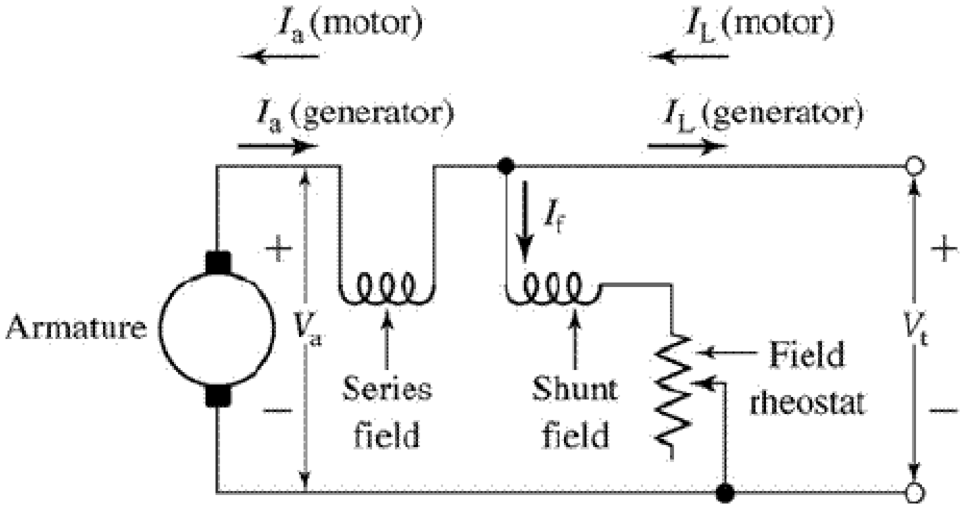


Figure 7.7 Motor or generator connection diagram with current directions.

- For compound machines, Fig.7.7 shows a long-shunt connection and the short-shunt connection is illustrated in Fig.7.8.

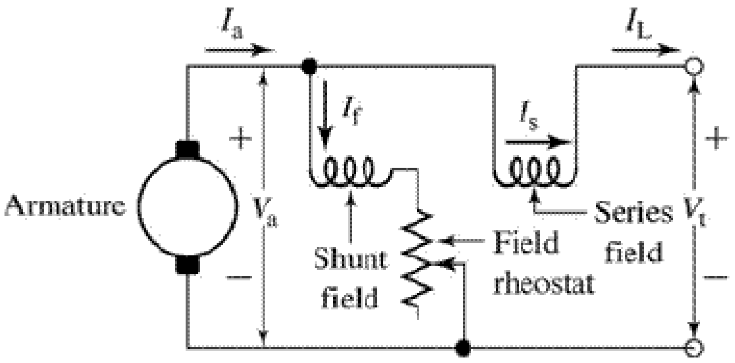


Figure 7.8 Short-shunt compound-generator connections.

§7.5 Analytical Fundamentals: Magnetic-Circuit Aspects

- The net flux per pole is that resulting from the combined mmf’s of the field and armature windings.

- First we consider the mmf intentionally placed on the stator main poles to create the working flux, i.e., the main-field mmf, and then we include armature-reaction effects.

§7.5.1 Armature Reaction Neglected

- With no load on the machine or with armature-reaction effects ignored, the resultant mmf is the algebraic sum of the mmf's acting on the main or direct axis.

$$\text{Main} - \text{field mmf} = N_f I_f \pm N_s I_s \text{ (7.20)}$$

$$\text{Gross mmf} = I_f + \left[\frac{N_s}{N_f}\right] I_s \text{equivalent shunt} - \text{field amperes (7.21)}$$

- An example of a no-load magnetization characteristic is given by the curve for $I_a = 0$ in Fig.7.9.
- The generated voltage E_a at any speed ω_m is given by

$$E_a = \left[\frac{\omega_m}{\omega_{m0}}\right] E_{a0} \text{ (7.22)}$$

$$E_a = \left[\frac{n}{n_0}\right] E_{a0} \text{ (7.23)}$$

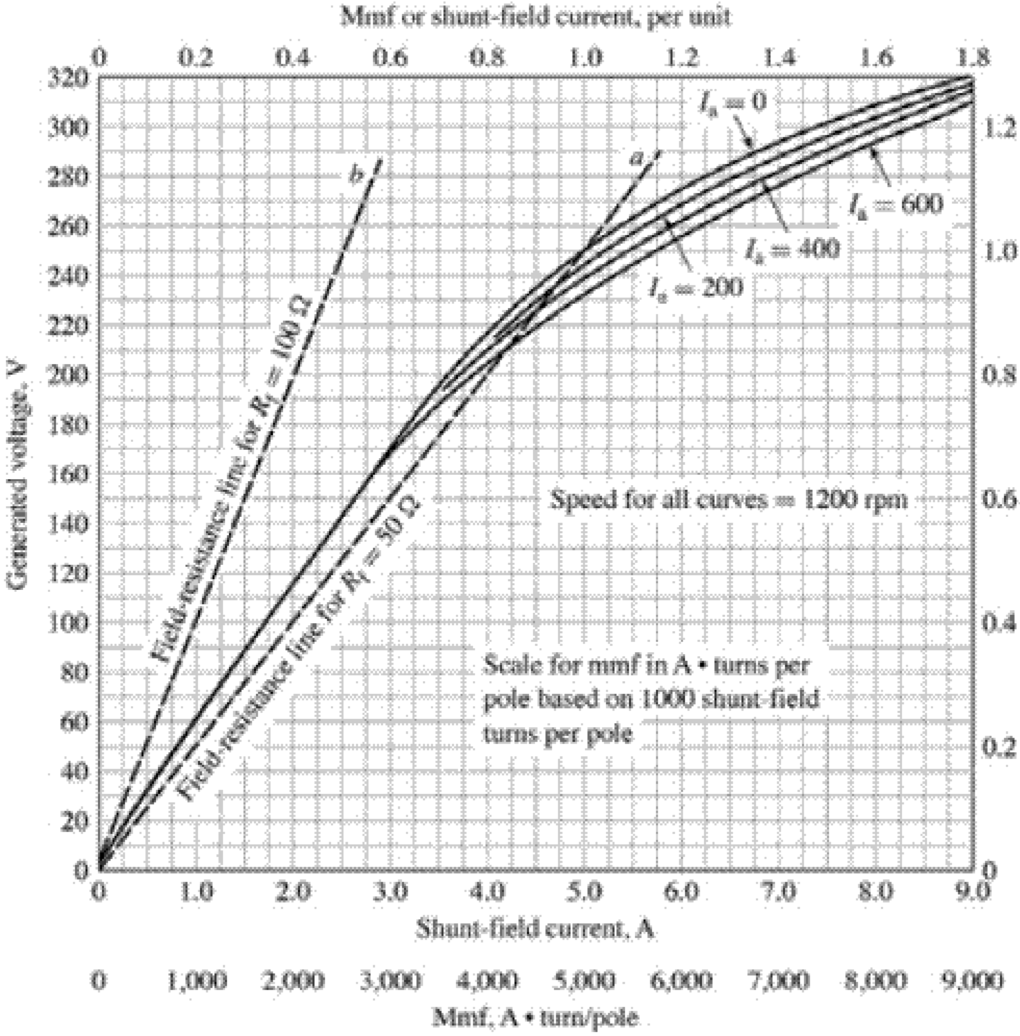


Figure 7.9 Magnetization curves for a 100-kW, 250-V, 1200-r/min dc machine.

Also shown are field-resistance lines for the discussion of self-excitation in §7.6.1.

§7.5.2 Effects of Armature Reaction Included

- Current in the armature winding gives rise to a demagnetizing effect caused by a cross-magnetizing armature reaction.

One common approach is to base analyses on the measured performance of the machine.

- Data are taken with both the field and armature excited, and the tests are conducted so that the effects on generated emf of varying both the main-field excitation and armature mmf can be noted.
- Refer to Fig.7.9. The inclusion of armature reaction becomes simply a matter of using the magnetization curve corresponding to the armature current involved.
- The load-saturation curves are displaced to the right of the no-load curve by an amount which is a function of I_a.
- The effect of armature reaction is approximately the same as a demagnetizing mmf F_{ar} acting on the main-field axis.

$$\text{Net mmf} = \text{gross mmf} - F_{\mathrm{ar}} = N_f I_f + N_s I_s - \mathrm{AR} \quad (7.24)$$

- Over the normal operating range, the demagnetizing effect of armature reaction may be assumed to be approximately proportional to the armature current.
- The amount of armature of armature reaction present in Fig.7.9 is definitely more than one would expect to find in a normal, well-designed machine operating at normal currents.

§7.6 Analysis of Steady-State Performance

- Generator operation and motor operation

- For a generator, the speed is usually fixed by the prime mover, and problems often encountered are to determine the terminal voltage corresponding to a specified load and excitation or to find the excitation required for a specified load and terminal voltage.
- For a motor, problems frequently encountered are to determine the speed corresponding to a specific load and excitation or to find the excitation required for specific load and speed conditions; terminal voltage is often fixed at the value of the available source.

§7.6.1 Generator Analysis

- Analysis is based on the type of field connection.

- Separately-excited generators are the simplest to analyze.

- Its main-field current is independent of the generator voltage.
- For a given load, the equivalent main-field excitation is given by (7.21) and the associated armature-generated voltage E_a is determined by the appropriate magnetization curve.
- The voltage E_a , together with (7.17) or (7.18), fixes the terminal voltage.

- Shunt-excited generators will be found to self-excite under properly chosen operating condition under which the generated voltage will build up spontaneously.

- The process is typically initiated by the presence of a small amount of residual magnetism in the field structure and the shunt-field excitation depends on the terminal voltage. Consider the field-resistance line, the line 0a in Fig. 7.9.
- The tendency of a shunt-connected generator to self-excite can be observed by examining the buildup of voltage for an unloaded shunt generator.

–Buildup continues until the volt-ampere relations represented by the magnetization curve and the field-resistance line are simultaneously satisfied.

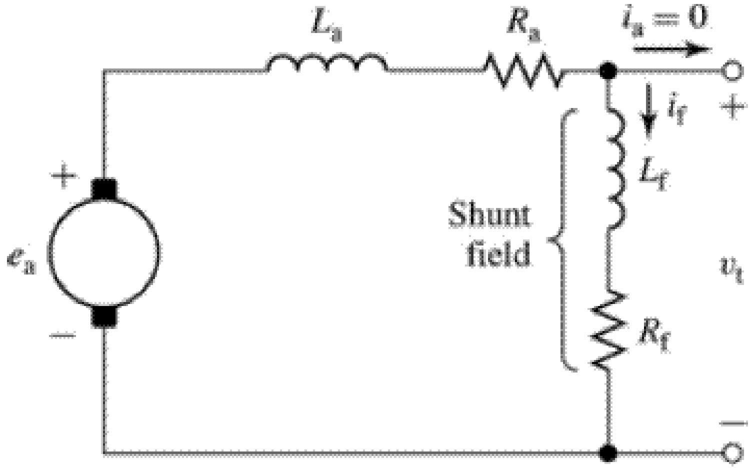


Figure 7.10 Equivalent circuit for analysis of voltage buildup in a self-excited dc generator.

Note that in Fig.7.10,

$(L_a + L_f)\frac{\mathrm{d}i_f}{\mathrm{d}t} = e_a - (R_a + R_f)i_f$ (7.25)

- The field resistance line should also include the armature resistance.
- Notice that if the field resistance is too high, as shown by line 0b in Fig.7.9, voltage buildup will not be achieved.
- The critical field resistance, corresponding to the slope of the field-resistance line tangent to the magnetization curve, is the resistance above which buildup will not be obtained.

§7.6.2 Motor Analysis

- The terminal voltage of a motor is usually held substantially constant or controlled to a specific value. Motor analysis is most nearly resembles that for separately-excited generators.

- Speed is an important variable and often the one whose value is to be found.

$V_a = E_a \pm I_a R_a$ (7.26)

$V_t = E_a \pm I_a(R_a + R_s)$ (7.27)

$\text{Gross mmf} = I_f + \left[\frac{N_s}{N_f}\right] I_s \text{equivalent shunt} - \text{field amperes}$ (7.28)

$T_{\text{mech}} = K_a \Phi_d I_a$ (7.29)

$E_a = K_a \Phi_d \omega_m$ (7.30)

$E_a = \left[\frac{\omega_m}{\omega_{m0}}\right] E_{a0}$ (7.31)

$E_a = \left[\frac{n}{n_0}\right] E_{a0}$ (7.32)

PROBLEMS - chapter 7

PROBLEMS

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

7.1 Consider a separately-excited dc motor. Describe the speed variation of the motor operating unloaded under the following conditions:

a. The armature terminal voltage is varied while the field current is held constant.

b. The field current is varied while the armature terminal voltage is held constant.

c. The field winding is connected in shunt directly to the armature terminals, and the armature terminal voltage is then varied.

7.2 A dc shunt motor operating at an armature terminal voltage of 125 V is observed to be operating at a speed of 1180 r/min. When the motor is operated unloaded at the same armature terminal voltage but with an additional resistance of 5 Ω in series with the shunt field, the motor speed is observed to be 1250 r/min.

a. Calculate the resistance of the series field.

b. Calculate the motor speed which will result if the series resistance is increased from 5 Ω to 15 Ω.

7.3 For each of the following changes in operating condition for a dc shunt motor, describe how the armature current and speed will vary:

a. Halving the armature terminal voltage while the field flux and load torque remain constant.

b. Halving the armature terminal voltage while the field current and load power remain constant.

c. Doubling the field flux while the armature terminal voltage and load torque remain constant.

d. Halving both the field flux and armature terminal voltage while the load power remains constant.

e. Halving the armature terminal voltage while the field flux remains constant and the load torque varies as the square of the speed. Only brief quantitative statements describing the general nature of the effect are required, for example, "speed approximately doubled."

7.4 The constant-speed magnetization curve for a 25-kW, 250-V dc machine at a speed of 1200 r/min is shown in Fig.7.1. This machine is separately excited and has an armature resistance of 0.14 Ω. This machine is to be operated as a dc generator while driven from a synchronous motor at constant speed.

a. What is the rated armature current of this machine?

b. With the generator speed held at 1200 r/min and if the armature current is limited to its rated value, calculate the maximum power output of the generator and the corresponding armature voltage for constant field currents of (i) 1.0 A, (ii) 2.0 A and (iii) 2.5 A.

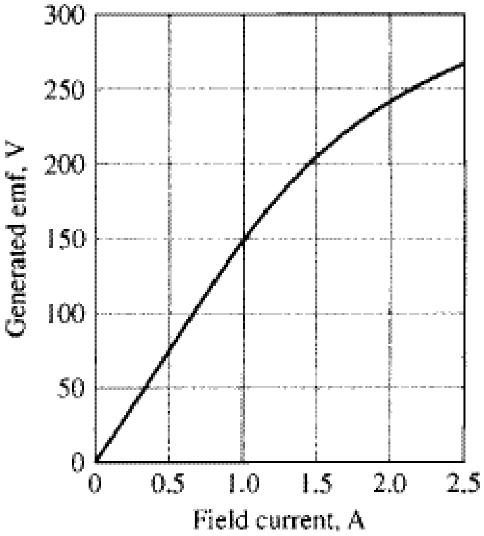


Figure 7.1 1200 r/min magnetization curve for the dc generator.

c. Repeat part (b) if the speed of the synchronous generator is reduced to 900 r/min.

7.5 The dc generator of Problem 7.4 is to be operated at a constant speed of 1200 r/min into a load resistance of 2.5 Ω.

a. Using the "spline0" function of MATLAB and the points of the magnetization curve of Fig. 7.1 at 0, 0.5, 1.0, 1.5, 2.0, and 2.5 A, create

a MATLAB plot of the magnetization curve of Fig. 7.1.

b. Using the "spline0" function as in part (a), use MATLAB to plot (i) the terminal voltage and (ii) the power delivered to the load as the generator field current is varied from 0 to 2.5A.

7.6 The dc machine of Problem 7.4 is to be operated as a motor supplied by a constant armature terminal voltage of 250 V. If saturation effects are ignored, the magnetization curve of Fig. 7.1 becomes a straight line with a constant slope of 150 volts per ampere of field current. For the purposes of this problem, you may assume that saturation effects can be neglected.

a. Assuming that the field current is held constant at 1.67 A, plot the motor speed as a function of motor shaft power as the shaft power varies from 0 to 25 kW.

b. Now assuming that the field current can be adjusted in order to maintain the motor speed constant at 1200 r/min, plot the required field current as a function of motor shaft power as the shaft power varies from 0 to 25 kW.

7.7 Repeat Problem 7.6 including the saturation effects represented by the saturation curve of Fig. 7.1. For part (a), set the field current equal to the value required to produce an open-circuit armature terminal voltage of 250 V at 1200 r/min. (Hint: This problem is most easily solved using MATLAB and its "spline0" function as in Problem 7.5.)

7.8 A 15-kW, 250-V, 1150 r/min shunt generator is driven by a prime mover whose speed is 1195 r/min when the generator delivers no load. The speed falls to 1140 r/min when the generator delivers 15 kW and may be assumed to decrease in proportion to the generator output. The generator is to be changed into a short-shunt compound generator by equipping it with a series field winding which will cause its voltage to rise from 230 V at no load to 250 V for a load of 61.5 A. It is estimated that the series field winding will have a resistance of 0.065 Ω. The armature resistance (including brushes) is 0.175 Ω. The shunt field winding has 500 turns per pole. To determine the necessary series-field turns, the machine is run as a separately-excited generator and the following load data are obtained:

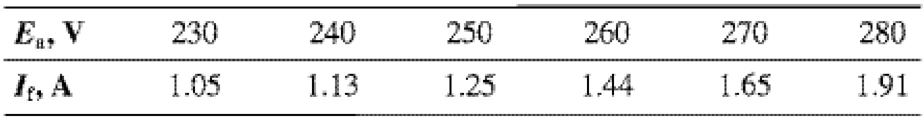
Armature terminal voltage = 254 V

Armature current = 62.7 A

Field current = 1.95 A

Speed = 1140 r/min

The magnetization curve at 1195 r/min is as follows:



E_a, V	230	240	250	260	270	280
I_f, A	1.05	1.13	1.25	1.44	1.65	1.91

Determine

a. the armature reaction in equivalent demagnetizing ampere-turns per pole for I_a = 62.7 A and

b. the necessary number of series-field turns per pole. (Hint: This problem can be solved either graphically or by use of the MATLAB "spline()" function to represent the magnetization curve.)

7.9 When operated from a 230-V dc supply, a dc series motor operates at 975 r/min with a line current of 90 A. Its armature-circuit resistance is 0.11 Ω and its series-field resistance is 0.08 Ω. Due to saturation effects, the flux produced by an armature current of 30 A is 48 percent of that at an armature current of 90 A. Find the motor speed when the armature voltage is 230 V and the armature current is 30 A.

7.10 A 250-V dc shunt-wound motor is used as an adjustable-speed drive over the range from 0 to 2000 r/min. Speeds from 0 to 1200 r/min are obtained by adjusting the armature terminal voltage from 0 to 250 V with the field current kept constant. Speeds from 1200 r/min to 2000 r/min are obtained by decreasing the field current with the armature terminal voltage remaining at 250 V. Over the entire speed range, the torque required by the load remains constant.

a. Sketch the general form of the curve of armature current versus speed over the entire range. Ignore machine losses and armature-reaction effects.

b. Suppose that, instead of operating with constant torque, the load torque at any given speed is adjusted to maintain the armature current at its rated value. Sketch the general form of the allowable torque as a function of speed assuming the motor is controlled as described above.

7.11 Two adjustable-speed dc shunt motors have maximum speeds of 1800 r/min and minimum speeds of 500 r/min. Speed adjustment is obtained by field-rheostat control. Motor A drives a load requiring constant power over the speed range; motor B drives one requiring constant torque. All losses and armature reaction may be neglected.

a. If the power outputs of the two motors are equal at 1800 r/min and the armature currents are each 125 A, what will the armature currents be at 500 r/min?

b. If the power outputs of the two motors are equal at 500 r/min and the armature currents are each 125 A, what will the armature current be at 1800 r/min?

c. Answer parts (a) and (b) with speed adjustment by armature-voltage control with conditions otherwise the same.

7.12 Consider a dc shunt motor connected to a constant-voltage source and driving a load requiring constant electromagnetic torque. Show that if $E_a > 0.5\mathrm{V}_t$ (the normal situation), increasing the resultant air-gap flux decreases the speed, whereas if $E_a < 0.5\mathrm{V}_t$ (as might be brought about by inserting a relatively high resistance in series with the armature), increasing the resultant air-gap flux increases the speed.

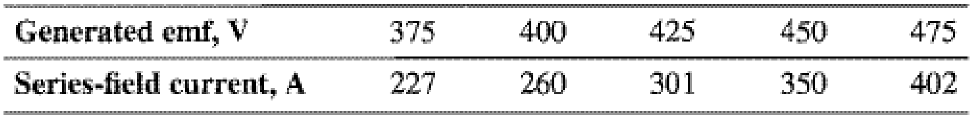
7.13 A separately-excited dc motor is mechanically coupled to a three-phase, four-pole, 30-kVA, 460-V, cylindrical-pole synchronous generator. The dc motor is connected to a constant 230-V dc supply, and the ac generator is connected to a 460-V, fixed-voltage, fixed-frequency, three-phase supply. The synchronous reactance of the synchronous generator is 5.13 Ω/phase. The armature resistance of the dc motor is 30 mA. The four-pole dc machine is rated 30 kW at 230 V. All unspecified losses are to be neglected.

a. If the two machines act as a motor-generator set receiving power from the dc source and delivering power to the ac supply, what is the excitation voltage of the ac machine in volts per phase (line-to-neutral) when it delivers 30 kW at unity power factor? What is the internal voltage of the dc motor?

b. Leaving the field current of the ac machine at the value corresponding to the condition of part (a), what adjustment can be made to reduce the power transfer between the two machines to zero? Under this condition of zero power transfer, what is the armature current of the dc machine? What is the armature current of the ac machine?

c. Leaving the field current of the ac machine as in parts (a) and (b), what adjustment can be made to cause the transfer of 30 kW from the ac source to the dc source? Under these conditions what are the armature current and internal voltage of the dc machine? What will be the magnitude and phase of the current of the ac machine?

7.14 A 150-kW, 600-V, 600 r/min dc series-wound railway motor has a combined field and armature resistance (including brushes) of 0.125 Ω. The full-load current at rated voltage and speed is 250 A. The magnetization curve at 400 r/min is as follows:

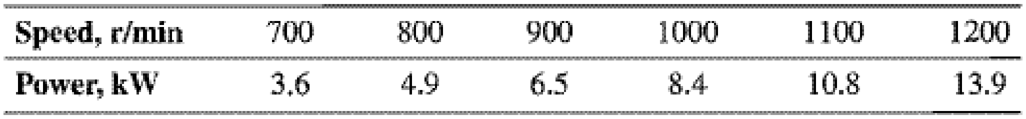


Generated emf, V	375	400	425	450	475
Series-field current, A	227	260	301	350	402

Determine the internal starting torque when the starting current is limited to 460 A. Assume the armature reaction to be equivalent to a demagnetizing mmf which varies as the square of the current. (Hint: This problem can be solved either graphically or by use of the MATLAB "spline0" function to represent the magnetization curve.)

7.15 A 25-kW, 230-V shunt motor has an armature resistance of 0.11 Ω and a field resistance of 117 Ω. At no load and rated voltage, the speed is 2150 r/min and the armature current is 6.35 A. At full load and rated voltage, the armature current is 115A and, because of armature reaction, the flux is 6 percent less than its no-load value. What is the full-load speed?

7.16 A 91-cm axial-flow fan is to deliver air at 16.1 m^3/sec against a static pressure of 120 P_a when rotating at a speed of 1165 r/min. The fan has the following speed-load characteristic



Speed, r/min	700	800	900	1000	1100	1200
Power, kW	3.6	4.9	6.5	8.4	10.8	13.9

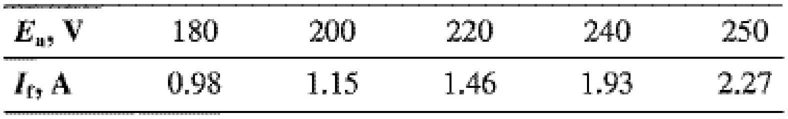
It is proposed to drive the fan by a 12.5 kW, 230-V, 46.9-A, four-pole dc shunt motor. The motor has an armature winding with two parallel paths and C_a = 666 active conductors. The armature-circuit resistance is 0.215 Ω. The armature flux per pole is $\varphi_d = 10^{-2}$ Wb and armature reaction effects can be neglected. No-load rotational losses (to be considered constant) are estimated to be 750 W. Determine the shaft power output and the operating speed of the motor when it is connected to the fan load and operated from a 230-V source. (Hint: This problem can be easily solved using MATLAB with the fan characteristic represented by the MATLAB "spline()" function.)

7.17 A shunt motor operating from a 230-V line draws a full-load armature current of 46.5 A and runs at a speed of 1300 r/min at both no load and full load. The following data is available on this motor:

Armature-circuit resistance (including brushes) = 0.17 Ω

Shunt-field turns per pole = 1500 turns

The magnetization curve taken with the machine operating as a motor at no load and 1300 r/min is



E_a, V	180	200	220	240	250
I_f, A	0.98	1.15	1.46	1.93	2.27

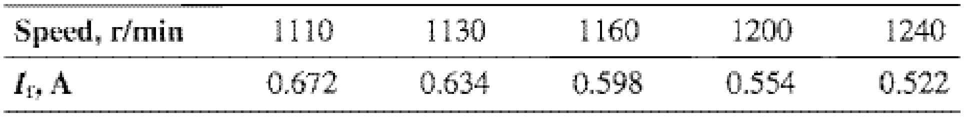
a. Determine the shunt-field current of this motor at no load and 1300 r/min when connected to a 230-V line. Assume negligible armature-circuit resistance and armature reaction at no load.

b. Determine the effective armature reaction at full load in ampere-turns per pole.

c. How many series-field turns should be added to make this machine into a long-shunt cumulatively compounded motor whose speed will be 1210 r/min when the armature current is 46.5 A and the applied voltage is 230 V? Assume that the series field has a resistance of 0.038 Ω.

d. If a series-field winding having 20 turns per pole and a resistance of 0.038 Ω is installed, determine the speed when the armature current is 46.5 A and the applied voltage is 230 V. (Hint: This problem can be solved either graphically or by use of the MATLAB "spline0" function to represent the magnetization curve.)

7.18 A 7.5-kW, 230-V shunt motor has 2000 shunt-field turns per pole, an armature resistance (including brushes) of 0.21 Ω, and a commutating-field resistance of 0.035 Ω. The shunt-field resistance (exclusive of rheostat) is 310 Ω. When the motor is operated at no load with rated terminal voltage and varying shunt-field resistance, the following data are obtained:



Speed, r/min	1110	1130	1160	1200	1240
I_f, A	0.672	0.634	0.598	0.554	0.522

The no-load armature current is negligible. When the motor is operating at full load and rated terminal voltage with a field current of 0.554 A, the armature current is 35.2 A and the speed is 1185 r/min.

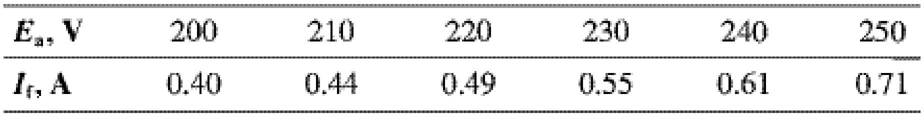
a. Calculate the full-load armature reaction in equivalent demagnetizating ampere-turns per pole.

b. Calculate the full-load electromagnetic torque at this operating condition.

c. What starting torque will the motor produce with maximum field current if the starting armature current is limited to 65 A? Assume that the armature reaction under these conditions is equal to 160 ampere-turns per pole.

d. Design a series field winding to give a speed of 1050 r/min when the motor is loaded to an armature current of 35.2 A and when the shunt field current is adjusted to give a no-load speed of 1200 r/min. Assume the series field will have a resistance of 0.05 Ω. (Hint: This problem can be solved either graphically or by use of the MATLAB "spline0" function to represent the magnetization curve.)

7.19 When operated at rated voltage, a 230-V shunt motor runs at 1750 r/min at full load and at no load. The full-load armature current is 70.8 A. The shunt field winding has 2000 turns per pole. The resistance of the armature circuit (including brushes and interpoles) is 0.15 Ω. The magnetization curve at 1750 r/min is



E_a, V	200	210	220	230	240	250
I_f, A	0.40	0.44	0.49	0.55	0.61	0.71

a. Compute the demagnetizing effect of the armature reaction at full load.

b. A long-shunt cumulative series field winding having four turns per pole and a resistance of 0.038 Ω is added to the machine. Compute the speed at full-load current and rated voltage. The shunt field current will remain equal to that of part (a).

c. With the series-field winding of part (b) installed, compute the internal starting torque in N. m if the starting armature current is limited to 125 A. Assume that the corresponding demagnetizating effect of armature reaction is 230 ampere-turns per pole. (Hint: This problem can be solved either graphically or by use of the MATLAB "spline0" function to represent the magnetization curve.)

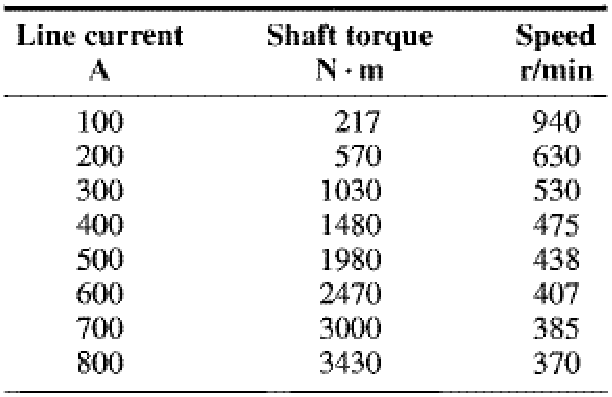
7.20 A 230-V dc shunt motor has an armature-circuit resistance of 0.23 Ω. When operating from a 230-V supply and driving a constant-torque load, the motor is observed to be drawing an armature current of 60 A. An external resistance of 1.0 Ω is now inserted in series with the armature while the shunt field current is unchanged. Neglecting the effects of rotational losses and armature reaction, calculate

a. the resultant armature current and

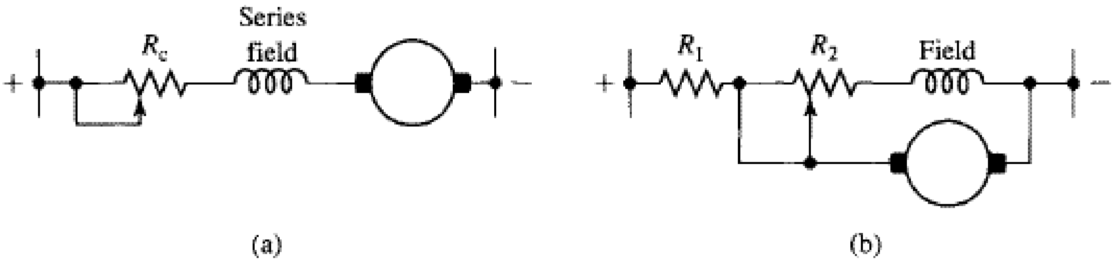
b. the fractional speed change of the motor.

7.21 A common industrial application of dc series motors is in crane and hoist drives. This problem relates to the computation of selected motor performance characterstics for such a drive. The specific motor concerned is a series-wound, 230-V, totally-enclosed motor having a 1/2-hour crane rating of 100 kW with a $75^0 C$ temperature rise. The performance characteristics

Table 7.1 Motor characteristics for Problem 7.22.



Line current A	Shaft torque N · m	Speed r/min
100	217	940
200	570	630
300	1030	530
400	1480	475
500	1980	438
600	2470	407
700	3000	385
800	3430	370



(a) (b)

Figure 7.2 Series crane motor : (a) hoisting connection and (b) lowering connection.

of the motor alone at 230 V as found in the manufacturer's catalog are listed in Table 7.1. The resistance of the armature (including brushes) plus commutating winding is 0.065 Ω and that of the series field winding is 0.027 Ω. Armature reaction effects can be ignored. The motor is to be connected as in Fig. 7.2a for hoisting and Fig. 7.2b for lowering. The former connection consists simply of series-resistance control. The latter connection provides dynamic breaking with the field reconnected in shunt with the addition of an adjustable series resistance. You will use MATLAB to plot some sample speed-torque curves (speed as a function of torque) to determine the suitability of the motor and control for the specified application. Plot all of the curves on a single set of axes coveting roughly the torque-magnitude range found in Table 7.1. Provide for both positive and negative values of speed, corresponding respectively to hoisting and lowering, as well as for both positive and negative values of torque, corresponding respectively to torque in the direction of raising the load and torque in the direction of lowering the load.

a. For the hoisting connection, plot speed-torque curves for the control resistor R_C set at 0, 0.3 and 0.6 Ω. If any of these curves extend into the fourth quadrant within the range of torques covered, interpret physically what operation in that regime means.

b. For the lowering connection, plot a speed-torque curve for $R_1 = 0.3\Omega$ and $R_2 = 0.3\Omega$. The most important portion of this curve is in the fourth quadrant, but if it extends into the third quadrant, that region should also be plotted and interpreted physically.

c. In part (b), what is the lowering speed corresponding to a torque of 1500 N. m? (Hint: This can be found easily using the MATLAB "spline0" function.)

7.22 A 25-kW, 230-V shunt motor has an armature resistance of 0.064 Ω and a field-circuit resistance of 95 Ω. The motor delivers rated output power at rated voltage when its armature current is 122 A. When the motor is operating at rated voltage, the speed is observed to be 1150 r/min when the machine is loaded such that the armature current is 69.5 A.

a. Calculate the rated-load speed of this motor. In order to protect both the motor and the dc supply under starting conditions, an external resistance will be connected in series with the armature winding (with the field winding remaining directly across the 230-V supply). The resistance will then be automatically adjusted in steps so that the armature current does not exceed 200 percent of rated current. The step size will be determined such that, until all the extemal resistance is switched out, the armature current will not be permitted to drop below rated value. In other words, the machine is to start with 200 percent of rated armature current and as soon as the current falls to rated value, sufficient series resistance is to be cut out to restore the current to 200 percent. This process will be repeated until all of the series resistance has been eliminated.

b. Find the maximum value of the series resistance.

c. How much resistance should be cut out at each step in the starting operation and at what speed should each step change occur?

7.23 The manufacturer's data sheet for a permanent-magnet dc motor indicates that it has a torque constant K_m = 0.21 V/(rad/sec) and an armature resistance of 1.9 Ω. For a constant applied armature voltage of 85 V dc, calculate

a. the no-load speed of the motor in r/min and

b. its stall (zero-speed) current and torque (in N. m).

c. Plot the motor torque as a function of speed.

7.24 Measurements on a small permanent-magnet dc motor indicate that it has an armature resistance of 4.6 Ω. With an applied armature voltage of 5 V, the motor is observed to achieve a no-load speed of 11,210 r/min while drawing an armature current of 12.5mA.

a. Calculate the motor torque constant K_m in V/(rad/sec).

b. Calculate the no-load rotational losses in mW.

Assume the motor to be operating from an applied armature voltage of 5V.

c. Find the stall current and torque of the motor.

d. At what speeds will the motor achieve an output power of 1 W? Estimate the motor efficiency under these operating conditions. Assume that the rotational loss varies as the cube of the speed.

7.25 Write a MATLAB script to calculate the parameters of a dc motor. The inputs will be the armature resistance and the no-load armature voltage, speed, and armature current. The output should be the no-load rotational loss and the torque constant Km.

7.26 The dc motor of Problem 7.24 will be used to drive a load which requires a power of 0.75 W at a speed of 8750 r/min. Calculate the armature voltage which must be applied to achieve this operating condition.

Chapter 8: Variable - Reluctance Machines and Stepping Motors

Variable - Reluctance Machines

and Stepping Motors

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- Variable-reluctance machines (VRMs) are perhaps the simplest of electrical machines. They consist of a stator with excitation windings and a magnetic rotor with saliency. Rotor conductors are not required because torque is produced by the tendency of the rotor to align with the stator- produced flux wave in such a fashion as to maximize the stator flux linkages that result from a given applied stator current. Torque production in these machines can be evaluated by using the techniques of Chapter 3 and the fact that the stator winding inductances are functions of the angular position of the rotor.
- Although the concept of the VRM has been around for a long time, only in the past few decades have these machines begun to see widespread use in engineering applications. This is due in large part to the fact that although they are simple in construction, they are somewhat complicated to control. The position of the rotor must be known in order to properly energize the phase windings to produce torque. It is the widespread availability and low cost of micro and power electronics that has made the VRM competitive with other motor technologies in a wide range of applications.
- By sequentially exciting the phases of a VRM, the rotor will rotate in a step- wise fashion, rotating through a specific angle per step. Stepper motors are designed to take advantage of this characteristic. Such motors often combine the use of a variable-reluctance geometry with permanent magnets to produce increased torque and precision position accuracy.

8.1 BASICS OF VRM ANALYSIS

- VRMs can be categorized into two types: singly- salient and doubly-salient. In both cases, their most noticeable features are that there are no windings or permanent magnets on their rotors and that their only source of excitation consists of stator windings.
- To produce torque, VRMs must be designed such that the stator-winding inductances vary with the position of the rotor.
- Figure 8.1a shows a cross-sectional view of a singly-salient VRM, which can be seen to consist of a nonsalient stator and a two-pole salient rotor, both constructed of high-permeability magnetic material. In the figure, a two-phase stator winding is shown although any number of phases is possible.
- Figure 8.2a shows the form of the variation of the stator inductances as a function of rotor angle θm for a singly-salient VRM of the form of Fig. 8.1 a. Notice that the inductance of each stator phase winding varies with rotor position such that the inductance is maximum when the rotor axis is aligned with the magnetic axis of that phase and minimum when the two axes are perpendicular. The mutual inductance between the phase windings is zero when the rotor is aligned with the magnetic axis of either phase but otherwise varies periodically with rotor position.
- Figure 8.lb shows the cross-sectional view of a two-phase doubly-salient VRM in which both the rotor and stator have salient poles. In this machine, the stator has four poles, each with a winding. However, the windings on opposite poles are of the same phase; they may be connected either in series or in parallel. Thus this machine is quite similar to that of Fig. 8.1a in that there is a two-phase stator winding and a two-pole salient rotor. Similarly, the phase inductance of this configuration varies from a maximum value when the rotor axis is aligned with the axis of that phase to a minimum when they are perpendicular.
- Unlike the singly-salient machine of Fig. 8.1 a, under the assumption of negligible iron reluctance the mutual inductances between the phases of the doubly-salient VRM of Fig. 8.1 b will be zero, with the exception of a small, essentially-constant component associated with leakage flux. In addition, the saliency of the stator enhances the difference between the maximum and minimum inductances, which in turn enhances the torque-producing characteristics of the doubly-

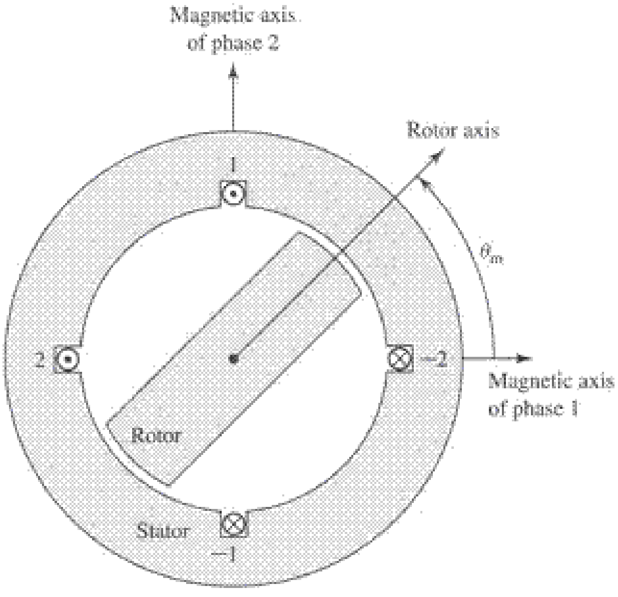
salient machine. Figure 8.2b shows the form of the variation of the phase inductances for the doubly-salient VRM of Fig. 8.lb.

- The relationship between flux linkage and current for the singly-salient VRM is of the form

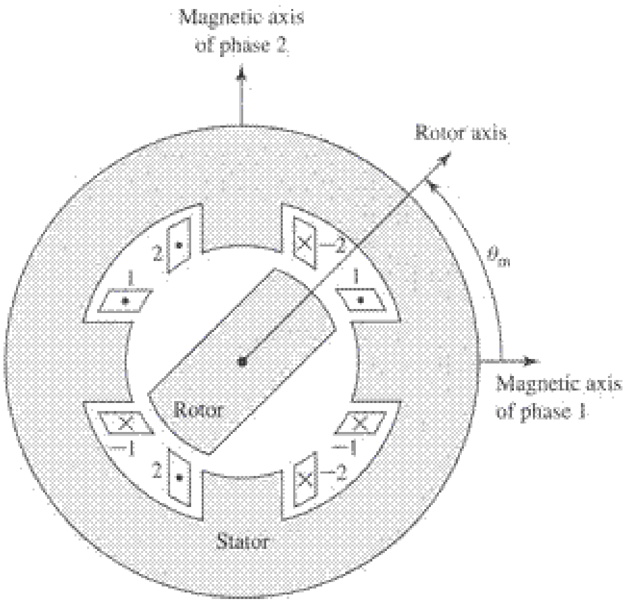
$$\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} = \begin{bmatrix}L_{11}(\theta_m) & L_{12}(\theta_m)\\ L_{12}(\theta_m) & L_{22}(\theta_m)\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix} \text{(8.1)}$$

Here $L_{11}(\theta_m)$ and $L_{22}(\theta_m)$ are the self-inductances of phases 1 and 2, respectively, and $L_{12}(\theta_m)$ is the mutual inductances. Note that, by symmetry

$$L_{22}(\theta_m) = L_{11}(\theta_m - 90^0) \text{(8.2)}$$



(a)



(b)

Figure 8.1 Basic two-phase VRMs: (a) singly-salient and (b) doubly-salient.

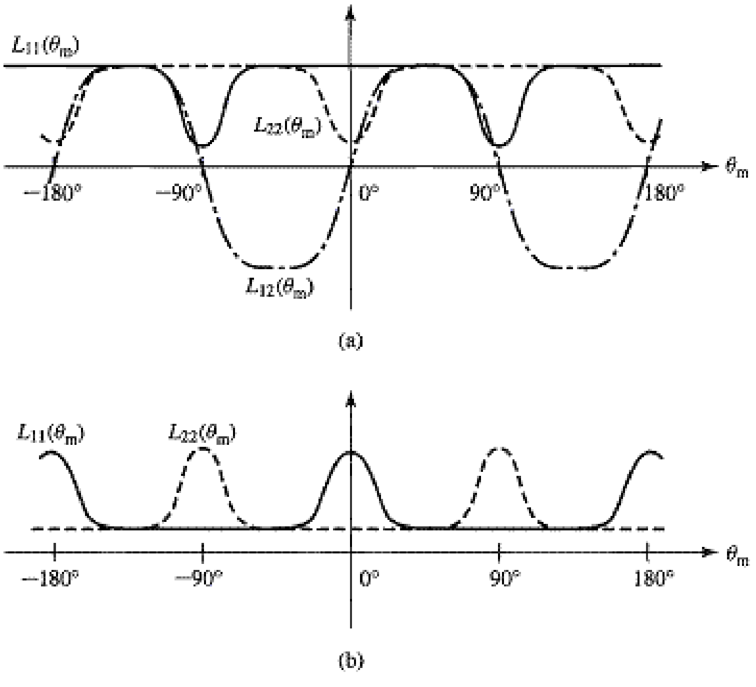


Figure 8.2 Plots of inductance versus 0m for (a) the singly-salient VRM of Fig. 8.1a and (b) the doubly-salient VRM of Fig. 8.1b.

- These inductances are periodic with a period of 180^0 because rotation of the rotor through 180^o from any given angular position results in no change in the magnetic circuit of the machine.
- The electromagnetic torque of this system can be determined from the coenergy as

$$T_{\text{mech}} = \frac{\partial W_{\text{fld}}^{'}(i_1,i_2,\theta_m)}{\partial \theta_m}(8.3)$$

where the partial derivative is taken while holding currents i_1 and i_2 constant. Here, the coenergy can be found,

$$W_{\text{fld}}^{'} = \frac{1}{2}L_{11}(\theta_m)i_1^2 + L_{12}(\theta_m)i_1i_2 + \frac{1}{2}L_{22}(\theta_m)i_2^2(8.4)$$

Thus, combining Eqs. 8.3 and 8.4 gives the torque as

$$T_{\text{mech}} = \frac{1}{2}i_1^2\frac{\mathrm{dL}_{11}(\theta_m)}{d\theta_m} + i_1i_2\frac{\mathrm{dL}_{12}(\theta_m)}{d\theta_m} + \frac{1}{2}i_2^2\frac{\mathrm{dL}_{22}(\theta_m)}{d\theta_m}(8.5)$$

- For the double-salient VRM of Fig. 8. lb, the mutual-inductance term $\mathrm{dL}_{12}(\theta_m)/d\theta_m$ is zero and the torque expression of Eq. 8.5 simplifies to

$$T_{\text{mech}} = \frac{1}{2}i_1^2\frac{\mathrm{dL}_{11}(\theta_m)}{d\theta_m} + \frac{1}{2}i_2^2\frac{\mathrm{dL}_{22}(\theta_m)}{d\theta_m}(8.6)$$

Substitution of Eq. 8.2 then gives

$$T_{\text{mech}} = \frac{1}{2}i_1^2\frac{\mathrm{dL}_{11}(\theta_m)}{d\theta_m} + \frac{1}{2}i_2^2\frac{\mathrm{dL}_{11}(\theta_m-90^0)}{d\theta_m}(8.7)$$

- Equations 8.6 and 8.7 illustrate an important characteristic of VRMs in which mutual-inductance effects are negligible. In such machines the torque expression consists of a sum of terms, each of which is proportional to the square of an individual phase current. As a result, the torque depends only on the magnitude of the phase currents and not on their polarity. Thus the electronics which supply the phase currents to these machines can be unidirectional; i.e., bidirectional currents are not required.

- Since the phase currents are typically switched on and off by solid-state switches such as transistors or thyristors and since each switch need only handle currents in a single direction, this means that the motor drive requires only half the number of switches (as well as half the corresponding control electronics) that would be required in a corresponding bidirectional drive. The result is a drive system which is less complex and may be less expensive.
- The assumption of negligible mutual inductance is valid for the doubly-salient VRM of Fig.8.1b both due to symmetry of the machine geometry and due to the assumption of negligible iron reluctance. In practice, even in situations where symmetry might suggest that the mutual inductances are zero or can be ignored because they are independent of rotor position (e.g., the phases are coupled through leakage fluxes), significant nonlinear and mutual-inductance effects can arise due to saturation of the machine iron.
- At the design and analysis stage, the winding flux-current relationships and the motor torque can be determined by using numerical-analysis packages which can account for the nonlinearity of the machine magnetic material. Once a machine has been constructed, measurements can be made, both to validate the various assumptions and approximations which were made as well as to obtain an accurate measure of actual machine performance.
- The symbol Ps is used to indicate the number of stator poles and P_r to indicate the number of rotor poles, and the corresponding machine is called a P_s/P_r machine.
- Example 8.1 examines a 4/2 VRM.

A 4/2 VRM is shown in Fig. 8.3. Its dimensions are

R = 3.8 cm $\alpha = \beta = 60^o = \pi/3$ rad

g = 2.54 x 10^{-2} cm D = 13.0 cm

and the poles of each phase winding are connected in series such that there are a total of N=100 turns (50 turns per pole) in each phase winding. Assume the rotor and stator to be of infinite magnetic permeability.

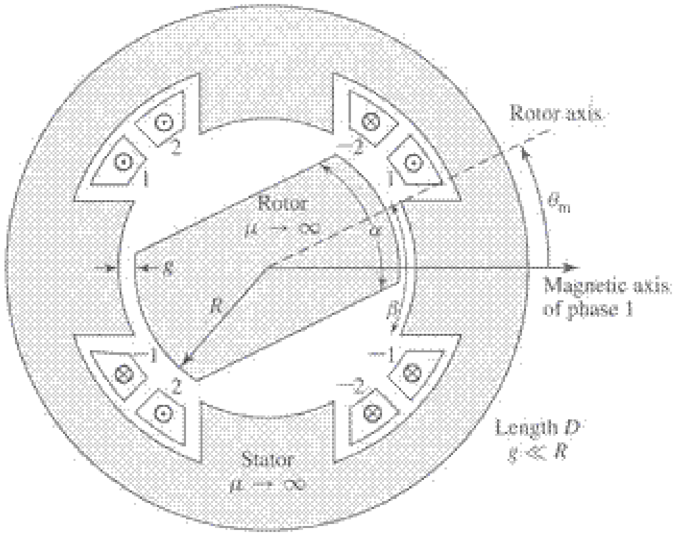


Figure 8.3 4/2 VRM for Example 8.1.

a. Neglecting leakage and fringing fluxes, plot the phase-1 inductance $L(\theta_m)$as a function of θ_m.

b. Plot the torque, assuming (i) $i_1 = I_1$and i_2 = 0 and (ii) i_1 = 0 and $i_2 = I_2$.

c. Calculate the net torque (in N. m) acting on the rotor when both windings are excited such that $i_1 = i_2 =$5 A and at angles (i) $\theta_m = 0^o$, (ii) $\theta = 45^o$, (iii) $\theta_m = 75^o$.

Solution

a. The maximum inductance L_{max} for phase 1 occurs when the rotor axis is aligned with the phase-1 magnetic axis. L_{max} is equal to

$$L_{\text{max}} = \frac{N^2\mu_o\alpha\text{RD}}{\text{2g}}$$

where αRD is the cross-sectional area of the air gap and 2g is the total gap length in the magnetic circuit. For the values given,

$$L_{\text{max}} = \frac{N^2\mu_o\alpha\text{RD}}{\text{2g}}$$

$$= \frac{(100)^2(4\pi\times10^{-7})(\pi/3)(3.8\times10^{-2})(0.13)}{2\times(2.54\times10^{-4})}$$

$$= 0.128H$$

Neglecting fringing, the inductance $L(\theta_m)$will vary linearly with the air-gap cross-sectional area as shown in Fig. 8.4a. Note that this idealization predicts that the inductance is zero when there is no overlap when in fact there will be some small value of inductance, as shown in Fig. 8.2.

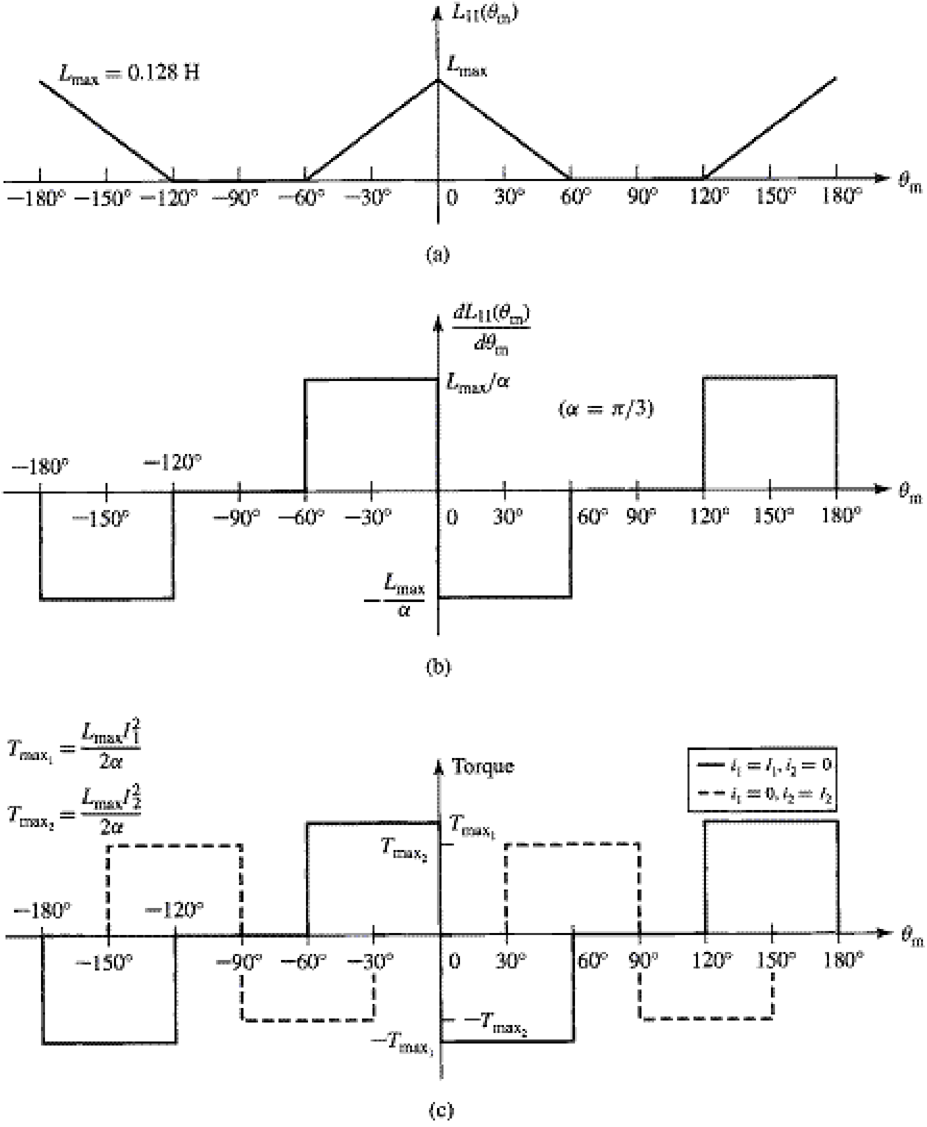


Figure 8.4 (a) $L_{11}(\theta_m)$ versus θ_m, (b) $dL_{11}(\theta_m)/d\theta_m$versus θ_m, and (c) torque versus θ_m.

b. From Eq. 8.7, the torque consists of two terms

$$T_{\text{mech}} = \frac{1}{2} i_1^2 \frac{\mathrm{dL}_{11}(\theta_m)}{d\theta_m} + \frac{1}{2} i_2^2 \frac{\mathrm{dL}_{11}(\theta_m - 90^o)}{d\theta_m}$$

and $\mathrm{dL}_{11}/d\theta_m$ can be seen to be the stepped waveform of Fig.8.4b whose maximum values are given by $\pm L_{\max}/\alpha$ (with α expressed in radians!). Thus the torque is as shown in Fig. 8.4c.

c. The peak torque due to each of the windings is given by

$$T_{\max} = \left[\frac{L_{\max}}{2\alpha}\right] i^2 = \left[\frac{0.128}{2(\pi/3)}\right] 5^2 = 1.53 N.m$$

(i) From the plot in Fig. 8.4c, at ***SORRY, THIS MEDIA TYPE IS NOT SUPPORTED.*** , the torque contribution from phase 2 is clearly zero. Although the phase-1 contribution appears to be indeterminate, in an actual machine the torque change from T_{max1},$\text{to} - T_{\text{max1}}$,$\text{at} \theta_m = 0^o$would have a finite slope and the torque would be zero at $\theta = 0^0$. Thus the net torque from phases 1 and 2 at this position is zero.

Notice that the torque at $\theta_m = 0$ is zero independent of the current levels in phases 1 and 2. This is a problem with the 4/2 configuration of Fig. 8.3 since the rotor can get "stuck" at this position (as well as at $\theta_m = \pm 90^o, \pm 180^o$), and there is no way that electrical torque can be produced to move it.

(ii) At $\theta_m = 45^o$ both phases are providing torque. That of phase 1 is negative while that of phase 2 is positive. Because the phase currents are equal, the torques are thus equal and opposite and the net torque is zero. However, unlike the case of $\theta_m = 0^o$, the torque at this point can be made either positive or negative simply by appropriate selection of the phase currents.

(iii) At $\theta_m = 75^o$ phase 1 produces no torque while phase 2 produces a positive torque of magnitude T_{max2}. Thus the net torque at this position is positive and of magnitude 1.53N.m. Notice that there is no combination of phase currents that will produce a negative torque at this position since the phase-1 torque is always zero while that of phase 2 can be only positive (or zero).

- Example 8.1 illustrates a number of important considerations for the design of VRMs. Clearly these machines must be designed to avoid the occurrence of rotor positions for which none of the phases can produce torque. This is of concern in the design of 4/2 machines which will always have such positions if they are constructed with uniform, symmetric air gaps.
- It is also clear that to operate VRMs with specified torque characteristics, the phase currents must be applied in a fashion consistent with the rotor position. For example, positive torque production from each phase winding in Example 8.1 can be seen from Fig.8.4c to occur only for specific values of θm. Thus operation of VRMs must include some sort of rotor-position sensing as well as a controller which determines both the sequence and the waveform of the phase currents to achieve the desired operation. This is typically implemented by using electronic switching devices (transistors, thyristors, gate-turn-off devices, etc.) under the supervision of a microprocessor-based controller.
- Although a 4/2 VRM such as in Example 8.1 can be made to work, as a practical matter it is not particularly useful because of undesirable characteristics such as its zero-torque positions and the fact that there are angular locations at which it is not possible to achieve a positive torque.
- The analysis of VRMs is conceptually straightforward. In the case of linear machine iron (no magnetic saturation), finding the torque is simply a matter of finding the stator-phase inductances (self and mutual) as a function of rotor position, expressing the coenergy in terms of these inductances, and then calculating the derivative of the coenergy with respect to angular position (holding the phase currents constant when taking the derivative). The electric terminal voltage for each of the phases can be found from the sum of the time derivative of the phase flux linkage and the iR drop across the phase resistance.
- In the case of nonlinear machine iron (where saturation effects are important), the coenergy can be found by appropriate integration of the phase flux linkages, and the torque can again be found from the

derivative of the coenergy with respect to the angular position of the rotor.
- Although VRMs are simple in concept and construction, their operation is some what complicated and requires sophisticated control and motor-drive electronics to achieve useful operating characteristics.

8.2 PRACTICAL VRM CONFIGURATIONS

- Practical VRM drive systems (the motor and its inverter) are designed to meet operating criteria such as

- Low cost.
- Constant torque independent of rotor angular position.
- A desired operating speed range.
- High efficiency.
- A large torque-to-mass ratio.
- A compromise must be made between the variety of options available to the designer. Because VRMs require some sort of electronics and control to operate, often the designer is concerned with optimizing a characteristic of the complete drive system, and this will impose additional constraints on the motor design.
- VRMs can be built in a wide variety of configurations. In Fig. 8.1, two forms of a 4/2 machine are shown: a singly-salient machine in Fig. 8.1a and a doubly-salient machine in Fig. 8. lb. Although both types of design can be made to work, a doubly-salient design is often the superior choice because it can generally produce a larger torque for a given frame size.
- This can be seen qualitatively (under the assumption of a high-permeability, nonsaturating magnetic structure) by reference to Eq. 8.7, which shows that the torque is a function of $\mathrm{dL}_{11}(\theta_m)/d\theta_m$, the derivative of the phase inductance with respect to angular position of the rotor. Clearly, all else being equal, the machine with the largest derivative will produce the largest torque.
- This derivative can be thought of as being determined by the ratio of the maximum to minimum phase inductances $L_{\max}/L_{\min}$.

$$\frac{\mathrm{dL}_{11}(\theta_m)}{d\theta_m} \simeq \frac{L_{\max}-L_{\min}}{\Delta\theta_m} = \frac{L_{\max}}{\Delta\theta_m}\left[1-\frac{L_{\min}}{L_{\max}}\right] \quad (8.8)$$

Where $\Delta\theta_m$ is the angular displacement of the rotor between the positions of maximum and minimum phase inductance. From Eq. 8.8, for a given $L_{\max}$and $\Delta\theta_m$, the largest value of $L_{\max}/L_{\min}$will give the largest torque. Because of its geometry, a doubly-salient structure will typically have a lower minimum inductance and thus a larger value of $L_{\max}/L_{\min}$; hence it will produce a larger torque for the same rotor structure.

- For this reason doubly-salient machines are the predominant type of VRM, and hence only doubly-salient VRMs are considered. In general, doubly-salient machines can be constructed with two or more poles on each of the stator and rotor. It should be pointed out that once the basic structure of a VRM is determined, $L_{\max}$is fairly well determined by such quantities as the number of turns, air-gap length, and basic pole dimensions. The challenge to the VRM designer is to achieve a small value of $L_{\min}$. This is a difficult task because $L_{\min}$ is dominated by leakage fluxes and other quantities which are difficult to calculate and analyze.
- The geometry of a symmetric 4/2 VRM with a uniform air gap gives rise to rotor positions for which no torque can be developed for any combination of excitation of the phase windings. These torque zeros can be seen to occur at rotor positions where all the stator phases are simultaneously at a position of either maximum or minimum inductance.

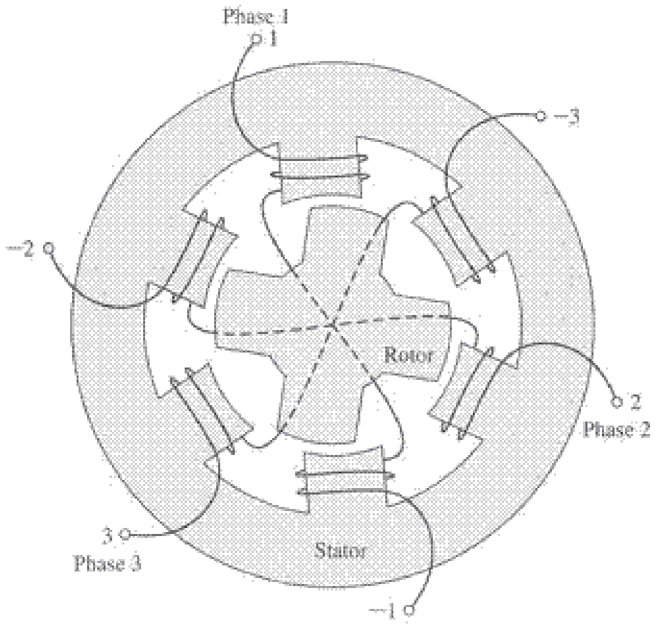


Figure 8.5 Cross-sectional view of a 6/4 three-phase VRM.

- Figure 8.5 shows a fundamental feature of the 6/4 machine is that no such simultaneous alignment of phase inductances is possible. As a result, this machine does not have any zero-torque positions. This is a significant point because it eliminates the possibility that the rotor might get stuck in one of these positions at standstill, requiring that it be mechanically moved to a new position before it can be started. In addition to the fact that there are not positions of simultaneous alignment for the 6/4 VRM, it can be seen that there also are no rotor positions at which only a torque of a single sign (either positive or negative) can be produced. Hence by proper control of the phase currents, it should be possible to achieve constant-torque, independent of rotor position.
- In the case of a symmetric VRM with P_s stator poles and P_r rotor poles, a simple test can be used to determine if zero-torque positions exist. If the ratio P_s/P_r(or alternatively P_r/P_s if P_r is larger than P_s) is an integer, there will be zero-torque positions. For example, for a 6/4 machine the ratio is 1.5, and there will be no zero- torque positions. However, the ratio is 2.0 for a 6/3 machine, and there will be zero-torque positions.

- In some instances, design constraints may dictate that a machine with an integral pole ratio is desirable. In these cases, it is possible to eliminate the zero-torque positions by constructing a machine with an asymmetric rotor. The rotor radius can be made to vary with angle as shown in grossly exaggerated fashion in Fig. 8.6a. This design, which also requires that the width of the rotor pole be wider than that of the stator, will not produce zero torque at positions of alignment because $\mathrm{dL}(\theta_m)/d\theta_m$ is not zero at these points, as can be seen with reference to Fig. 8.6b.

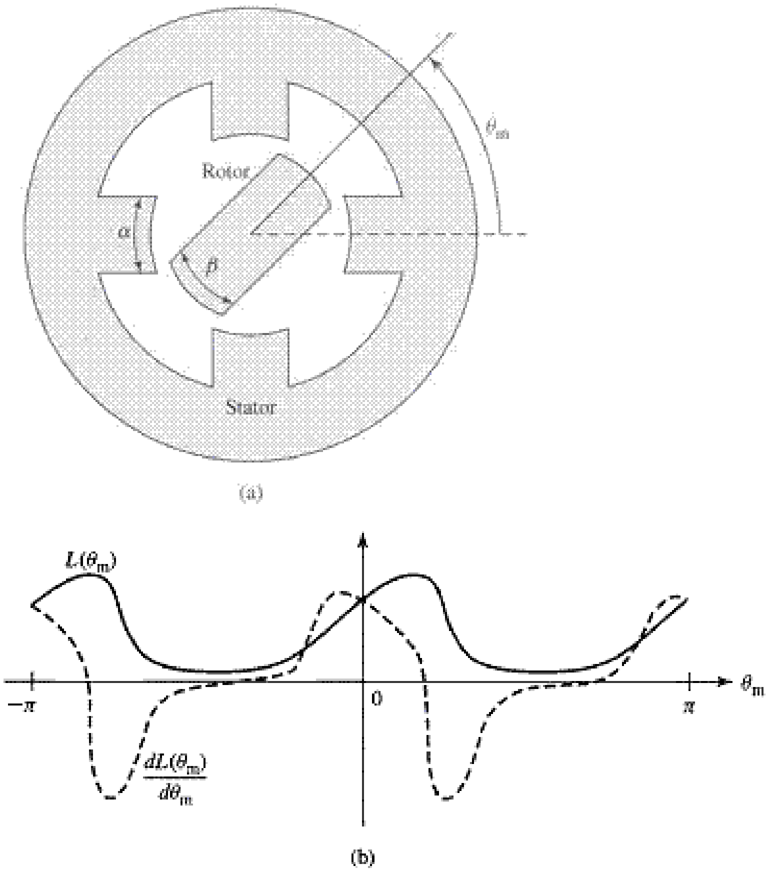


Figure 8.6 A 4/2 VRM with nonuniform air gap: (a) exaggerated schematic view and (b) plots of $L(\theta_m)$ and $\mathrm{dL}(\theta_m)/d\theta_m$versus θ_m.

- An alternative procedure for constructing an integral-pole-ratio VRM without zero-torque positions is to construct a stack of two or more VRMs in series, aligned such that each of the VRMs is displaced in angle from the others and with all rotors sharing a common shaft. In this fashion, the zero-torque positions of the individual machines will not align, and thus the complete machine will not have any torque zeros. For example, a series stack of two two-phase, 4/2 VRMs such as that of Example 8.1 (Fig. 8.3) with a 45^o angular displacement between the individual VRMs will result in a four-phase VRM without zero-torque positions.

- Generally VRMs are wound with a single coil on each pole. Although it is possible to control each of these windings separately as individual phases, it is common practice to combine them into groups of poles which are excited simultaneously. For example, the 4/2 VRM of Fig. 8.3 is shown connected as a two-phase machine. As shown in Fig. 8.5, a 6/4 VRM is commonly connected as a three-phase machine with opposite poles connected to the same phase and in such a fashion that the windings drive flux in the same direction through the rotor.
- In some cases, VRMs are wound with a parallel set of windings on each phase. This configuration, known as a bifilar winding, in some cases can result in a simple inverter configuration and thus a simple, inexpensive motor drive.
- When a given phase is excited, the torque is such that the rotor is pulled to the nearest position of maximum flux linkage. As excitation is removed from that phase and the next phase is excited, the rotor "follows" as it is then pulled to a new maximum flux-linkage position. Thus, the rotor speed is determined by the frequency of the phase currents.
- However, unlike the case of a synchronous machine, the relationship between the rotor speed and the frequency and sequence of the phase winding excitation can be quite complex, depending on the number of rotor poles and the number of stator poles and phases.
- EXAMPLE 8.2

Consider a four-phase, 8/6 VRM. If the stator phases are excited sequentially, with a total time of To sec required to excite the four phases (i.e., each phase is excited for a time of To/4 sec), find the angular velocity of the stator flux wave and the corresponding angular velocity of the rotor. Neglect any system dynamics and assume that the rotor will instantaneously track the stator excitation.

Solution

Figure 8.7 shows in schematic form an 8/6 VRM. The details of the pole shapes are not of importance for this example and thus the rotor and stator poles are shown simply as arrows indicating their locations. The figure shows the rotor aligned with the stator phase-1 poles. This position corresponds to that which would occur if there were no load on the rotor and the stator phase-1 windings were excited, since it corresponds to a position of maximum phase-1 flux linkage.

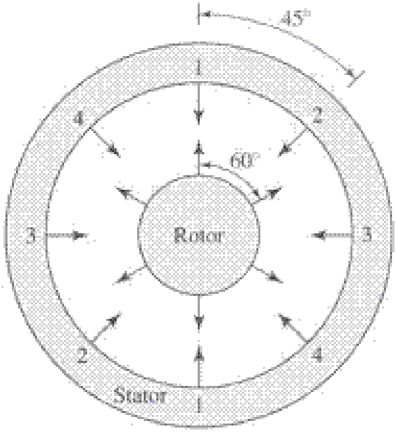


Figure 8.7 Schematic view of a four-phase 8/6 VRM. Pole locations are indicated by arrows.

Consider next that the excitation on phase 1 is removed and phase 2 is excited. At this point, the stator flux wave has rotated 45^o in the clockwise direction. Similarly, as the excitation on phase 2 is removed and phase 3 is excited, the stator flux wave will move an additional 45^o clockwise. Thus the angular velocity ω_s of the stator flux wave can be calculated quite simply as $\pi/4$ rad divided by T_o/4 sec, or $\omega_s = \pi/T_o$rad/sec.

Note, however, that this is not the angular velocity of the rotor itself. As the phase-1 excitation is removed and phase 2 is excited, the rotor will move in such a fashion as to maximize the phase-2 flux linkages. In this case, Fig. 8.7 shows that the rotor will move 15^o counterclockwise since the nearest rotor poles to phase 2 are actually 15^0 ahead of the phase-2 poles. Thus the angular velocity of the rotor can be calculated as $-\pi/12$ rad (15^o, with the minus sign indicating counterclockwise rotation) divided by $T_o/4$ see, or $\omega_m = -\pi/(3\mathrm{T}_o)$rad/sec.

In this case, the rotor travels at one-third the angular velocity of the stator excitation and in the opposite direction.

- Example 8.2 illustrates the complex relationship that can exist between the excitation frequency of a VRM and the "synchronous" rotor frequency. This relationship is directly analogous to that between two mechanical gears for which the choice of different gear shapes and configurations gives rise to a wide range of speed ratios. It is difficult to derive a single rule which will describe this relationship for the immense variety of VRM configurations which can be envisioned.
- Further variations on VRM configurations are possible if the main stator and rotor poles are subdivided by the addition of individual teeth (which can be thought of as a set of small poles excited simultaneously by a single winding). The basic concept is illustrated in Fig. 8.8, which shows a schematic view of three poles of a three-phase VRM with a total of six main stator poles. Such a machine, with the stator and rotor poles subdivided into teeth, is known as a castleated VRM.
- In Fig. 8.8 each stator pole has been divided into four subpoles by the addition of four teeth of width $6\frac{3}{7}^o$ (indicated by the angle β in the figure), with a slot of the same width between each tooth. The same tooth/slot spacing is chosen for the rotor, resulting in a total of 28 teeth on the rotor. Notice that this number of rotor teeth and the corresponding value of β were chosen so that when the rotor teeth are aligned with those of the phase- 1 stator pole, they are not aligned with those of phases 2 and 3. In this fashion, successive excitation of the stator phases will result in a rotation of the rotor.
- Castleation further complicates the relationship between the rotor speed and the frequency and sequence of the stator-winding excitation. For example, from Fig. 8.8

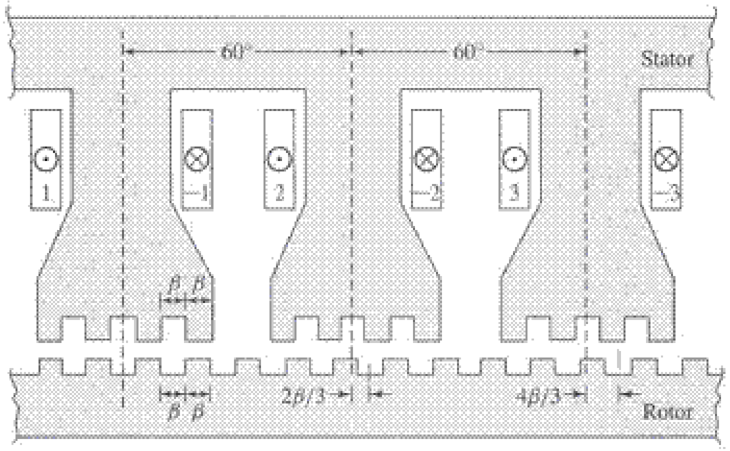


Figure 8.8 Schematic view of a three-phase castleated VRM with

six stator poles and four teeth per pole and 28 rotor poles.

it can be seen that for this configuration, when the excitation of phase 1 is removed and phase 2 is excited (corresponding to a rotation of the stator flux wave by 60^0in the clockwise direction), the rotor will rotate by an angle of $\frac{2\beta}{3} = 4\frac{2}{7}^{o}$ in the counterclockwise direction.

- The technique of castleation can be used to create VRMs capable of operating at low speeds (and hence producing high torque for a given stator power input) and with very precise rotor position accuracy. The machine of Fig. 8.8 can be rotated precisely by angular increments of $2\beta/3$. The use of more teeth can further increase the position resolution of these machines. Such machines can be found in applications where low speed, high torque, and precise angular resolution are required. This castleated configuration is one example of a class of VRMs commonly referred to as stepping motors because of their capability to produce small steps in angular resolution.

8.3 CURRENT WAVEFORMS FOR TORQUE PRODUCTION

- The torque produced by a VRM in which saturation and mutual-inductance effects can be neglected is determined by the summation of terms consisting of the derivatives of the phase inductances with respect to the rotor angular position, each multiplied by the square of the corresponding phase current. The torque of the two-phase, 4/2 VRM of Fig. 8.1b is given by

$$T_{\text{mech}} = \frac{1}{2} i_1^2 \frac{\mathrm{dL}_{11}(\theta_m)}{d\theta_m} + \frac{1}{2} i_2^2 \frac{\mathrm{dL}_{22}(\theta_m)}{d\theta_m} = \frac{1}{2} i_1^2 \frac{\mathrm{dL}_{11}(\theta_m)}{d\theta_m} + \frac{1}{2} i_2^2 \frac{\mathrm{dL}_{11}(\theta_m - 90^0)}{d\theta_m}$$

(8.9)

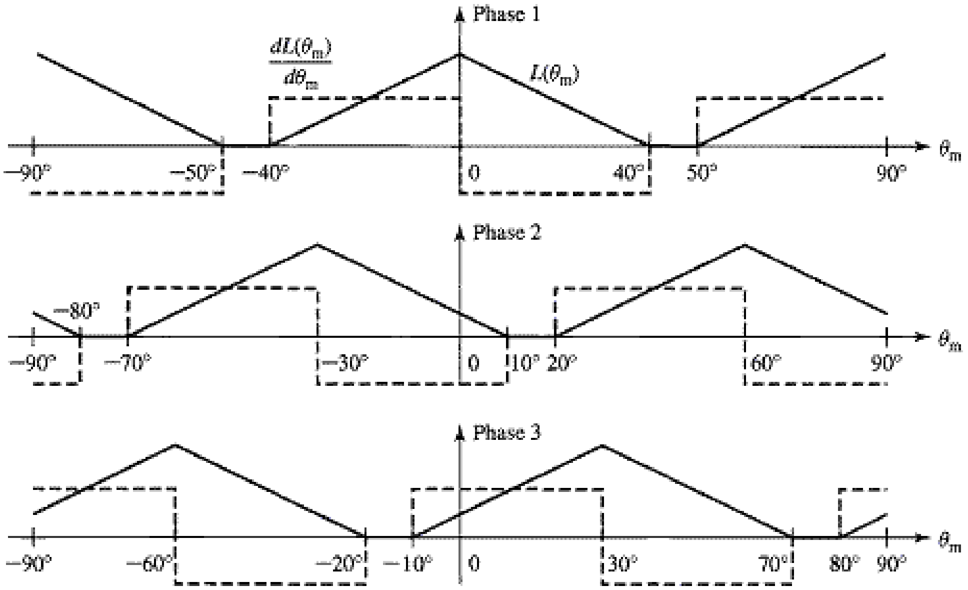


Figure 8.9 Idealized inductance and $\mathrm{dL}/d\theta_m$ curves for a three-phase 6/4 VRM with

40^o rotor and stator poles.

- For each phase of a VRM, the phase inductance is periodic in rotor angular position, and thus the area under the curve of $\mathrm{dL}/d\theta_m$ calculated over a complete period of $L(\theta_m)$ is zero, i.e.,

$$\int_0^{2\pi/\mathrm{Pr}} \frac{\mathrm{dL}(\theta_m)}{d\theta_m} d\theta_m = L(2\pi/P_r) - L(0) = 0(8.10)$$

where P_r is the number of rotor poles.

- The average torque produced by a VRM can be found by integrating the torque equation (Eq.8.9) over a complete period of rotation. Clearly, if the stator currents are held constant, Eq. 8.10 shows that the average torque will be zero. Thus, to produce a time-averaged torque, the stator currents must vary with rotor position. The desired average output torque for a VRM depends on the nature of the application. For example, motor operation requires a positive time-averaged shaft torque. Similarly, braking or generator action requires negative time-averaged torque.

- Positive torque is produced when a phase is excited at angular positions with positive $\mathrm{dL}/d\theta_m$for that phase, and negative torque is produced by excitation at positions at which $\mathrm{dL}/d\theta_m$ is negative. Consider a three-phase, 6/4 VRM (similar to that shown in Fig. 8.5) with 40 ° rotor and stator poles. The inductance versus rotor position for this machine will be similar to the idealized representation shown in Fig. 8.9.
- Operation of this machine as a motor requires a net positive torque. Alternatively, it can be operated as a generator under conditions of net negative torque. Noting that

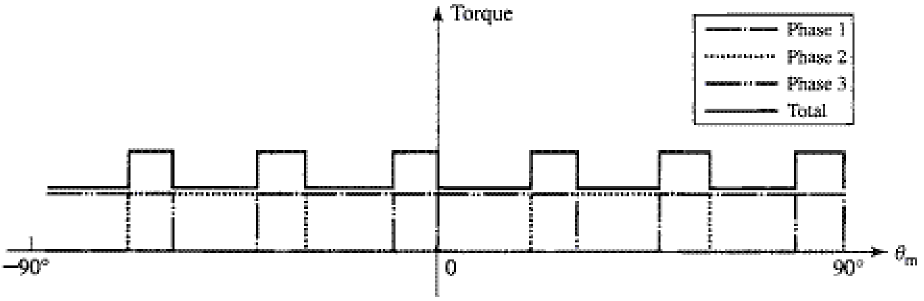


Figure 8.10 Individual phase torques and total torque for the motor of Fig. 8.9. Each

phase is excited with a constant current I_0 only at positions where $\mathrm{dL}/d\theta_m > 0$.

positive torque is generated when excitation is applied at rotor positions at which $\mathrm{dL}/d\theta_m$ is positive, we see that a control system is required that determines rotor position and applies the phase-winding excitations at the appropriate time. It is, in fact, the need for this sort of control that makes VRM drive systems more complex than might perhaps be thought, considering only the simplicity of the VRM itself.

- One of the reasons that VRMs have found application in a wide variety of situations is because the widespread availability and low cost of microprocessors and power electronics have brought the cost of the sensing and control required to successfully operate VRM drive systems down to a level where these systems can be competitive with competing technologies. Although the control of VRM drives is more complex than that required for dc, induction, and permanent-magnet ac motor systems, in many applications the overall VRM drive system turns out to be less expensive and more flexible than the competition.
- Assuming that the appropriate rotor-position sensor and control system is available, the question still remains as to how to excite the armature phases. From Fig. 8.9, one possible excitation scheme would be to apply a constant current to each phase at those angular positions at which $\mathrm{dL}/d\theta_m$ is positive and zero current otherwise.
- If this is done, the resultant torque waveform will be that of Fig. 8.10. Note that because the torque waveforms of the individual phases overlap, the resultant torque will not be constant but rather will have a pulsating component on top of its average value. In general, such pulsating torques are to be avoided both because they may produce damaging stresses in the VRM and because they may result in the generation of excessive vibration and noise.
- Consideration of Fig.8.9 shows that there are alternative excitation strategies which can reduce the torque pulsations of Fig. 8.10. Perhaps the simplest strategy is to excite each phase for only 30^0 of angular position instead of the 40^o which resulted in Fig. 8.9. Thus, each phase would simply be turned off as the next phase is turned on, and there would be no torque overlap between phases.
- Although this strategy would be an ideal solution to the problem, as a practical matter it is not possible to implement. The problem is that because each phase winding has a self-inductance, it is not possible to instantaneously switch on or off the phasecurrents.
- For a VRM with independent (uncoupled) phases, 2 the voltage-current relationship of the jth phase is given by

$$v_j = R_j i_j + \frac{d\lambda_j}{\mathrm{dt}} (8.11)$$

where

$\lambda_i = L_{\text{jj}}(\theta_m)i_j$(8.12)

Thus,

$v_j = R_j i_j + \frac{d}{\text{dt}}[L_{\text{jj}}(\theta_m)i_j]$(8.13)

Equation 8.13 can be rewritten as

$v_j = \left\{R_j + \frac{d}{\text{dt}}[L_{\text{jj}}(\theta_m)]\right\}i_j + L_{\text{jj}}(\theta_m)\frac{\text{di}_j}{\text{dt}}$(8.14)

or

$v_j = \left[R_j + \frac{\text{dL}_{\text{jj}}(\theta_m)}{d(\theta_m)}\frac{d\theta_m}{\text{dt}}\right]i_j + L_{\text{jj}}(\theta_m)\frac{\text{di}_j}{\text{dt}}$(8.15)

- Eqs. 8.13 through 8.15 are mathematically complex, they clearly indicate that some time is required to build up currents in the phase windings following application of voltage to that phase. A similar analysis can be done for conditions associated with removal of the phase currents. The delay time associated with current build up can limit the maximum achievable torque while the current decay time can result in negative torque if current is still flowing when $\text{dL}(\theta_m)/d\theta_m$ reverses sign.
- It is clear that it is not possible to readily apply phase currents of arbitrary waveshapes. Winding inductances (and their time derivatives) significantly affect the current waveforms that can be achieved for a given applied voltage.
- Figure 8.12a shows the cross- sectional view of a 4/2 VRM similar to that of Fig. 8.3 with the exception that the rotor pole angle has been increased from 60^o to 75^0, with the result that the rotor pole overhangs that of the stator by 15^0. As can be seen from Fig. 8.12b, this results in a region of constant inductance separating the positive and negative $\text{dL}(\theta_m)/d\theta_m$ regions, which in turn provides additional time for the phase current to be turned off before the region of negative torque production is reached.

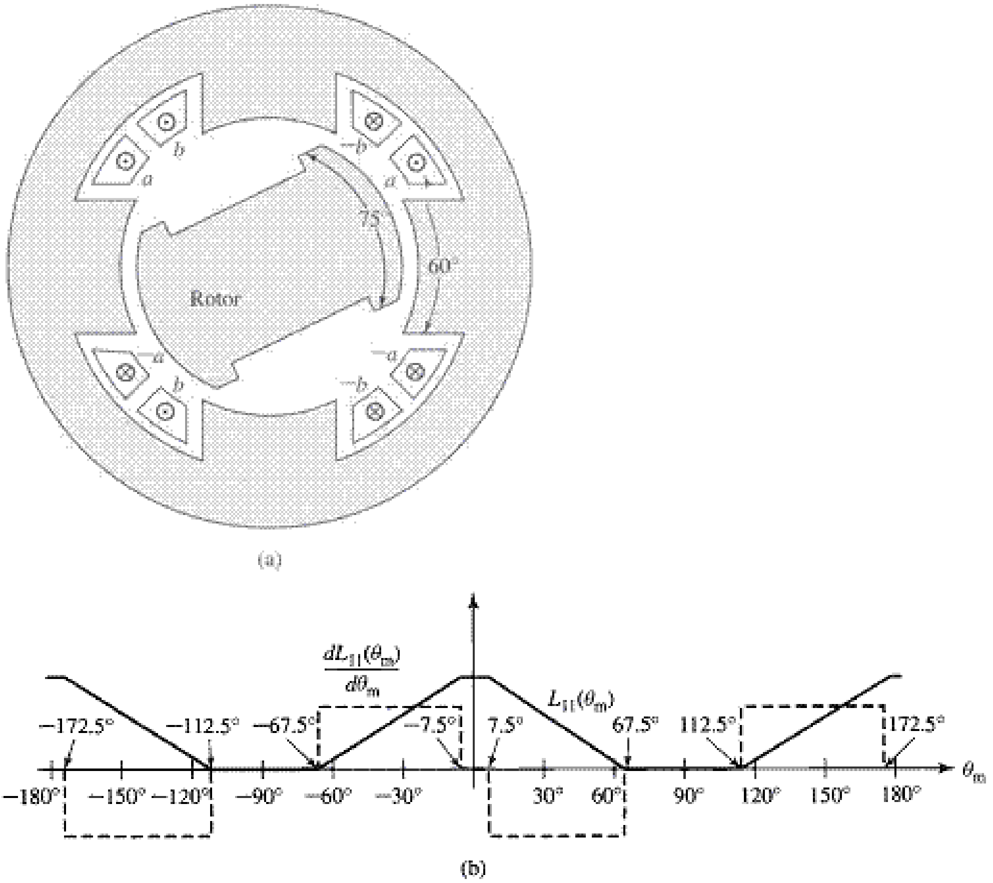


Figure 8.12 A 4/2 VRM with 15^0 rotor overhang: (a) cross-sectional view and (b) plots of $L_{11}(\theta_m)$ and $L_{11}(\theta_m)/d\theta_m$ versus θ_m.

8.4 NONLINEAR ANALYSIS

- To obtain the maximum benefit from the magnetic material, practical VRMs are operated with the magnetic flux density high enough so that the magnetic material is in saturation under normal operating conditions.
- The actual operating flux density is determined by trading off such quantities as cost, efficiency, and torque-to-mass ratio. However, because the VRM and its drive electronics are quite closely interrelated, VRMs design typically involves additional trade-offs that in turn affect the choice of operating flux density.

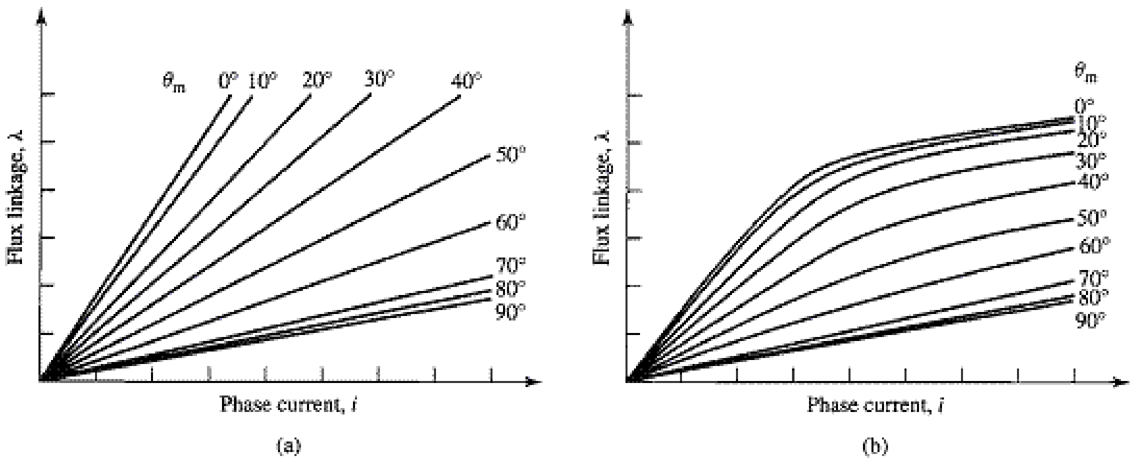


Figure 8.13 Plots of λ versus i for a VRM with (a)linear and (b) nonlinear magnetics.

- A representation of the flux-linkage versus current characteristic of a VRM is shown in Fig. 8.13. This representation consists of a series of plots of the flux linkage versus current at various rotor angles. In this figure, the curves correspond to a machine with a two-pole rotor such as in Fig. 8.1, and hence a plot of curves from 0^0 to 90^0 is sufficient to completely characterize the machine.
- Figure 8.13a shows the set of λ-i characteristics which would be measured in a machine with linear magnetics, i.e., constant magnetic permeability and no magnetic saturation. For each rotor angle, the curve is a straight line whose slope corresponds to the inductance $L(\theta_m)$at that angular position. In fact, a plot of L(θ_m) versus θ_m such as in Fig. 8.2 is an equivalent representation to that of Fig. 8.13a.
- In practice, VRMs do operate with their magnetic material in saturation and their λ-i characteristics take on the form of Fig. 8.13b. Note that saturation effects are maximum at $\theta_m = 0^o$(for which the rotor and stator poles are aligned) and minimal for higher angles as the rotor approaches the nonaligned position.
- Saturation has two important, somewhat contradictory effects on VRM performance. On the one hand, saturation limits flux densities for a given current level and thus tends to limit the amount of torque available from the VRM. On the other hand, it can be shown that saturation tends to lower the required inverter volt-ampere rating

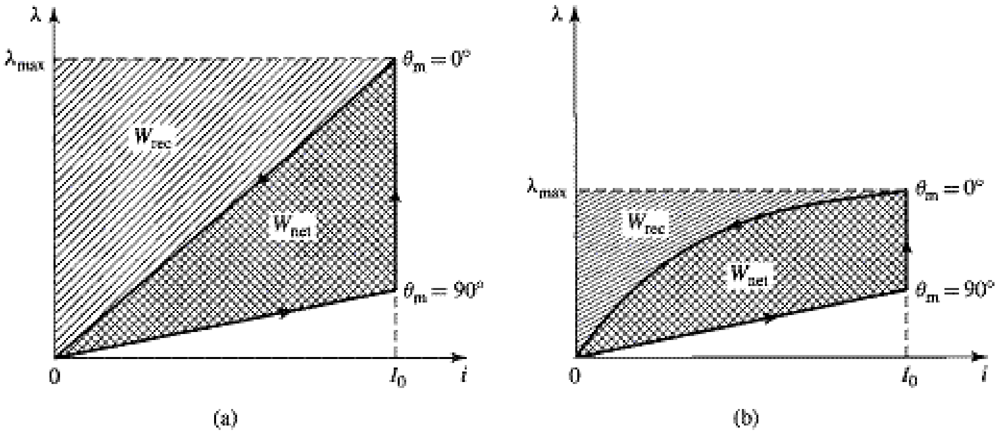


Figure 8.14 (a) Flux-linkage-current trajectory for the (a) linear and (b) nonlinear

machines of Fig. 8.13.

for a given VRM output power and thus tends to make the inverter smaller and less costly. A well-designed VRM system will be based on a trade-off between the two effects.

- These effects of saturation can be investigated by considering the two machines of Figs. 8.13a and b operating at the same rotational speed and under the same operating condition. Assume a somewhat idealized condition in which the phase-1 current is instantaneously switched on to a value I_0 at $\theta_m = -90^o$ (the unaligned position for phase 1) and is instantaneously switched off at $\theta_m = 0^0$ (the aligned position).
- Because of rotor symmetry, the flux linkages for negative rotor angles are identical to those for positive angles. Thus, the flux linkage-current trajectories for one current cycle can be determined from Figs.8.13a and b and are shown for the two machines in Figs.8.14a and b.
- As each trajectory is traversed, the power input to the winding is given by its volt-ampere product

$$p_{\text{in}} = \text{iv} = i\frac{d\lambda}{\text{dt}} \text{(8.16)}$$

The net electric energy input to the machine (the energy that is converted to mechanical work) in a cycle can be determined by integrating Eq. 8.16

around the trajectory

$$\text{Net work} = \int p_{\text{in}} \text{dt} = \oint \text{id}\lambda (8.17)$$

This can be seen graphically as the area enclosed by the trajectory, labeled Wnet in Figs.8.14a and b. Note that the saturated machine converts less useful work per cycle than the unsaturated machine. As a result, to get a machine of the same power output, the saturated machine will have to be larger than a corresponding (hypothetical) unsaturated machine. This analysis demonstrates the effects of saturation in lowering torque and power output.

- The peak energy input to the winding from the inverter can also be calculated.:

$$\text{Peak energy} = \int_0^{\lambda_{\max}} \text{id}\lambda (8.18)$$

This is the total area under the λ-i curve, shown in Fig. 8.14a and b as the sum of the areas labeled W_{rec} and W_{net}.

- The energy represented by the area W_{net} corresponds to useful output energy, it is clear that the energy represented by the area W_{rec} corresponds to energy input that is required to make the VRM operate (i.e., it goes into creating the magnetic fields in the VRM). This energy produced no useful work; rather it must be recycled back into the inverter at the end of the trajectory.
- The inverter volt-ampere rating is determined by the average power per phase processed by the inverter as the motor operates, equal to the peak energy input to the VRM divided by the time T between cycles. Similarly, the average output power per phase of the VRM is given by the net energy input per cycle divided by T. The ratio of the inverter volt-ampere rating to power output is

$$\frac{\text{Inverter volt}-\text{ampere rating}}{\text{Net output area}} = \frac{\text{area}(W_{\text{rec}} + W_{\text{net}})}{\text{area}(W_{\text{net}})} \; (8.19)$$

- The inverter volt-ampere rating determines its cost and size. Thus, for a given power output from a VRM, a smaller ratio of inverter volt-ampere rating to output power means that the inverter will be both smaller and cheaper. Comparison of Figs. 8.14a and b shows that this ratio is smaller in the machine which saturates; the effect of saturation is to lower the amount of energy which must be recycled each cycle and hence the volt-ampere rating of the inverter required to supply the VRM.

8.5 STEPPING MOTORS

- When the phases of a VRM are energized sequentially in an appropriate step-wise fashion, the VRM will rotate a specific angle for each step. Motors designed specifically to take advantage of this characteristic are referred to as stepping motors or stepper motors. Frequently stepping motors are designed to produce a large number of steps per revolution, for example 50, 100, or 200 steps per revolution (corresponding to a rotation of 7.2^0,3.6^0 and 1.8^oper step).
- An important characteristic of the stepping motor is its compatibility with digital- electronic systems. These systems are common in a wide variety of applications and continue to become more powerful and less expensive. Typical applications include paper-feed and print-head-positioning motors in printers and plotters, drive and head-positioning motors in disk drives and CD players, and worktable and tool positioning in numerically controlled machining equipment. In many applications, position information can be obtained simply by keeping count of the pulses sent to the motor, in which case position sensors and feedback control are not required.
- The angular resolution of a VRM is determined by the number of rotor and stator teeth and can be greatly enhanced by techniques such as castleation. Stepping motors come in a wide variety of designs and configurations. In addition to variable-reluctance configurations, these include permanent-magnet and hybrid configurations. The use of permanent magnets in combination with a variable-reluctance geometry can significantly enhance the torque and positional accuracy of a stepper motor.
- The VRM configurations discussed in Sections 8.1 through 8.3 consist of a single rotor and stator with multiple phases. A stepping motor of this configuration is called a single-stack, variable-reluctance stepping

motor. An alternate form of variable- reluctance stepping motor is known as a multistack variable-reluctance stepping motor. In this configuration, the motor can be considered to be made up of a set of axially displaced, single-phase VRMs mounted on a single shaft.

- Figure 8.16 shows a multistack variable-reluctance stepping motor. This type of motor consists of a series of stacks, each axially displaced, of identical geometry and each excited by a single phase winding, as shown in Fig. 8.17. The motor of Fig. 8.16 has three stacks and three phases, although motors with additional phases and stacks are common. For an ns-stack motor, the rotor or stator (but not both) on each stack is displaced by 1 / ns times the pole-pitch angle. In Fig. 8.16, the rotor poles are aligned, but the stators are offset in angular displacement by one-third of the pole pitch. By successively exciting the individual phases, the rotor can be turned in increments of this displacement angle.
- A schematic diagram of a two-phase stepping motor with a permanent-magnet, two-pole rotor is shown in Fig. 8.18. Note that this machine is in fact a two-phase synchronous machine, similar for example to the three-phase permanent-magnet ac machine of Fig.5.29. The distinction between such a stepping motor and a synchronous motor arises not from the construction of the motor but rather from how the motor is operated. The synchronous motor is typically intended to drive a load at a specified speed, and the stepping motor is typically intended to control the position of a load.

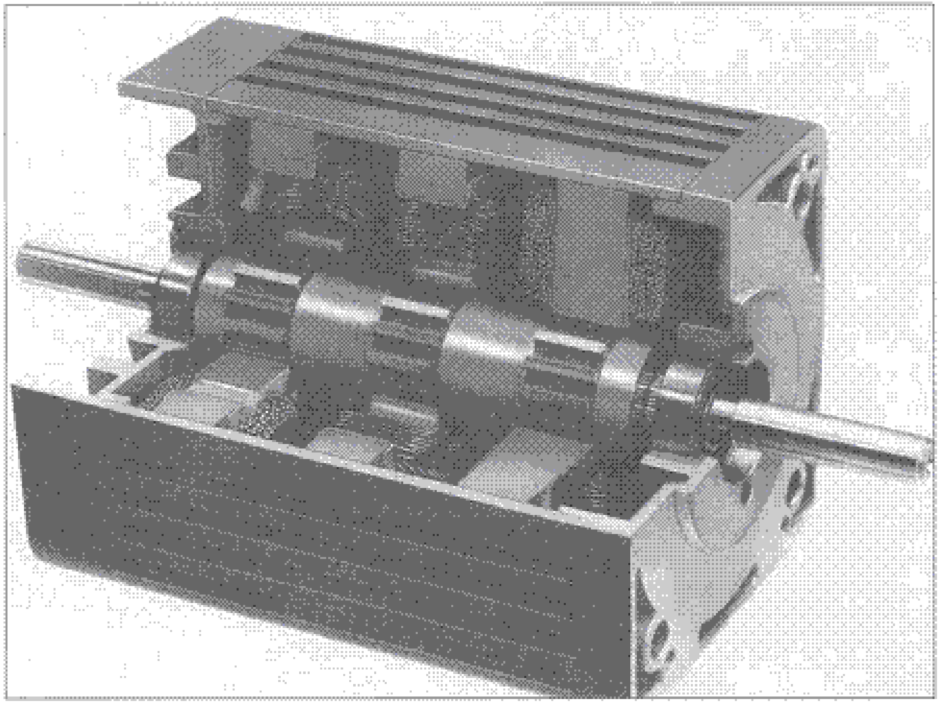


Figure 8.16 Cutaway view of a three-phase, three-stack variable-reluctance stepping

motor. (Warner Electric.)

- The rotor of the stepping motor of Fig.8.18 assumes the angles $\theta_m = 0$,45^0,90^0 ... as the windings are excited in the sequence:

1. Positive current in phase 1 alone.

2. Equal-magnitude positive currents in phase 1 and phase 2.

3. Positive current in phase 2 alone.

4. Equal-magnitude negative current in phase 1 and positive current in phase 2.

5. Negative current in phase 1 alone.

6. And so on.

Note that if a ferromagnetic rotor were substituted for the permanent-magnet rotor, the rotor would move in a similar fashion.

- The stepping motor of Fig.8.18 can also be used for 90^0steps by exciting the coils singly. In the latter case, only a permanent-magnet rotor can be used. This can be seen from the torque-angle curves for the two types of rotors shown in Fig.8.19. Whereas the permanent-magnet rotor produces peak torque when the excitation is shifted 90^0, the ferromagnetic rotor produces zero torque and may move in either direction.
- The rotor position in the permanent-magnet stepping motor of Fig.8.18 is defined by the winding currents with no ambiguity and depends on the direction of the phase

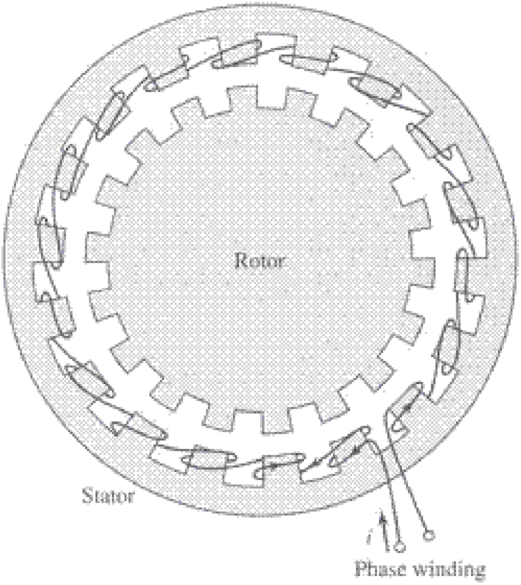


Figure 8.17 Diagram of one stack and phase of a multiphase, multistack variable-reluctance stepping motor, such as that in Fig. 8.16. For an n_s-stack motor, the rotor or stator (but not both) on each stack is displaced by 1/ n_s times the pole pitch.

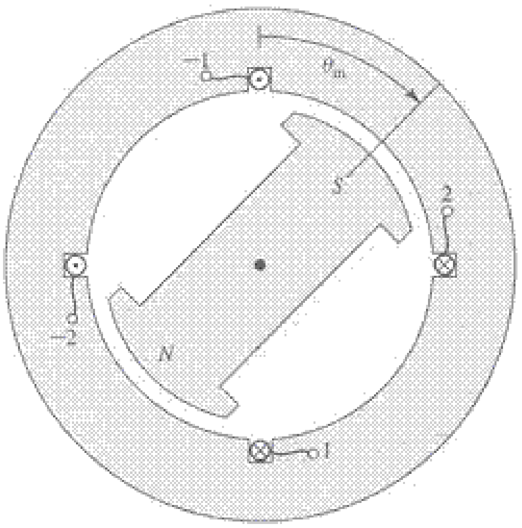


Figure 8.18 Schematic diagram of a two-phase permanent-magnet stepping motor.

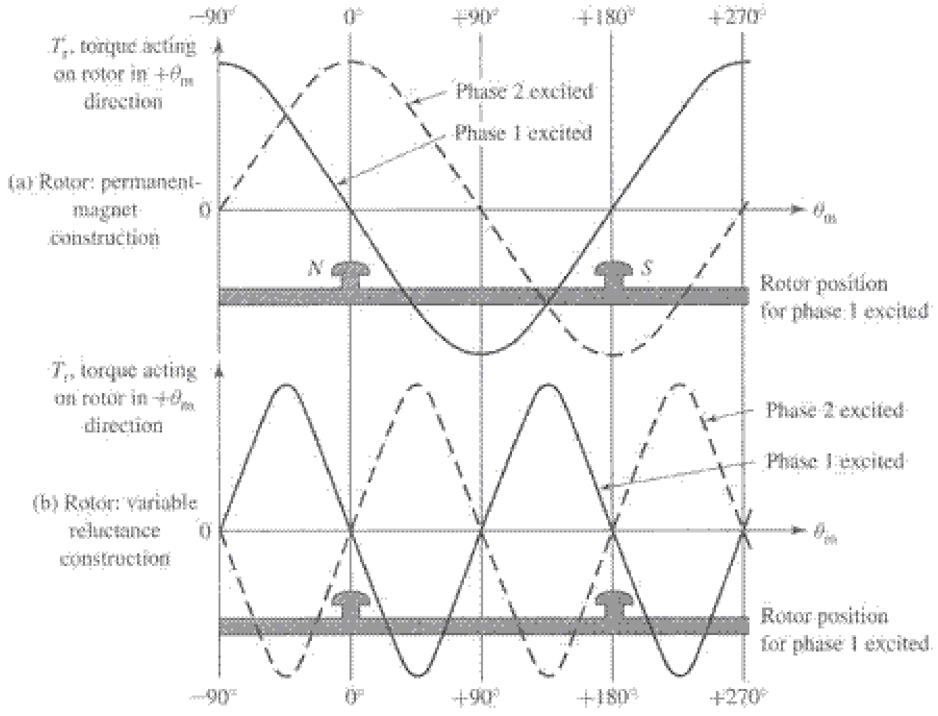


Figure 8.19 Torque-angle curves for the stepping motor of Fig. 8.18:

(a) permanent-magnet rotor and (b) variable-reluctance rotor.

currents.

- Reversing the phase currents will cause the rotor to reverse its orientation. This is in contrast to VRM configuration with a ferromagnetic rotor, in which two rotor positions are equally stable for any particular set of phase currents, and hence the rotor position cannot be determined uniquely.
- Permanent-magnet stepping motors are also unlike their VRM counterparts in that torque tending to align the rotor with the stator poles will be generated even when there is no excitation applied to the phase windings. Thus the rotor will have preferred unexcited rest positions, a fact which can be used to advantage in some applications.

- Stable equilibrium positions of an unloaded stepping motor satisfy the conditions that there is zero torque, i.e.,

$$T_{\text{mech}} = 0 \quad (8.20)$$

and that there is positive restoring torque, i.e.,

$$\frac{\partial T_{\text{mech}}}{\partial \theta_m} \bigg|_{i_1,i_2} < 0 \quad (8.21)$$

- In practice, there will of course be a finite load torque tending to perturb the stepping motor from these idealized positions. For open-loop control systems (i.e., control systems in which there is no mechanism for position feedback), a high-degree of position control can be achieved by designing the stepping motor to produce large restoring torque (i.e., a large magnitude of $\partial T_{\text{mech}}/\partial \theta_m$). In such a stepping motor, load torques will result in only a small movement of the rotor from the idealized positions which satisfies Eqs.8.20 and 8.21.
- Carefully controlled combinations of phase currents can enhance the resolution of a stepper motor. This technique, referred to as microstepping, can be used to achieve increased step resolution of a wide variety of stepper motors. Microstepping can be used to produce extremely fine position resolution. The increased resolution comes, however, at the expense of an increase in complexity of the stepping-motor drive electronics and control algorithms, which must accurately control the distribution of current to multiple phases simultaneously.

- The hybrid stepping motor combines characteristics of the variable-reluctance and permanent-magnet stepping motors. A photo of a hybrid stepping motor is shown in Fig. 8.20, and a schematic view of a hybrid stepping motor is shown in Fig. 8.21. The hybrid-stepping-motor rotor configuration appears much like that of a multistack variable-reluctance stepping motor. In the rotor of Fig. 8.21 a, two identical rotor stacks are displaced axially along the rotor and displaced in angle by one-half the rotor pole pitch, while the stator pole structure is continuous along the length of the rotor. Unlike the multistack variable-reluctance stepping motor, in the hybrid stepping motor, the rotor stacks are separated by an axially-directed permanent magnet. As a result, in Fig. 8.21 a one end of the rotor can be considered to have a north magnetic pole and

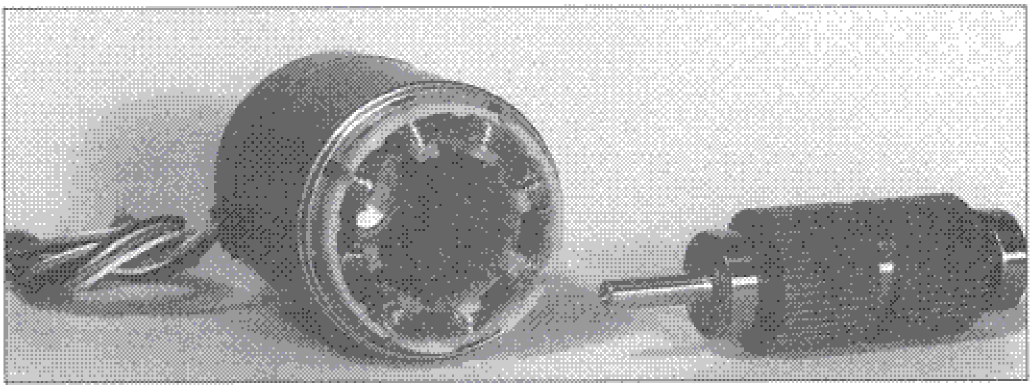
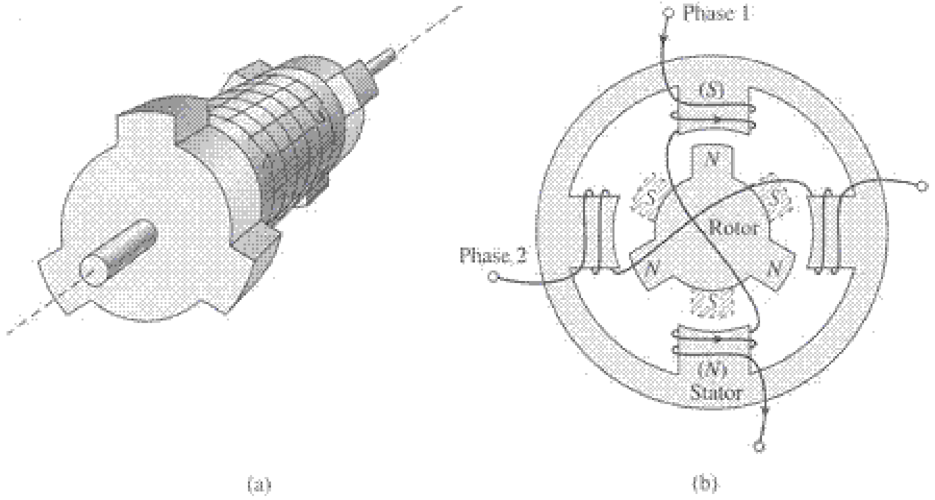


Figure 8.20 Disassembled 1.8^0/step hybrid stepping motor. (Oriental Motor.)



(a)

(b)

Figure 8.21 Schematic view of a hybrid stepping motor. (a) Two-stack rotor showing the axially-directed permanent magnet and the pole pieces displaced by one-half the pole pitch. (b) End view from the rotor north poles and showing the rotor south poles at the far end (shown crosshatched). Phase 1 of the stator is energized to align the rotor as shown.

the other end a south magnetic pole. Figure 8.2 lb shows a schematic end view of a hybrid stepping motor. The stator has four poles with the phase-1 winding wound on the vertical poles and the phase-2 winding wound on the horizontal poles. The rotor is shown with its north-pole end at the near end of the motor and the south-pole end (shown crosshatched) at the far end.

- In Fig. 8.21b, phase 1 is shown excited such that the top stator pole is a south pole while the bottom pole is a north pole. This stator excitation interacts with the permanent-magnet flux of the rotor to align the rotor with a pole on its north-pole end vertically upward and a pole on its south-pole end vertically downward, as shown in the figure. Note that if the stator excitation is removed, there will still be a permanent-magnet torque tending to maintain the rotor in the position shown.
- To turn the rotor, excitation is removed from phase 1, and phase 2 is excited. If phase 2 is excited such that the fight-hand stator pole is a south pole and the left-hand one is a north pole, the rotor will rotate 30^0 counterclockwise. Similarly, if the opposite excitation is applied to the phase-2 winding, a 30^0 rotation in the clockwise direction will

occur. Thus, by alternately applying phase-1 and phase-2 excitation of the appropriate polarity, the rotor can be made to rotate in either direction by a specified angular increment.

- Practical hybrid stepping motors are generally built with more rotor poles than are indicated in the schematic motor of Fig. 8.21, in order to give much better angular resolution. Correspondingly, the stator poles are often castleated (see Fig.8.8) to further increase the angular resolution. In addition, they may be built with more than two stacks per rotor.
- The hybrid stepping motor design offers advantages over the permanent-magnet design discussed earlier. It can achieve small step sizes easily and with a simple magnet structure while a purely permanent-magnet motor would require a multipole permanent magnet. In comparison with the variable-reluctance stepping motor, the hybrid design may require less excitation to achieve a given torque because some of the excitation is supplied by the permanent magnet. In addition, the hybrid stepping motor will tend to maintain its position when the stator excitation is removed, as does the permanent-magnet design.

PROBLEMS - chapter 8

PROBLEMS

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

8.1 Repeat Example 8.1 for a machine identical to that considered in the example except that the stator pole-face angle is, β = 45°.

8.2 In the paragraph preceeding Eq. 8.1, the text states that "under the assumption of negligible iron reluctance the mutual inductances between the phases of the doubly-salient VRM of Fig. 8.lb will be zero, with the exception of a small, essentially constant component gyyassociated with leakage flux." Neglect any leakage flux effects and use magnetic circuit techniques to show that this statement is true.

8.3 Use magnetic-circuit techniques to show that the phase-to-phase mutual inductance in the 6/4 VRM of Fig. 8.5 is zero under the assumption of infinite rotor- and stator-iron permeability. Neglect any contributions of leakage flux.

8.4 A 6/4 VRM of the form of Fig. 8.5 has the following properties:

Stator pole angle $\beta = 30^o$

Rotor pole angle $\alpha = 30^o$

Air-gap length g = 0.35 mm

Rotor outer radius R = 5.1 cm

Active length D = 7 cm

This machine is connected as a three-phase motor with opposite poles connected in series to form each phase winding. There are 40 turns per pole (80 turns per phase). The rotor and stator iron can be considered to be of infinite permeability and hence mutual-inductance effects can be neglected.

a. Defining the zero of rotor angle θ_m at the position when the phase-1 inductance is maximum, plot and label the inductance of phase 1 as a function of rotor angle.

b. On the plot of part (a), plot the inductances of phases 2 and 3.

c. Find the phase-1 current I_0 which results in a magnetic flux density of 1.0 T in the air gap under the phase-1 pole face when the rotor is in a position of maximum phase-1 inductance.

d. Assuming that the phase-1 current is held constant at the value found in part (c) and that there is no current in phases 2 and 3, plot the torque as a function of rotor position. The motor is to be driven from a three-phase current-source inverter which can be switched on or off to supply either zero current or a constant current of magnitude I_0 in phases 2 and 3; plot the torque as a function of rotor position.

e. Under the idealized assumption that the currents can be instantaneously switched, determine the sequence of phase currents (as a function of rotor position) that will result in constant positive motor torque, independent of rotor position.

f. If the frequency of the stator excitation is such that a time T_0 = 35 msec is required to sequence through all three phases under the excitation conditions of part (e), find the rotor angular velocity and its direction of rotation.

8.5 In Section 8.2, when discussing Fig. 8.5, the text states: "In addition to the fact that there are not positions of simultaneous alignment for the 6/4 VRM, it can be seen that there also are no rotor positions at which only a torque of a single sign (either positive or negative) can be produced." Show that this statement is true.

8.6 Consider a three-phase 6/8 VRM. The stator phases are excited sequentially, requiting a total time of 15 msec. Find the angular velocity of the rotor in r/min.

8.7 The phase windings of the castleated machine of Fig. 8.8 are to be excited by turning the phases on and off individually (i.e., only one phase can be on at any given time).

a. Describe the sequence of phase excitations required to move the rotor to the fight (clockwise) by an angle of approximately 21.4^o.

b. The stator phases are to be excited as a regular sequence of pulses. Calculate the phase order and the time between pulses required to produce a steady-state rotor rotation of 125 r/min in the counterclockwise direction.

8.8 Replace the 28-tooth rotor of Problem 8.7 with a rotor with 26 teeth.

a. Phase 1 is excited, and the rotor is allowed to come to rest. If the excitation on phase 1 is removed and excitation is applied to phase 2, calculate the resultant direction and magnitude (in degrees) of rotor rotation.

b. The stator phases are to be excited as a regular sequence of pulses. Calculate the phase order and the time between pulses required to produce a steady-state rotor rotation of 80 r/min in the counterclockwise direction.

8.9 The three-phase 6/4 VRM of Problem 8.4 has a winding resistance of 0.15 Ω/phase and a leakage inductance of 4.5 mH in each phase. Assume that the rotor is rotating at a constant angular velocity of 1750 ffmin.

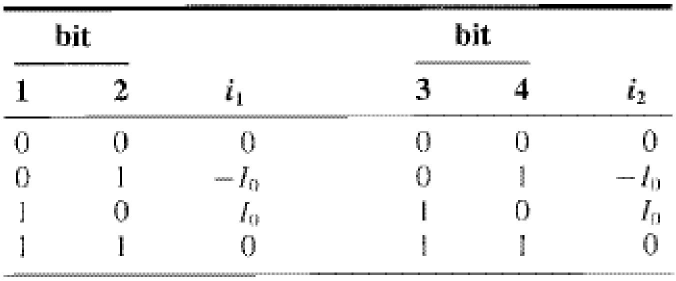
a. Plot the phase-1 inductance as a function of the rotor angle θ_m.

b. A voltage of 75 V is applied to phase 1 as the rotor reaches the position $\theta_m = -30^o$ and is maintained constant until $\theta_m = 0^o$. Calculate and plot the phase-1 current as a function of time during this period.

c. When the rotor reaches $\theta = 0^o$, the applied voltage is reversed so that a voltage of -75 V is applied to the winding. This voltage is maintained until the winding current reaches zero, at which point the winding is open-circuited. Calculate and plot the current decay during the time until the current decays to zero.

d. Calculate and plot the torque during the time periods investigated in parts (b) and (c).

8.10 Consider a two-phase stepper motor with a permanent-magnet rotor such as shown in Fig.8.18 and whose torque-angle curve is as shown in Fig.8.19a. This machine is to be excited by a four-bit digital sequence corresponding to the following winding excitation:



bit			bit		
1	2	i_1	3	4	i_2
0	0	0	0	0	0
0	1	$-I_0$	0	1	$-I_0$
1	0	I_0	1	0	I_0
1	1	0	1	1	0

a. Make a table of 4-bit patterns which will produce rotor angular positions of 0, 45^o,..., 315^o.

b. By sequencing through the bit pattern found in part (a) the motor can be made to rotate. What time interval (in milliseconds) between bit-pattern changes will result in a rotor speed of 1200 r/min?

8.11 Figure 8.22 shows a two-phase hybrid stepping motor with castleated poles on the stator. The rotor is shown in the position it occupies when current is flowing into the positive lead of phase 1.

a. If phase one is turned off and phase 2 is excited with current flowing into its positive lead, calculate the corresponding angular rotation of the rotor. Is it in the clockwise or counterclockwise direction?

b. Describe an excitation sequence for the phase windings which will result in a steady rotation of the rotor in the clockwise direction.

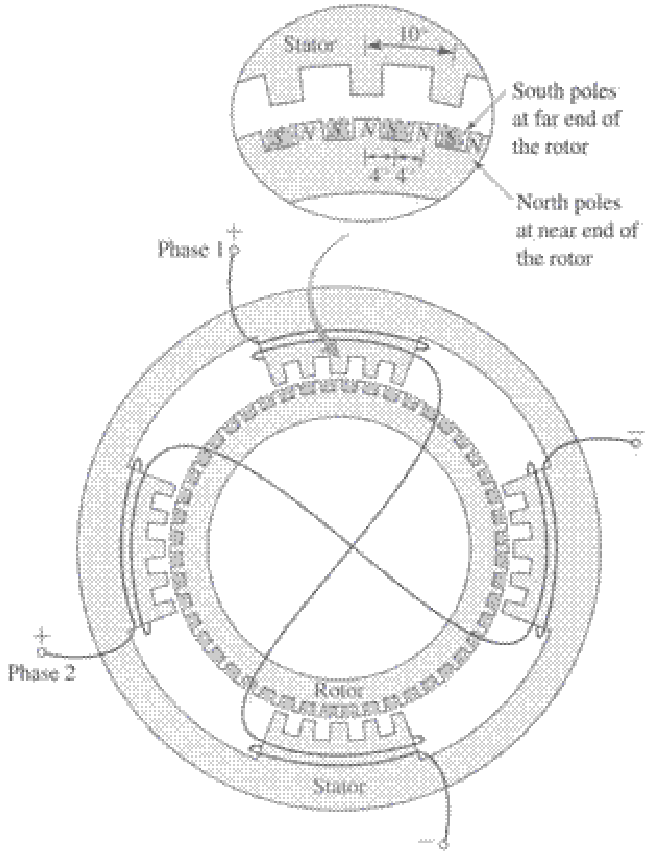

Stator
10°
South poles at far end of the rotor
N
N
N
N
4°
4°
North poles at near end of the rotor
Phase 1
+
−
+
Phase 2
Rotor
Stator
−

Figure 8.22 Castleated hybrid stepping motor

c. Determine the frequency of the phase currents required to achieve a rotor speed of 8 r/min.

8.12 Consider a multistack, multiphase variable-reluctance stepping motor, such as that shown schematically in Fig. 8.17, with 14 poles on each of the rotor and stator stacks and three stacks with one phase winding per stack. The motor is built such that the stator poles of each stack are aligned.

a. Calculate the angular displacement between the rotor stacks.

b. Determine the frequency of phase currents required to achieve a rotor speed of 900 r/min.

Chapter 9: Single- and Two-Phase Motors

Single- and Two-Phase

Motors

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- This chapter discusses single-phase motors. While focusing on induction motors, synchronous-reluctance, hysteresis, and shaded-pole induction motors are also discussed.
- Most induction motors of fractional-kilowatt (fractional horsepower) rating are single-phase motors. In residential and commercial applications, they are found in a wide range of equipment including refrigerators, air conditioners and heat pumps, fans, pumps, washers, and dryers.
- Most single-phase induction motors are actually two-phase motors with unsymmetrical windings; the two windings are typically quite different, with different numbers of turns and/or winding distributions.
- 9.1 SINGLE-PHASE INDUCTION MOTORS:

QUALITATIVE EXAMINATION

- Structurally, the most common types of single-phase induction motors resemble polyphase squirrel-cage motors except for the arrangement of the stator windings. An induction motor with a squirrel-cage rotor and a single-phase stator winding is represented schematically in Fig. 9.1.
- Instead of being a concentrated coil, the actual stator winding is distributed in slots to produce an approximately sinusoidal space distribution of mmf. A single-phase winding produces equal forward- and backward-rotating mmf waves. By symmetry, it is clear that such a motor inherently will produce no starting torque since at standstill, it will produce equal torque in both directions.
- If it is started by auxiliary

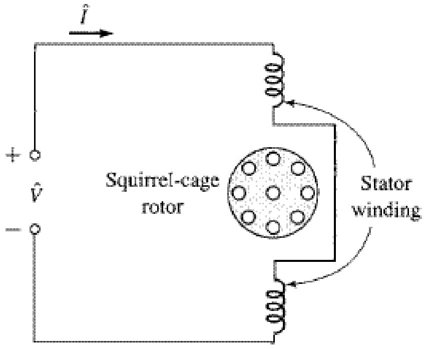


Figure 9.1 Schematic view of a single-phase induction motor.

means, the result will be a net torque in the direction in which it is started, and hence the motor will continue to run.

- We will discuss the basic properties of the schematic motor of Fig. 9.1. If the stator current is a cosinusoidal function of time, the resultant air-gap mmf is given by

$F_{\mathrm{agl}} = F_{\max} \cos(\theta_{\mathrm{ae}}) \cos \omega_e t$(9.1)

which, can be written as the sum of positive- and negative traveling mmf waves of equal magnitude. The positive-traveling wave is given by

$F_{\mathrm{agl}}^{+} = \frac{1}{2} F_{\max} \cos(\theta_{\mathrm{ae}} - \omega_e t)$(9.2)

and the negative-traveling wave is given by

$F_{\mathrm{agl}}^{-} = \frac{1}{2} F_{\max} \cos(\theta_{\mathrm{ae}} + \omega_e t)$(9.3)

- Each of these component mmf waves produces induction-motor action, but the corresponding torques are in opposite directions. With the rotor at rest, the forward and backward air-gap flux waves created by the combined mmf's of the stator and rotor currents are equal, the component torques are equal, and no starting torque is produced. If the forward and backward air-gap flux waves were to remain equal when the rotor revolves, each of the component fields would produce a torque-speed characteristic similar to that of a polyphase motor with negligible stator leakage impedance, as illustrated by the dashed curves f and b in Fig. 9.2a. The resultant torque-speed characteristic, which is the algebraic sum of the two component curves, shows that if the motor were started by auxiliary means, it would produce torque in whatever direction it was started.
- The assumption that the air-gap flux waves remain equal when the rotor is in motion is a rather drastic simplification of the actual state of affairs. First, the effects of stator leakage impedance are ignored. Second, the effects of induced rotor currents are not properly accounted for.

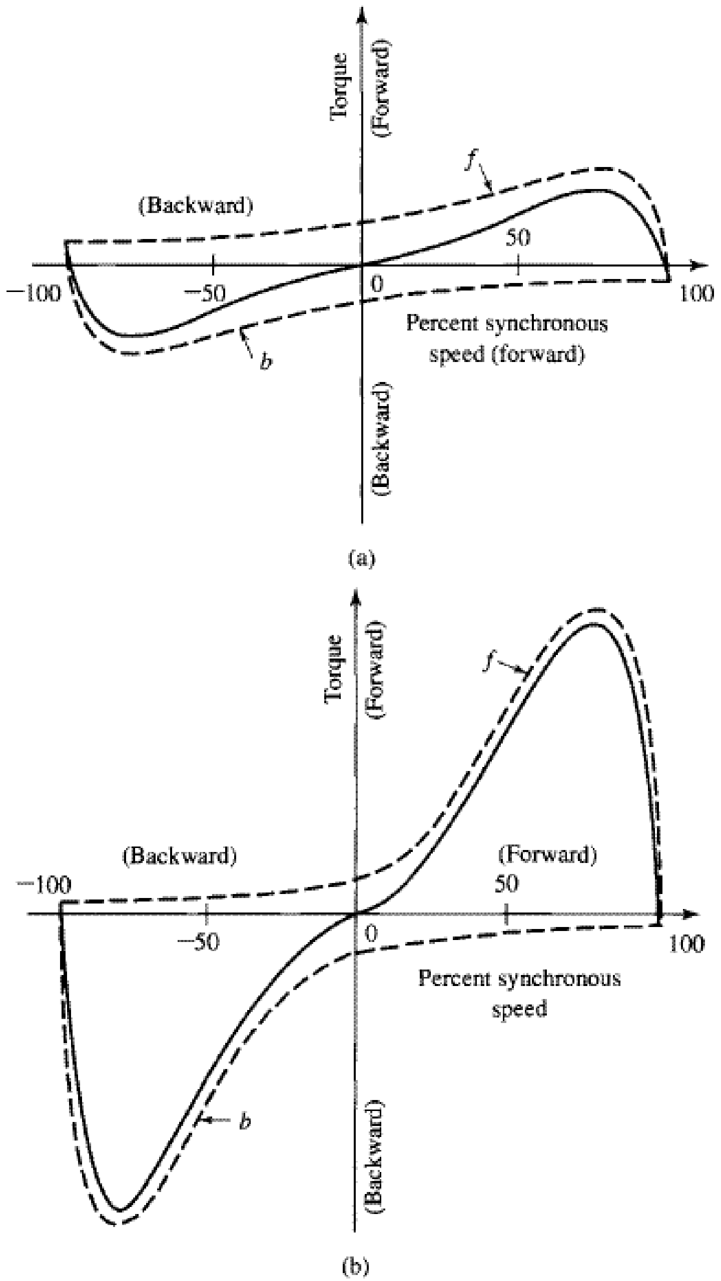


Figure 9.2 Torque-speed characteristic of a single-phase induction motor

(a) on the basis of constant forward and backward flux waves,

(b) taking into account changes in the flux waves.

- When the rotor is in motion, the component rotor currents induced by the backward field are greater than at standstill, and their power factor is lower. Their mmf, which opposes that of the stator current, results in a reduction of the backward flux wave. Conversely, the magnetic effect of the component currents induced by the forward field is less than at standstill because the rotor currents are less and their power factor is higher. As speed increases, therefore, the forward flux wave increases while the backward flux wave decreases. The sum of these flux waves must remain roughly constant since it must induce the stator counter emf, which is approximately constant if the stator leakage-impedance voltage drop is small.

- Hence, with the rotor in motion, the torque of the forward field is greater and that of the backward field less than in Fig. 9.2a, the true situation being about that shown in Fig. 9.2b. In the normal running region at a few percent slip, the forward field is several times greater than the backward field, and the flux wave does not differ greatly from the constant-amplitude revolving field in the air gap of a balanced polyphase motor.
- In the normal running region, therefore, the torque-speed characteristic of a single-phase motor is not too greatly inferior to that of a polyphase motor having the same rotor and operating with the same maximum air-gap flux density.
- In addition to the torques shown in Fig. 9.2, double-stator-frequency torque pulsations are produced by the interactions of the oppositely rotating flux and mmf waves which rotate past each other at twice synchronous speed. These interactions produce no average torque, but they tend to make the motor noisier than a polyphase motor.

9.2 STARTING AND RUNNING PERFORMANCE OF SINGLE-PHASE INDUCTION AND SYNCHRONOUS MOTORS

Single-phase induction motors are classified in accordance with their starting methods and are usually referred to by names descriptive of these methods.

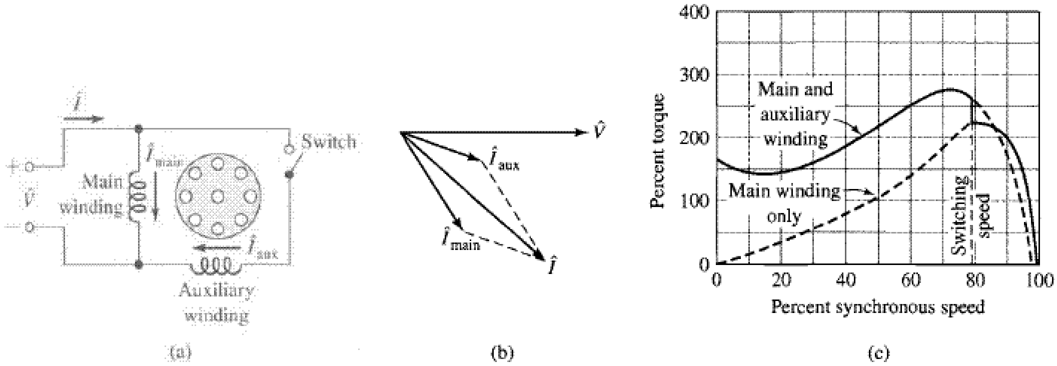


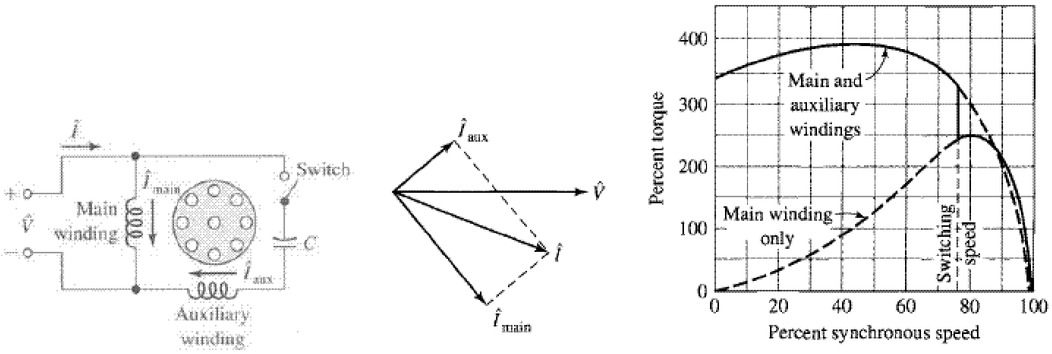
Figure 9.3 Split-phase motor: (a) connections, (b) phasor diagram at starting, and (c) typical torque-speed characteristic.

9.2.1 Split-Phase Motors

- Split-phase motors have two stator windings, a main winding (also referred to as the run winding) which we will refer to with the subscript 'main' and an auxiliary winding (also referred to as the start winding) which we will refer to with the subscript 'aux'. As in a two-phase motor, the axes of these windings are displaced 90 electrical degrees in space, and they are connected as shown in Fig. 9.3a. The auxiliary winding has a higher resistance-to-reactance ratio than the main winding, with the result that the two currents will be out of phase, as indicated in the phasor diagram of Fig. 9.3b, which is representative of conditions at starting. Since the auxiliary-winding current $\hat{I}_{\text{aux}}$ leads the main-winding current $\hat{I}_{\text{main}}$, the stator field first reaches a maximum along the axis of the auxiliary winding and then somewhat later in time reaches a maximum along the axis of the main winding.
- The winding currents are equivalent to unbalanced two-phase currents, and the motor is equivalent to an unbalanced two-phase motor. The result is a rotating stator field which causes the motor to start. After the motor starts, the auxiliary winding is disconnected, usually by means of a centrifugal switch that operates at about 75 percent of synchronous speed. The simple way to obtain the high resistance-to-reactance ratio for the auxiliary winding is to wind it with smaller wire than the main winding, a permissible procedure because this winding operates only during starting. Its reactance can be reduced somewhat by placing it in the tops of the slots. A typical torque-speed characteristic for such a motor is shown in Fig. 9.3c.
- Split-phase motors have moderate starting torque with low starting current. Typical applications include fans, blowers, centrifugal pumps, and office equipment. Typical ratings are 50 to 500 watts; in this range they are the lowest-cost motors available.

9.2.2 Capacitor-Type Motors

- Capacitors can be used to improve motor starting performance, running performance, or both, depending on the size and connection of the capacitor. The capacitor-start



(a) (b) (c)

Figure 9.4 Capacitor-start motor: (a) connections, (b) phasor diagram at starting, and (c) typical torque-speed characteristic.

motor is also a split-phase motor, but the time-phase displacement between the two currents is obtained by means of a capacitor in series with the auxiliary winding, as shown in Fig. 9.4a. Again the auxiliary winding is disconnected after the motor has started, and consequently the auxiliary winding and capacitor can be designed at minimum cost for intermittent service.

- By using a starting capacitor of appropriate value, the auxiliary-winding current $\hat{I}_{\text{aux}}$ at standstill can be made to lead the main-winding current $\hat{I}_{\text{main}}$ by 90 electrical degrees, as it would in a balanced two-phase motor (see Fig. 9.4b). In practice, the best compromise between starting torque, starting current, and cost typically results with a phase angle somewhat less than 90^o. A typical torque-speed characteristic is shown in Fig. 9.4c, high starting torque being an outstanding feature. These motors are used for compressors, pumps, refrigeration and air-conditioning equipment, and other hard-to-start loads. A cutaway view of a capacitor-start motor is shown in Fig. 9.5.
- In the permanent-split-capacitor motor, the capacitor and auxiliary winding are not cut out after starting; the construction can be simplified by omission of the switch, and the power factor, efficiency, and torque pulsations improved. For example, the capacitor and auxiliary winding could be designed for perfect two-phase operation (i.e., no backwards flux wave) at any one desired load. The losses due to the backward field at this operating point would then be eliminated, with resulting improvement in efficiency. The double-stator-frequency torque pulsations would also be eliminated, with the capacitor serving as an energy storage reservoir for smoothing out the pulsations in power input from the single-phase line, resulting in quieter operation.

- Starting torque must be sacrificed because the choice of capacitance is necessarily a compromise between the best starting and running values. The resulting torque-speed characteristic and a schematic diagram are given in Fig. 9.6.
- If two capacitors are used, one for starting and one for running, theoretically optimum starting and running performance can both be obtained. One way of accomplishing this result is shown in Fig. 9.7a. The small value of capacitance required for

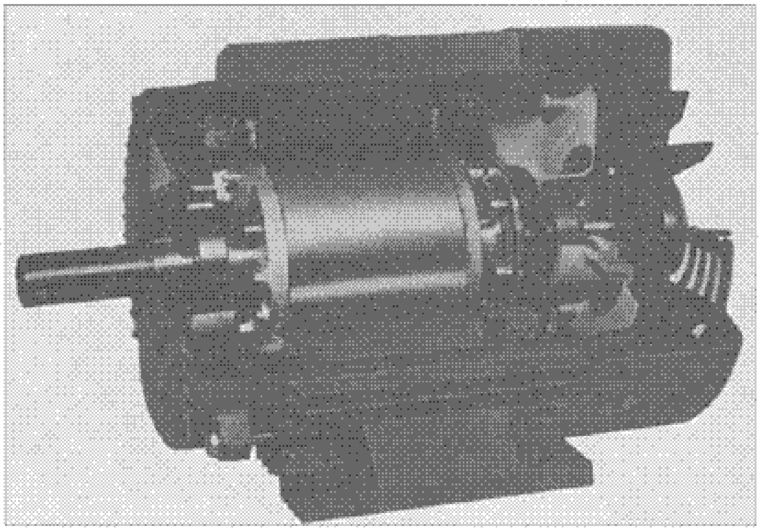


Figure 9.5 Cutaway view of a capacitor-start induction motor.

The starting switch is at the right of the rotor. The motor is of

drip-proof construction. (General Electric Company.)

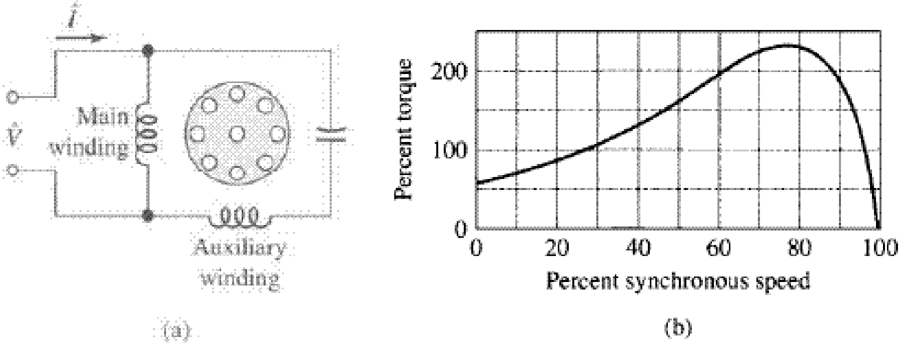
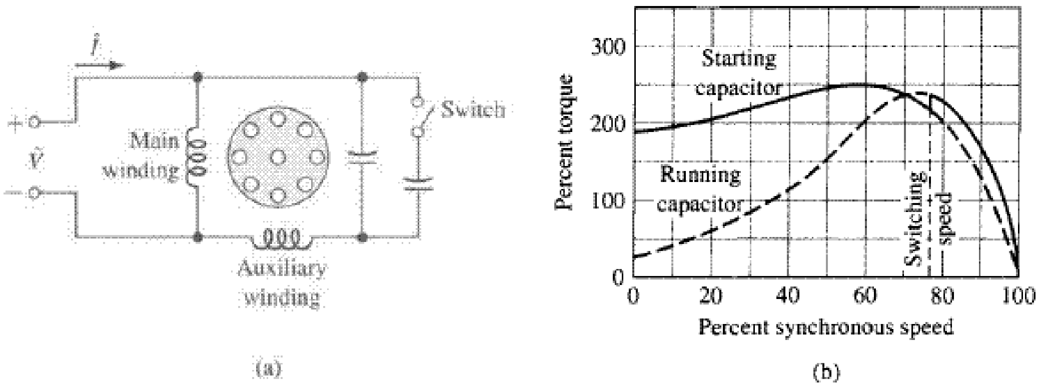


Figure 9.6 Permanent-split-capacitor motor

and typical torque-speed characteristic.



(a)

(b)

Figure 9.7 Capacitor-start, capacitor-run motor

and typical torque-speed characteristic.

optimum running conditions is permanently connected in series with the auxiliary winding, and the much larger value required for starting is obtained by a capacitor connected in parallel with the running capacitor via a switch with opens as the motor comes up to speed. Such a motor is known as a capacitor-start, capacitor-run motor.

- The capacitor for a capacitor-start motor has a typical value of 300 μF for a 500-W motor. Since it must carry current for just the starting time, the capacitor is a special compact ac electrolytic type made for motor-starting duty. The capacitor for the same motor permanently connected has a typical rating of 40 μF, and since it operates continuously, the capacitor is an ac paper, foil, and oil type.

9.2.3 Shaded-Pole Induction Motors

- In Fig.9.8a, the shaded-pole induction motor usually has salient poles with one portion of each pole surrounded by a short-circuited turn of copper called a shading coil. Induced currents in the shading coil cause the flux in the shaded portion of the pole to lag the flux in the other portion. The result is similar to a rotating field moving in the direction from the unshaded to the shaded portion of the pole; currents are induced in the squirrel-cage rotor and a low starting torque is produced. A typical torque-speed characteristic is shown in Fig. 9.8b. Their efficiency is low, but shaded-pole motors are the least expensive type of subfractional-kilowatt motor. They are found in ratings up to about 50 watts.

9.2.4 Self-Starting Synchronous-Reluctance Motors

- Any one of the induction-motor types described above can be made into a self-starting synchronous-reluctance motor. Anything which makes the reluctance of the air gap a function of the angular position of the rotor with respect to the stator coil axis will produce reluctance torque when the rotor is revolving at synchronous speed. For

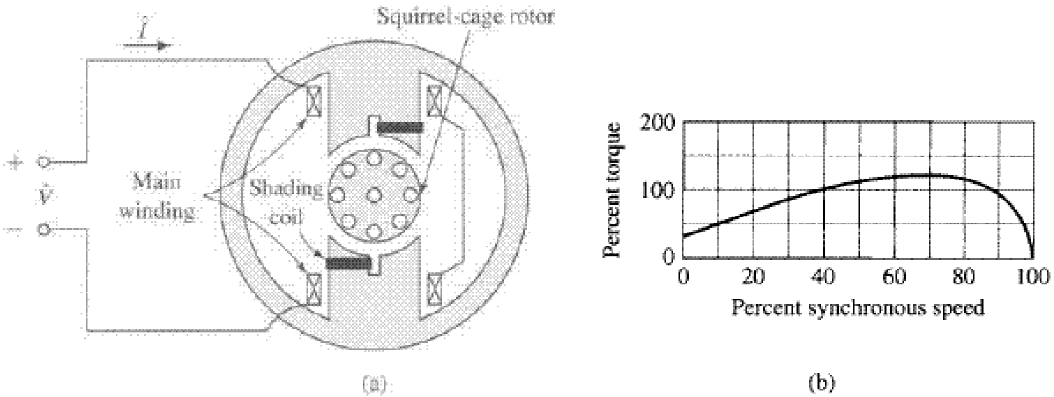


Figure 9.8 Shaded-pole induction motor and typical torque-speed characteristic.

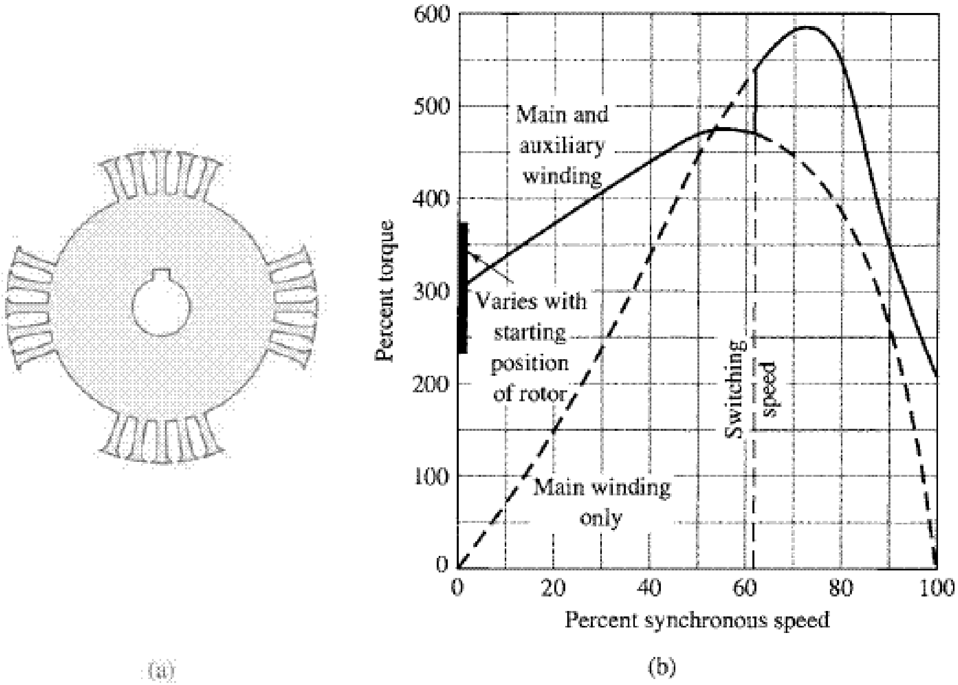


Figure 9.9 Rotor punching for four-pole synchronous-reluctance motor

and typical torque-speed characteristic.

example, suppose some of the teeth are removed from a squirrel-cage rotor, leaving the bars and end tings intact, as in an ordinary squirrel-cage induction motor. Figure 9.9a shows a lamination for such a rotor designed for use with a four-pole stator. The stator may be polyphase or any one of the single-phase types described above.

- The motor will start as an induction motor and at light loads will speed up to a small value of slip. The reluctance torque arises from the tendency of the rotor to try to align itself in the minimum-reluctance position with respect to the synchronously revolving forward air-gap flux wave, in accordance with the principles discussed in Chapter 3. At a small slip, this torque alternates slowly in direction; the rotor is accelerated during a positive half cycle of the torque variation and decelerated during the succeeding negative half cycle. If the moment of inertia of the rotor and its mechanical load are sufficiently small, the rotor will be accelerated from slip speed up to synchronous speed during an accelerating half cycle of the reluctance torque. The rotor will then pull into synchronism and continue to run at synchronous speed.
- A typical torque-speed characteristic for a split-phase-start synchronousreluctance motor is shown in Fig. 9.9b.

9.2.5 Hysteresis Motors

- In its simplest form, the rotor of a hysteresis motor is a smooth cylinder of magnetically hard steel, without windings or teeth. It is placed inside a slotted stator carrying distributed windings designed to produce as nearly as possible a sinusoidal space distribution of flux, since undulations in the flux wave greatly increase the losses. In single-phase motors, the stator windings usually are of the permanent-split-capacitor type, as in Fig. 9.6. The capacitor is chosen so as to result in approximately balanced two-phase conditions within the motor windings. The stator then produces a primarily space-fundamental air-gap field revolving at synchronous speed.
- Instantaneous magnetic conditions in the air gap and rotor are indicated in Fig. 9.10a for a two-pole stator. The axis S $S^{'}$ of the stator-mmf wave revolves at synchronous speed. Because of hysteresis, the magnetization of the rotor lags behind the inducing mmf wave, and therefore the axis R $R^{'}$ of the rotor flux wave lags behind the axis of the stator-mmf wave by the hysteretic lag angle δ (Fig. 9.10a). If the rotor is stationary, starting torque is produced proportional to the product of the fundamental components of the stator mmf and rotor flux and the sine of the torque angle δ. The rotor then accelerates if the torque of the load is less than the developed torque of the motor.
- As long as the rotor is turning at less than synchronous speed, each region of the rotor is subjected to a repetitive hysteresis cycle at slip frequency. While the rotor accelerates, the lag angle δ remains constant if the flux is constant, since the angle δ

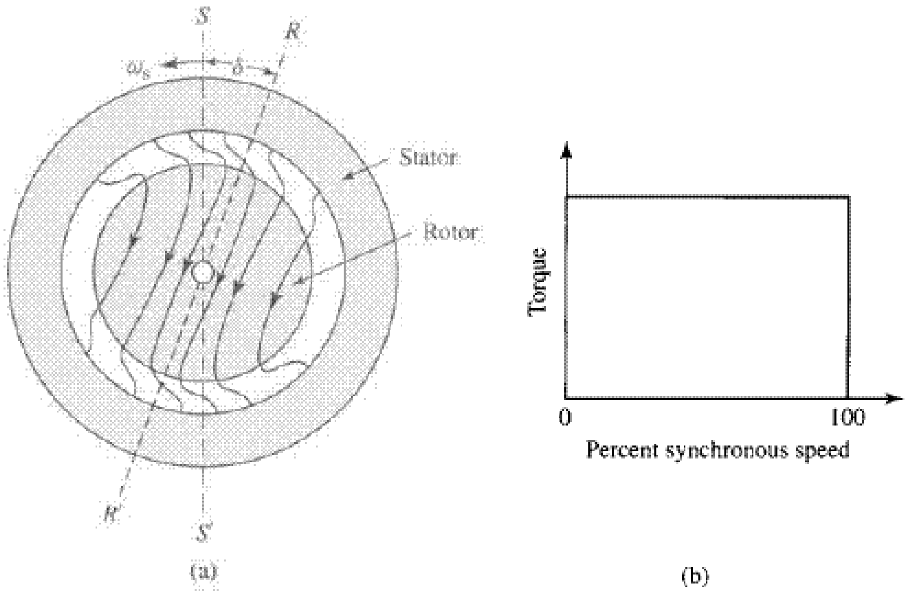


Figure 9.10 (a) General nature of the magnetic field in the air gap and rotor of a hysteresis motor; (b) idealized torque-speed characteristic.

depends merely on the hysteresis loop of the rotor material and is independent of the rate at which the loop is traversed. The motor therefore develops constant torque right up to synchronous speed, as shown in the idealized torque-speed characteristic of Fig. 9.10b.

- This feature is one of the advantages of the hysteresis motor. In contrast with a reluctance motor, which must "snap" its load into synchronism from an induction-motor torque-speed characteristic, a hysteresis motor can synchronize any load which it can accelerate, no matter how great the inertia. After reaching synchronism, the motor continues to run at synchronous speed and adjusts its torque angle so as to develop the torque required by the load. The hysteresis motor is inherently quiet and produces smooth rotation of its load.

9.3 REVOLVING-FIELD THEORY OF SINGLE-PHASE INDUCTION MOTORS

- Consider conditions with the rotor stationary and only the main stator winding excited. The motor then is equivalent to a transformer with its secondary shortcircuited. The equivalent circuit is shown in Fig. 9.11 a, where $R_{1,\text{main}}$ and $X_{1,\text{main}}$ are, respectively, the resistance and leakage reactance of the main winding, $X_{m,\text{main}}$ is the magnetizing reactance, and $R_{2,\text{main}}$ and $X_{2,\text{main}}$ are the standstill values of the rotor resistance and leakage reactance referred to the main stator winding by use of the appropriate turns ratio. Core loss, which is omitted here, will be accounted for later as if it were a rotational loss. The applied voltage is $\hat{V}$, and the main-winding current is $\hat{I}_{\text{main}}$. The voltage $\hat{E}_{\text{main}}$ is the counter emf generated in the main winding by the stationary pulsating air-gap flux wave produced by the combined action of the stator and rotor currents.

- In accordance with the double-revolving-field concept of Section 9.1, the stator mmf can be resolved into half-amplitude forward and backward rotating fields.
- At

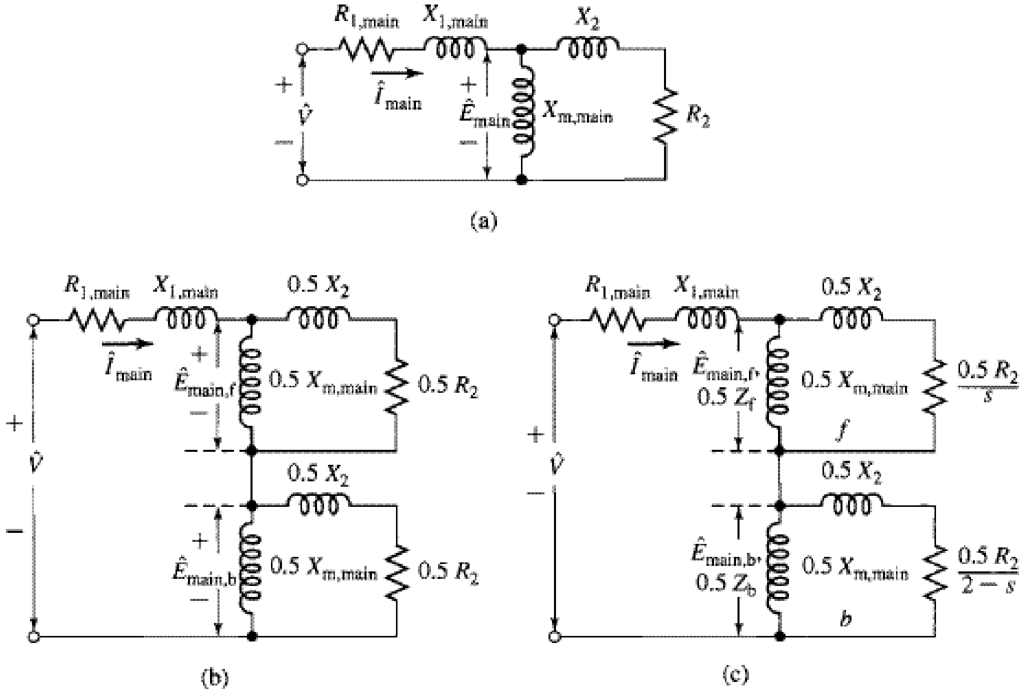


Figure 9.11 Equivalent circuits for a single-phase induction motor: (a) rotor blocked

(b) rotor blocked, showing effects of forward and backward fields; (c) running conditions.

standstill the amplitudes of the forward and backward resultant air-gap flux waves both equal half the amplitude of the pulsating field. In Fig. 9.11 b the portion of the equivalent circuit representing the effects of the air-gap flux is split into two equal portions, representing the effects of the forward and backward fields, respectively.

- Now consider conditions after the motor has been brought up to speed by some auxiliary means and is running on only its main winding in the direction of the forward field at a per-unit slip s. The rotor currents induced by the forward field are of slip frequency sf_e, where $f_e$is the stator applied electrical frequency. Just as in any polyphase motor with a symmetric polyphase or squirrel-cage rotor, these rotor currents produce an mmf wave traveling forward at slip speed with respect to the rotor and therefore at synchronous speed with respect to the stator. The resultant of the forward waves of stator and rotor mmf creates a resultant forward wave of air-gap flux, which generates a counter emf $\hat{E}_{\text{main},f}$ in the main winding of the stator. The reflected effect of the rotor as viewed from the stator is like that in a polyphase motor and can be represented by an impedance $0.5\mathrm{R}_{2,\text{main}}/s + j0.5\mathrm{X}_{2,\text{main}}$ in parallel with j0.5 $X_{m,\text{main}}$ as in the portion of the equivalent circuit of Fig. 9.1 lc labeled 'f'. The factors of 0.5 come from the resolution of the pulsating stator mmf into forward and backward components.

- Now consider conditions with respect to the backward field. The rotor is still turning at a slip s with respect to the forward field, and its per-unit speed n in the direction of the forward field is n = 1 - s. The relative speed of the rotor with respect to the backward field is 1 + n, or its slip with respect to the backward field is 1 + n = 2 - s. The backward field then induces rotor currents whose frequency is (2 - s) f_e.
- For small slips, these rotor currents are of almost twice stator frequency.
- By use of the equivalent circuit of Fig. 9.11 c, the stator current, power input, and power factor can be computed for any assumed value of slip when the applied voltage and the motor impedances are known. To simplify the notation, let

$$Z_f \equiv R_f + \mathrm{j}\mathrm{X}_f \equiv \left[\frac{R_{2,\text{main}}}{s} + \mathrm{j}\mathrm{X}_{2,\text{main}}\right] \text{in parallel with } \mathrm{j}\mathrm{X}_{\text{m,main}} \quad (9.4)$$

And

$$Z_b \equiv R_b + \mathrm{j}\mathrm{X}_b \equiv \left[\frac{R_{2,\text{main}}}{2-s} + \mathrm{j}\mathrm{X}_{2,\text{main}}\right] \text{in parallel with } \mathrm{j}\mathrm{X}_{\text{m,main}} \quad (9.5)$$

The impedances representing the reactions of the forward and backward fields from the viewpoint of the single-phase main stator winding are 0.5 Z_f and 0.5 Z_b, respectively, in Fig. 9.11 c.

- Examination of the equivalent circuit (Fig. 9.11c) confirms the conclusion, reached by qualitative reasoning in Section 9.1 (Fig. 9.2b), that the forward airgap flux wave increases and the backward wave decreases when the rotor is set in motion. When the motor is running at a small slip, the reflected effect of the rotor resistance in the forward field, 0.5 $R_{2,\text{main}}/s$, is much larger than its standstill value, while the corresponding effect in the backward field, 0.5 $R_{2,\text{main}}/(2-s)$, is smaller.
- The forward-field impedance therefore is larger than its standstill value, while that of the backward field is smaller. The forward-field counter emf $\hat{E}_{\text{main},f}$ therefore is larger than its standstill value, while the backward-field counter emf

$\hat{E}_{\text{main},b}$ is smaller; i.e., the forward air-gap flux wave increases, while the backward flux wave decreases.

- The electromagnetic torque $T_{\text{main},f}$ of the forward field in newtonmeters equals $1/\omega_s$ times the power $P_{\text{gap},f}$ in watts delivered by the stator winding to the forward field, where ω_s is the synchronous angular velocity in mechanical radians per second; thus

$$T_{\text{main},f} = \frac{1}{\omega_s} P_{\text{gap},f} (9.6)$$

When the magnetizing impedance is treated as purely inductive, $P_{\text{gap},f}$ is the power absorbed by the impedance 0.5 Z_f; that is,

$$P_{\text{gap},f} = I^2(0.5\mathrm{R}_f)(9.7)$$

where R_f is the resistive component of the forward-field impedance defined in Eq. 9.4.

- The internal torque $T_{\text{main},b}$ of the backward field is

$$T_{\text{main},b} = \frac{1}{\omega_s} P_{\text{gap},b} (9.8)$$

where $P_{\text{gap},b}$ is the power delivered by the stator winding to the backward field, or

$$P_{\text{gap},b} = I^2(0.5\mathrm{R}_b)(9.9)$$

where R_b is the resistive component of the backward-field impedance Z_b defined in Eq. 9.5.

- The net internal torque T_{mech} is

$$T_{\text{mech}} = T_{\text{main},f} - T_{\text{main},b} = \frac{1}{\omega_s}(P_{\text{gap},f} - P_{\text{gap},b})(9.10)$$

- The rotor I^2R loss caused by a rotating field equals the slip of the field times the power absorbed from the stator. Thus

$$\text{Forward} - \text{field rotor} I^2R = \mathrm{sP}_{\text{gap},f}(9.11)$$

$$\text{Backward} - \text{field rotor} I^2R = (2 - s)P_{\text{gap},b}(9.12)$$

$$\text{Total rotor} I^2R = \mathrm{sP}_{\text{gap},f} + (2 - s)P_{\text{gap},b}(9.13)$$

- The internal power P_{mech} converted to mechanical form, in watts, is

$P_{\text{mech}} = (1-s)\omega_s T_{\text{mech}} = (1-s)(P_{\text{gap},f} - P_{\text{gap},b})$(9.14)

As in the polyphase motor, the internal torque T_{mech} and internal power P_{mech} are not the output values because rotational losses remain to be accounted for. It is obviously correct to subtract friction and windage losses from T_{mech} or P_{mech} and it is usually assumed that core losses can be treated in the same manner. For the small changes in speed encountered in normal operation, the rotational losses are often assumed to be constant.

- Corresponding to a slightly greater approximation, the shunting effect of j $X_{m,\text{main}}$ on the backward-field impedance can often be neglected, whence

$Z_b \approx \frac{R_{2,\text{main}}}{2-s} + \text{jX}_{2,\text{main}}$(9.15)

for small slips

$Z_b \approx \frac{R_{2,\text{main}}}{2} + \text{jX}_{2,\text{main}}$(9.16)

9.4 TWO-PHASE INDUCTION MOTORS

- Most single-phase induction motors are actually constructed in the form of two-phase motors, with two stator windings in space quadrature. The main and auxiliary windings are typically quite different, with a different number of turns, wire size, and turns distribution. This difference, in combination with the capacitor that is typically used in series with the auxiliary winding, guarantees that the mmfs produced by the two winding currents will be quite unbalanced; at best they may be balanced at one specific operating point.
- Under balanced operating conditions, a symmetrical two-phase motor can be analyzed using techniques developed for three-phase motors, modified only slightly to take into account the fact that there are two phases rather than three.

9.4.1 Unbalanced Operation of Symmetrical

Two-Phase Machines; The Symmetrical-Component Concept

- When operating from the main winding alone, the single-phase motor is the extreme case of a motor operating under unbalanced stator-current conditions. In some cases, unbalanced voltages or currents are produced in the supply network to a motor, e.g., when a line fuse is blown. In other cases, unbalanced voltages are produced by the starting impedances of single-phase motors, as described in Section 9.2.

- When balanced two-phase voltages are applied to the stator terminals of a two-phase machine having a uniform air gap, a symmetrical polyphase or cage rotor, and two identical stator windings α and β in space quadrature, the stator currents are equal in magnitude and in time quadrature.

When the current in winding α is at its instantaneous maximum, the current in winding β is zero and the stator-mmf wave is centered on the axis of winding α. Similarly, the stator-mmf wave is centered on the axis of winding β at the instant when the current in winding β is at its instantaneous maximum.

- The stator-mmf wave therefore travels 90 electrical degrees in space in a time interval corresponding to a 90^o phase change of the applied voltage, with the direction of its travel depending on the phase sequence of the currents. The traveling wave has constant amplitude and constant angular velocity.
- If the rotor is turning at a slip s in the direction from winding α toward winding β , the terminal impedance per phase is given by the equivalent circuit of Fig. 9.12a when the applied voltage $\hat{V}_\beta$ lags the applied voltage $\hat{V}_\alpha$ by 90^o. This phase sequence is called positive sequence and is designated by the subscript 'f' since positive sequence currents result in a forward field. With the rotor running at the same speed and in the same direction, the terminal impedance per phase is given by the equivalent circuit of Fig. 9.12b when $\hat{V}_\beta$ leads $\hat{V}_\alpha$ by 90^o. This phase sequence is called negative sequence and is designated by subscript 'b', since negative-sequence currents produce a backward field.

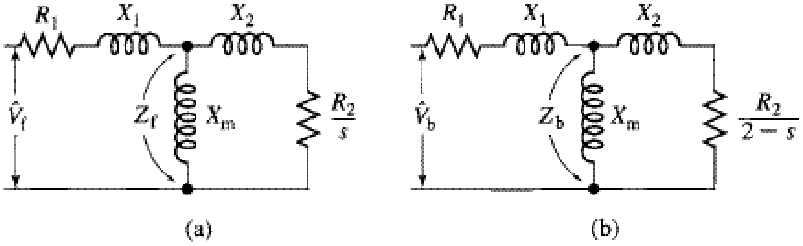


Figure 9.12 Single-phase equivalent circuits for a two-phase motor

under unbalanced conditions (a) forward field and (b) backward field.

- Suppose now that two balanced two-phase voltage sources of opposite phase sequence are connected in series and applied simultaneously to the motor, as indicated in Fig. 9.13a, where phasor voltages $\hat{V}_f$ and j $\hat{V}_f$ applied, respectively, to windings α and β form a balanced system of positive sequence, and phasor voltages $\hat{V}_b$ and -j $\hat{V}_b$ form another balanced system but of negative sequence.
- The resultant voltage V_α applied to winding α is, as a phasor,

$$\hat{V}_\alpha = \hat{V}_f + \hat{V}_b \quad (9.17)$$

and that applied to winding fl is

$$\hat{V}_\beta = j\hat{V}_f - j\hat{V}_b \quad (9.18)$$

Fig. 9.13b shows a generalized phasor diagram in which the forward, or positivesequence, system is given by the phasors $\hat{V}_f$ and j $\hat{V}_f$ and the backward, or negativesequence, system is given by the phasors $\hat{V}_b$ and -j $\hat{V}_b$. The resultant voltages, given by the phasors $\hat{V}_\alpha$ and $\hat{V}_\beta$are not, in general, either equal in magnitude or in time

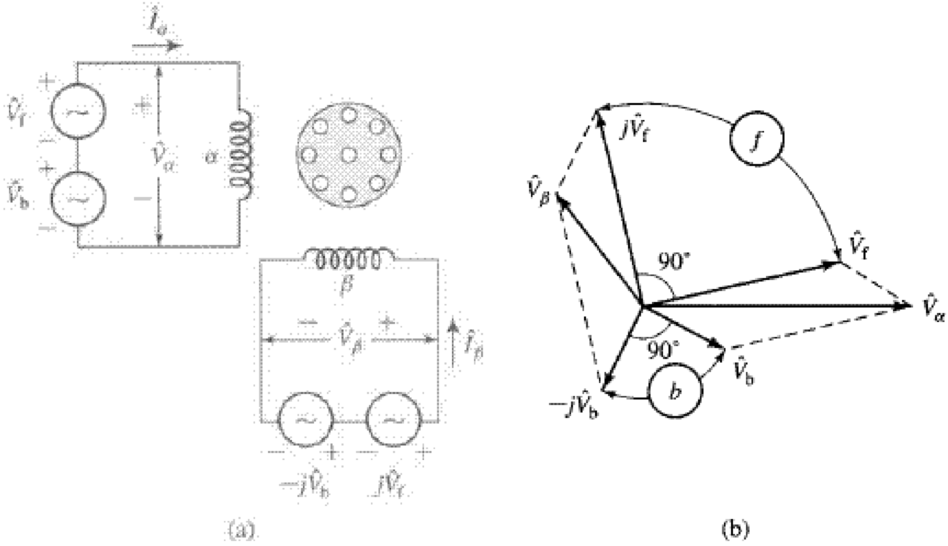


Figure 9.13 Synthesis of an unbalanced two-phase system

from the sum of two balanced systems of opposite phase sequence.

quadrature.

- The symmetrical-component systems are, however, much easier to work with than their unbalanced resultant system. Thus, it is easy to compute the component currents produced by each symmetrical-component system of applied voltages because the induction motor operates as a balanced two-phase motor for each component system.
- By superposition, the actual current in a winding then is the sum of its components.
- If $\hat{I}_f$ and $\hat{I}_b$ are, respectively, the positive- and negative-sequence component phasor currents in winding α, then the corresponding positive- and negative-

sequence component phasor currents in winding β are, respectively, j $\hat{I}_f$ and -j $\hat{I}_b$, and the actual winding currents $\hat{I}_\alpha$ and $\hat{I}_\beta$ are

$$\hat{I}_\alpha = \hat{I}_f + \hat{I}_b \quad (9.19)$$

$$\hat{I}_\beta = j\hat{I}_f - j\hat{I}_b \quad (9.20)$$

- The inverse operation of finding the symmetrical components of specified voltages or currents must be performed often. Solution of Eqs. 9.17 and 9.18 for the phasor components $\hat{V}_f$ and $\hat{V}_b$ in terms of known phasor voltages $\hat{V}_\alpha$ and $\hat{V}_\beta$ gives

$$\hat{V}_f = \frac{1}{2}(\hat{V}_\alpha - j\hat{V}_\beta) \quad (9.21)$$

$$\hat{V}_b = \frac{1}{2}(\hat{V}_\alpha + j\hat{V}_\beta) \quad (9.22)$$

These operations are illustrated in the phasor diagram of Fig. 9.14. Obviously, similar relations give the phasor symmetrical components $\hat{I}_f$ and $\hat{I}_b$ of the current in winding ot in terms of specified phasor current $\hat{I}_m$and $\{\hat{I}_a$in the two phases; thus

$$\hat{I}_f = \frac{1}{2}(\hat{I}_\alpha - j\hat{I}_\beta) \quad (9.23)$$

$$\hat{I}_b = \frac{1}{2}(\hat{I}_\alpha + j\hat{I}_\beta) \quad (9.24)$$

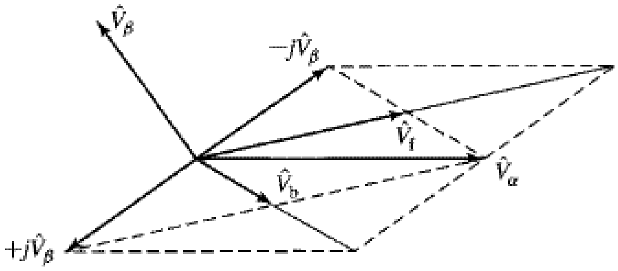


Figure 9.14 Resolution of unbalanced two-phase voltages

into symmetrical components.

9.4.2 The General Case: Unsymmetrical Two-Phase Induction Machines

- A single-phase induction motor with a main and auxiliary winding is an example of an unsymmetrical two-phase induction motor. In this section we will develop a model for such a two-phase motor, using notation appropriate to the single-phase motor. We will assume, as is commonly the case, that the windings are in space quadrature but that they are unsymmetrical in that they may have a different number of turns, a different winding distribution, and so on.
- To start with flux-linkage/current relationships for the rotor and stator of the form

$$\begin{matrix} \lambda_{\text{main}} \\ \lambda_{\text{aux}} \\ \lambda_{r1} \\ \lambda_{r2} \end{matrix} = \begin{matrix} L_{\text{main}} & 0 & L_{\text{main},r1}(\theta_{\text{me}}) & L_{\text{main},r2}(\theta_{\text{me}}) \\ 0 & L_{\text{aux}} & L_{\text{aux},r1}(\theta_{\text{me}}) & L_{\text{aux},r2}(\theta_{\text{me}}) \\ L_{\text{main},r1}(\theta_{\text{me}}) & L_{\text{aux},r1}(\theta_{\text{me}}) & L_r & 0 \\ L_{\text{main},r2}(\theta_{\text{me}}) & L_{\text{aux},r2}(\theta_{\text{me}}) & 0 & L_r \end{matrix} \quad \begin{matrix} i_{\text{main}} \\ i_{\text{aux}} \\ i_{r1} \\ i_{r2} \end{matrix}$$

(9.25)

where θ_{me} is the rotor angle measured in electrical radians.

$L_{\text{main}} =$ Self-inductance of the main winding

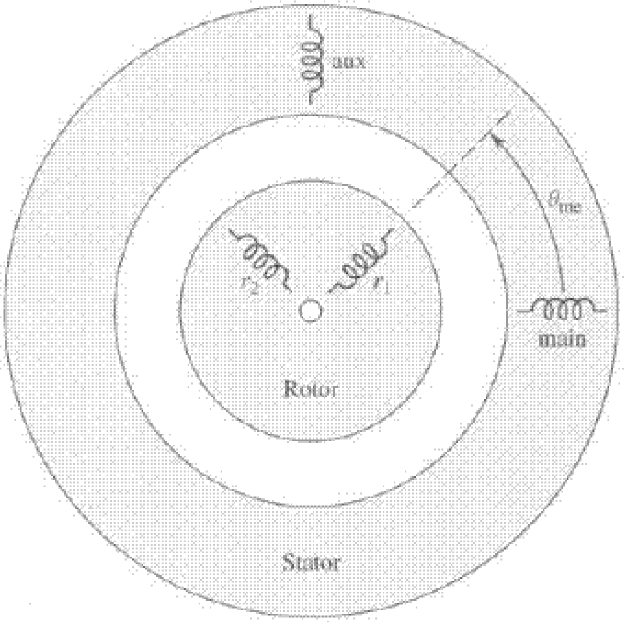


Figure 9.16 Schematic representation of a two-phase induction motor

with an equivalent two-phase rotor.

$L_{\text{aux}} =$Self-inductance of the auxiliary winding

$L_r =$Self-inductance of the equivalent rotor windings

$L_{\text{main},\,r1}(\theta_{\text{me}})$ =Mutual inductance between the main winding and equivalent rotor winding 1

$L_{\text{main},\,r2}(\theta_{\text{me}})$ =Mutual inductance between the main winding and equivalent rotor winding 2

$L_{\text{aux},\,r1}(\theta_{\text{me}})$ =Mutual inductance between the auxiliary winding and rotor winding 1

$L_{\text{aux},\,r2}(\theta_{\text{me}})$ =Mutual inductance between the auxiliary winding and rotor winding 2

- Assuming a sinusoidal distribution of air-gap flux, the mutual inductances between the main winding and the rotor will be of the form

$L_{\text{main},\,r1}(\theta_{\text{me}}) = L_{\text{main},r}\text{cos}\theta_{\text{me}}$(9.26)

And

$L_{\text{main},\,r2}(\theta_{\text{me}}) = -L_{\text{main},r}\text{sin}\theta_{\text{me}}$(9.27)

where $L_{\text{main},r}$ is the amplitude of the mutual inductance.

- The mutual inductances between the auxiliary winding will be of the same form with the exception that the auxiliary winding is displaced by 90 electrical degrees in space from the main winding.

$L_{\text{aux},\,r1}(\theta_{\text{me}}) = L_{\text{aux},r}\text{sin}\theta_{\text{me}}$(9.28)

And

$L_{\text{aux},\,r2}(\theta_{\text{me}}) = L_{\text{aux},r}\text{cos}\theta_{\text{me}}$(9.29)

- Note that the auxiliary winding will typically have a different number of turns (and perhaps a different winding distribution) from that of the main winding. Thus, for modeling purposes, it is often convenient to write

$L_{\text{aux},r} = \text{aL}_{\text{main},r}$(9.30)

Where

$a = \text{Turns ratio} = \frac{\text{Effective turns of auxiliary winding}}{\text{Effective turns of main winding}}$(9.31)

- Similarly, if we write the self-inductance of the magnetizing branch as the sum of a leakage inductance $L_{\text{main},1}$ and a magnetizing inductance L_m

$$L_{\text{main}} = L_{\text{main},1} + L_m (9.32)$$

then the self-inductance of the auxiliary winding can be written in the form

$$L_{\text{aux}} = L_{\text{aux},1} + a^2 L_m (9.33)$$

- The voltage equations for this machine can be written in terms of the winding currents and flux linkage as

$$v_{\text{main}} = i_{\text{main}} R_{\text{main}} + \frac{d\lambda_{\text{main}}}{\text{dt}} (9.34)$$

$$v_{\text{aux}} = i_{\text{aux}} R_{\text{aux}} + \frac{d\lambda_{\text{aux}}}{\text{dt}} (9.35)$$

$$v_{r1} = 0 = i_{r1} R_r + \frac{d\lambda_{r1}}{\text{dt}} (9.36)$$

$$v_{r2} = 0 = i_{r2} R_r + \frac{d\lambda_{r2}}{\text{dt}} (9.37)$$

Where R_{main}, R_{aux} and R_r are the resistances of the main, auxiliary, and rotor windings respectively. Note that the rotor-winding votages are set equal to zero because the rotor windings of an induction motor are internally shorted.

- Finally, the electro-magnetic torque of this motor can be written as

$$T_{\text{mech}} = i_{\text{main}} i_{r1} \frac{dL_{\text{main},\,r1}(\theta_{\text{me}})}{d\theta_m} + i_{\text{main}} i_{r2} \frac{dL_{\text{main},\,r2}(\theta_{\text{me}})}{d\theta_m}$$

$$+ i_{\text{aux}} i_{r1} \frac{dL_{\text{aux},\,r1}(\theta_{\text{me}})}{d\theta_m} + i_{\text{aux}} i_{r2} \frac{dL_{\text{aux}.\,r2}(\theta_{\text{me}})}{d\theta_m}$$

$$= \left[\frac{\text{poles}}{2}\right] [-L_{\text{main},r}(i_{\text{main}} i_{r1} \sin\theta_{\text{me}} + i_{\text{main}} i_{r2} \cos\theta_{\text{me}})$$
$$+ L_{\text{aux},r}(i_{\text{aux}} i_{r1} \cos\theta_{\text{me}} - i_{\text{aux}} i_{r2} \sin\theta_{\text{me}})]$$

where $\theta_m = (2/\text{poles})\theta_{\text{me}}$is the rotor angle in radians.

- The equations derived in this section can be further developed by assuming steady-state operation, with constant mechanical speed ω_{me}, corresponding to a slip s, and constant electrical supply frequency ω_e. Consistent with this assumption the rotor currents will be at frequencies $\omega_r = \omega_e - \omega_{\text{me}} = s\omega_{\text{me}}$ (produced by the stator positive-sequence field) and $\omega_r = \omega_e + \omega_{\text{me}} = (2 - s)\omega_e$ (produced by the stator negative-sequence field).
- After considerable algebraic manipulation, which includes using Eqs. 9.36 and 9.37 to eliminate the rotor currents, the main- and auxiliary-winding flux-linkage/current relationships of Eq. 9.25 can be written as phasor equations

$\hat{\lambda}_{\mathrm{main}} = [L_{\mathrm{main}} - \mathrm{jL}^2_{\mathrm{main},r}(\hat{K}^+ + \hat{K}^-)]\hat{I}_{\mathrm{main}} + L_{\mathrm{main},r}L_{\mathrm{aux},r}(\hat{K}^+ - \hat{K}^-)\hat{I}_{\mathrm{aux}}$
(9.39)

And

$\hat{\lambda}_{\mathrm{aux}} = -L_{\mathrm{main},r}L_{\mathrm{aux},r}(\hat{K}^+ - \hat{K}^-)\hat{I}_{\mathrm{main}} + [L_{\mathrm{aux}} - \mathrm{jL}^2_{\mathrm{aux},r}(\hat{K}^+ + \hat{K}^-)]\hat{I}_{\mathrm{aux}}$(9.40)

Where

$\hat{K}^+ = \frac{s\omega_e}{2(R_r + \mathrm{js}\omega_e L_r)}$(9.41)

and

$\hat{K}^- = \frac{(2-s)\omega_e}{2(R_r + j(2-s)\omega_e L_r)}$(9.42)

The voltage equations, Eqs. 9.34 and 9.35 become

$\hat{V}_{\mathrm{main}} = \hat{I}_{\mathrm{main}}R_{\mathrm{main}} + j\omega_e\hat{\lambda}_{\mathrm{main}}$(9.43)

$\hat{V}_{\mathrm{aux}} = \hat{I}_{\mathrm{aux}}R_{\mathrm{aux}} + j\omega_e\hat{\lambda}_{\mathrm{aux}}$(9.44)

The rotor currents will each consist of positive- and negative-sequence components. The complex amplitudes of the positive sequence components (at frequency $s\omega_e$) are given by

$\hat{I}^+_{r1} = \frac{-\mathrm{js}\omega_e[L_{\mathrm{main},r}\hat{I}_{\mathrm{main}} + \mathrm{jL}_{\mathrm{aux},r}\hat{I}_{\mathrm{aux}}]}{2(R_r + \mathrm{js}\omega_e L_r)}$(9.45)

And

$\hat{I}^+_{r2} = -j\hat{I}^+_{r1}$(9.46)

while the complex amplitudes of the negative sequence components (at frequency (2 - s) ω_e) are given by

$\hat{I}^-_{r1} = \frac{-j(2-s)\omega_e[L_{\mathrm{main},r}\hat{I}_{\mathrm{main}} - \mathrm{jL}_{\mathrm{aux},r}\hat{I}_{\mathrm{aux}}]}{2(R_r + j(2-s)\omega_e L_r)}$(9.47)

And

$\hat{I}^-_{r2} = j\hat{I}^-_{r1}$(9.48)

- Finally, the time-averaged electromagnetic torque can be shown to be given by

$$T_{\text{mech}}>=\left[\frac{\text{poles}}{2}\right]\text{Re}[(L^2_{\text{main},r}\hat{I}_{\text{main}}\hat{I}_{\text{main}}+L^2_{\text{aux},r}\hat{I}_{\text{aux}}\hat{I}_{\text{aux}})(\hat{K}^{+}-\hat{K}^{-})$$

$$+\text{jL}_{\text{main},r}L_{\text{aux},r}(\hat{I}_{\text{main}}\hat{I}_{\text{aux}}-\hat{I}_{\text{main}}\hat{I}_{\text{aux}})(\hat{K}^{+}+\hat{K}^{-})\,\text{(9.49)}$$

where the symbol Re[ ] again indicates the real part of a complex number and the superscript* indicates the complex conjugate. Note that Eq.9.49 is derived based upon the assumption that the various currents are expressed as rms quantities.

PROBLEMS - chapter 9

PROBLEMS

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

9.1 A 1-kW, 120-V, 60-Hz capacitor-start motor has the following parameters for the main and auxiliary windings (at starting):

$Z_{\text{main}} = 4.82 + j7.25\Omega$main winding

$Z_{\text{aux}} = 7.95 + j9.21\Omega$ auxiliary winding

a. Find the magnitude and the phase angles of the currents in the two windings when rated voltage is applied to the motor under starting conditions.

b. Find the value of starting capacitance that will place the main- and auxiliary-winding currents in time quadrature at starting.

c. Repeat part (a) when the capacitance of part (b) is inserted in series with the auxiliary winding.

9.2 Repeat Problem 9.1 if the motor is operated from a 120-V, 50-Hz source.

9.3 Given the applied electrical frequency and the corresponding impedances Z_{main} and Z_{aux} of the main and auxiliary windings at starting, write a MATLAB script to calculate the value of the capacitance, which, when connected in series with the starting winding, will produce a starting winding current which will lead that of the main winding by 90^o.

9.4 A 500-W, four-pole, 115-V, 60-Hz single-phase induction motor has the following parameters (resistances and reactances in Ω/phase):

$R_{1,\text{main}} = 1.68 R_{2,\text{main}} = 2.96 X_{1,\text{main}} = 1.87 X_{m,\text{main}} = 60.6 X_{2,\text{main}} = 1.72$

$\text{Coreloss} = 38\text{W Friction and windage} = 11.8\text{W}$

Find the speed, stator current, torque, power output, and efficiency when the motor is operating at rated voltage and a slip of 4.2 percent.

9.5 Write a MATLAB script to produce plots of the speed and efficiency of the single-phase motor of Problem 9.4 as a function of output power over the range 0 P_{out} $\leq$500 W.

9.6 At standstill the rms currents in the main and auxiliary windings of a four-pole, capacitor-start induction motor are I_{main} = 20.7 A and I_{aux} = 11.1 A respectively. The auxiliary-winding current leads the main-winding current by 53^o. The effective turns per pole (i.e., the number of turns corrected for the effects of winding distribution) are $N_{\text{main}} = 42$ and $N_{\text{aux}} = 68$. The windings are in space quadrature.

a. Determine the peak amplitudes of the forward and backward stator-mmf waves.

b. Suppose it were possible to adjust the magnitude and phase of the auxiliary-winding current. What magnitude and phase would produce a purely forward mmf wave?

9.7 The equivalent-circuit parameters of an 8-kW, 230-V, 60-Hz, four-pole, two-phase, squirrel-cage induction motor in ohms per phase are

$$R_1 = 0.253 X_1 = 1.14 X_m = 32.7 R_2 = 0.446 X_2 = 1.30$$

This motor is operated from an unbalanced two-phase, 60-Hz source whose phase voltages are, respectively, 223 and 190 V, the smaller voltage leading the larger by 73^o. For a slip of 0.045, find

a. the phase currents in each of the windings and

b. the internal mechanical power.

9.8 The induction motor of Problem 9.7 is supplied from an unbalanced two phase source by a four-wire feeder having an impedance Z = 0.32 + j1.5 Ω /phase. The source voltages can be expressed as

$$\hat{V}_\alpha = 235\angle 0^o \{ \hat{V}_\alpha = 212\angle 78^o$$

For a slip of 5 percent, show that the induction-motor terminal voltages correspond more nearly to a balanced two-phase set than do those of the source.

9.9 The equivalent-circuit parameters in ohms per phase referred to the stator for a two-phase, 1.0 kW, 220-V, four-pole, 60-Hz, squirrel-cage induction motor are given below. The no-load rotational loss is 65 W.

$R_1 = 0.78 R_2 = 4.2 X_1 = X_2 = 5.3 X_m = 93$

a. The voltage applied to phase α is 220 $\angle 0^o$ V and that applied to phase β is 220 $\angle 65^o$ V. Find the net air-gap torque at a slip s = 0.035.

b. What is the starting torque with the applied voltages of part (a)?

c. The applied voltages are readjusted so that $\hat{V}_\alpha = 220\angle 65^o$V and $\hat{V}_\beta = 220\angle 90^o$V. Full load on the machine occurs at s = 0.048. At what slip does maximum internal torque occur? What is the value of the maximum torque?

d. While the motor is running as in part (c), phase β is open-circuited. What is the power output of the machine at a slip s = 0.04?

e. What voltage appears across the open phase- β terminals under the conditions of part (d)?

9.10 A 120-V, 60-Hz, capacitor-run, two-pole, single-phase induction motor has the following parameters:

$L_{\text{main}} = 47.2\text{mH} R_{\text{main}} = 0.38\Omega L_{\text{aux}} = 102\text{mH} R_{\text{aux}} = 1.78\Omega$

$L_r = 2.35 \mu H R_r = 17.2\text{µ}\Omega L_{\text{main},r} = 0.342\text{mH} L_{\text{aux},r} = 0.530\text{mH}$

You may assume that the motor has 48 W of core loss and 23 W of rotational losses. The motor windings are connected with the polarity shown in Fig. 9.1 with a 40 μF run capacitor.

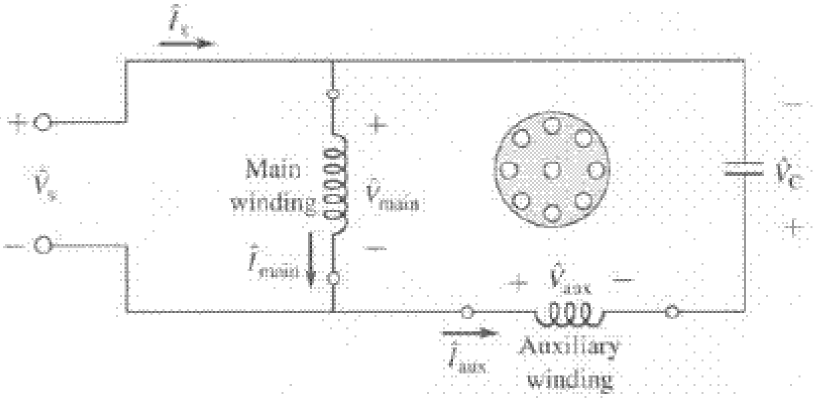


Figure 9.1 Permanent-split-capacitor induction-motor connections

a. Calculate the motor starting torque.

With the motor operating at a speed of 3490 r/min, calculate

b. the main and auxiliary-winding currents,

c. the total line current and the motor power factor,

d. the output power and

e. the electrical input power and the efficiency.

Note that this problem is most easily solved using MATLAB.

9.11 Consider the single-phase motor of Problem 9.10. Write a MATLAB script to search over the range of capacitor values from 25 μF to 75 μF to find the value which will maximize the motor efficiency at a motor speed of 3490 r/min. What is the corresponding maximum efficiency?

9.12 In order to raise the starting torque, the single-phase induction motor of Problem 9.10 is to be converted to a capacitor-start, capacitor-run motor. Write a MATLAB script to find the minimum value of starting capacitance required to raise the starting torque to 0.5 N. m.

Speed and Torque Control
- The objective of this chapter is to discuss various techniques for the control of electric machines. Basic techniques for speed and torque control will be illustrated with typical configurations of drive electronics that are used to implement the control algorithms. - The discussion of this chapter is limited to steady-state operation. The steady-state picture presented here is quite adequate for a wide variety of electricmachine applications. - In the discussion of torque control for synchronous and induction machines, the techniques of field-oriented or vector control are introduced and the analogy is made with torque control in dc motors. This material is somewhat more sophisticated mathematically than the speed-control discussion and requires application of the dq0 transformations.

CONTROL OF DC MOTORS

Speed Control

- The three most common speed-control methods for dc motors are adjustment of the flux, usually by means of field-current control, adjustment of the resistance associated with the armature circuit, and adjustment of the armature terminal voltage.

Field-Current Control: In part because it involves control at a relatively low power level, field-current control is frequently used to control the speed of a dc motor with separately excited or shunt field windings. The equivalent circuit for a separately excited dc machine is in Fig. 10.1. The shunt field current can be adjusted by means of a variable resistance in series with the shunt field. Alternatively, the field current can be supplied by power-electronic circuits which can be used to rapidly change the field current in response to a wide variety of control signals.

- Figure 10.2a shows a switching scheme for pulse-width modulation of the field voltage. It consists of a rectifier which rectifies the ac input voltage, a dc-link capacitor which filters the rectified voltage, producing a dc voltage V_{dc}, and a pulse-width modulator.
- In this system, because only a unidirectional field current is required, the pulsewidth modulator consists of a single switch and a free-

wheeling diode. Assuming both the switch and diode to be ideal, the average voltage across the field winding will be equal to

$$V_f = \mathrm{DV}_{\mathrm{dc}} \quad (10.1)$$

where D is the duty cycle of the switching waveform; i.e., D is the fraction of time that the switch S is on. Figure 10.2b shows the resultant field current.

- In the steady-state the average voltage across the inductor must equal zero, the average field current I_f will thus be equal to

$$I_f = \frac{V_f}{R_f} = D\left[\frac{V_{\mathrm{dc}}}{R_f}\right] \quad (10.2)$$

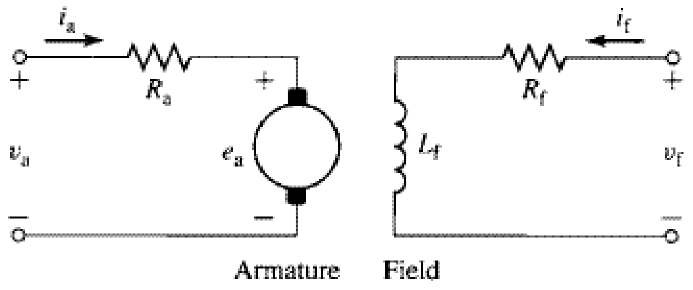
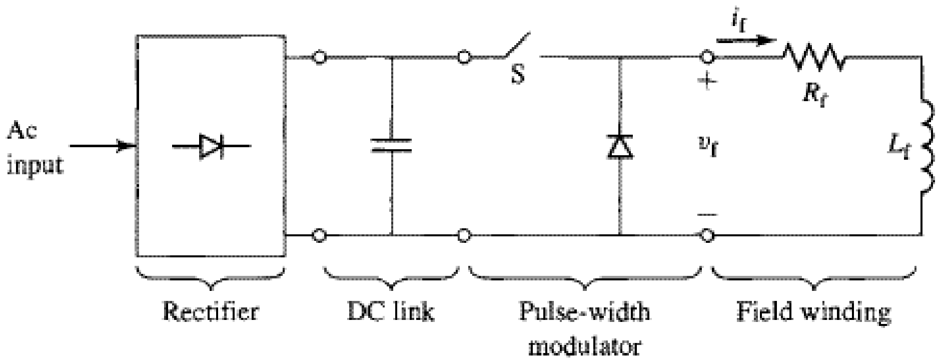


Figure 10.1 Equivalent circuit for a separately excited dc motor.



(a)

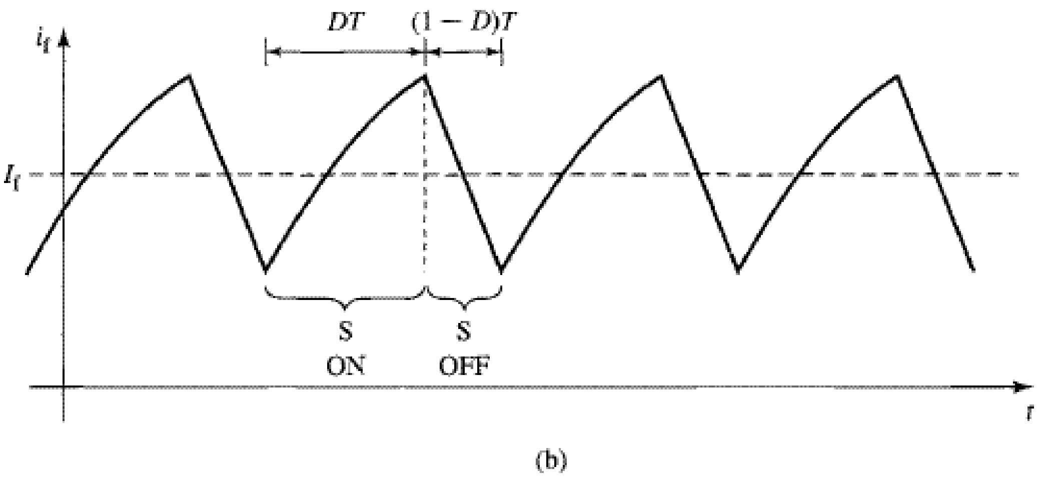


Figure 10.2 (a) Pulse-width modulation system for a dc-machine field winding.

(b) Field-current waveform.

- Thus, the field current can be controlled simply by controlling the duty cycle of the pulse-width modulator. If the field-winding time constant $L_f/R_f$is long compared to the switching time, the ripple current Δi_f will be small compared to the average current I_f.
- To examine the effect of field-current control, let us begin with the case of a dc motor driving a load of constant torque T_{load}. The generated voltage:

$E_a = K_f I_f \omega_m$(10.3)

where I_f is the average field current, ω_m is the angular velocity in rad/sec, and $K_f = K_a P_d N_f$ is a geometric constant which depends upon the dimensions of the motor, the properties of the magnetic material used to construct the motor, as well as the number of turns in the field winding.

- The electromagnetic torque is given as

$T_{\text{mech}} = \frac{E_a I_a}{\omega_m} = K_f I_f I_a$(10.4)

and the armature current is given by

$I_a = \frac{V_a - E_a}{R_a}$(10.5)

- Setting the motor torque equal to T_{load}, solve for ω_m

$\omega_m = \frac{V_a - I_a R_a}{K_f I_f} = \frac{V_a - \frac{T_{\text{load}} R_a}{K_f I_f}}{K_f I_f}$(10.6)

- From Eq. 10.6, recognizing that the armature resistance voltage drop $I_a R_a$ is generally quite small in comparison to the armature voltage V_a , for a given load torque, the motor speed will increase with decreasing field current and decrease as the field current is increased.
- The lowest speed obtainable is that corresponding to maximum field current (the field current is limited by heating considerations); the highest speed is limited mechanically by the mechanical integrity of the rotor and electrically by the effects of armature reaction under weak-field conditions giving rise to poor commutation.
- Armature current is typically limited by motor cooling capability. In many dc motors, cooling is aided by a shaft-driven fan whose cooling capacity is a function of motor speed. Under constant-terminal-voltage operation with varying field current, the $E_a I_a$ product, and hence the allowable motor output power, remain substantially constant as the speed is varied. A dc motor controlled in this fashion is referred to as a constant-power drive.
- Torque, however, varies directly with field flux and therefore has its highest allowable value at the highest field current and hence lowest speed.
- Field-current control is thus best suited to drives requiring increased torque at low speeds.

Armature-Circuit Resistance Control: Armature-circuit resistance control provides a means of obtaining reduced speed by the insertion of external series resistance in the armature circuit.

- It can be used with series, shunt, and compound motors; for the last two types, the series resistor must be connected between the shunt field and the armature, not between the line and the motor.

- Depending upon the value of the series armature resistance, the speed may vary significantly with load, since the speed depends on the voltage drop in this resistance and hence on the armature current demanded by the load.
- The disadvantage of poor speed regulation may not be important in a series motor, which is used only where varying-speed service is required or can be tolerated.
- A significant disadvantage of this method of speed control is that the power loss in the external resistor is large, especially when the speed is greatly reduced.
- Armature-resistance control results in a constant-torque drive because both the field-flux and, to a first approximation, the allowable armature current remain constant as speed changes.
- A variation of this control scheme is given by the shunted-armature method, which may be applied to a series motor, as in Fig. 10.3a, or a shunt motor, as in Fig. 10.3b.

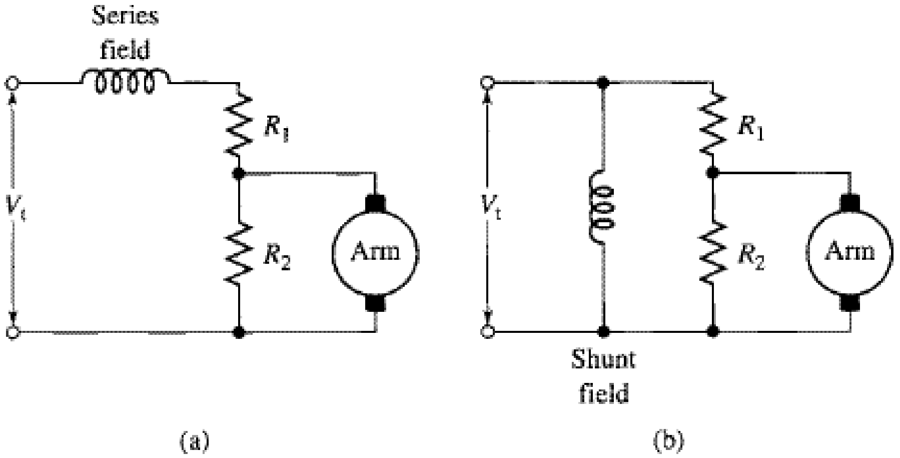


Figure 10.3 Shunted-armature method of speed control applied

to (a) a series motor and (b) a shunt motor.

Armature-Terminal Voltage Control Armature-terminal voltage control can be accomplished with the use of power-electronic systems.

- In Fig. 10.4a, a phase-controlled rectifier in combination with a dclink filter capacitor can be used to produce a variable dc-link voltage which can be applied directly to the armature terminals of the dc motor.
- In Fig. 10.4b, a constant dc-link voltage is produced by a diode rectifier in combination with a dc-link filter capacitor. The armature terminal voltage is then varied by a pulse-width modulation scheme in which switch S is alternately opened and closed.
- When switch S is closed, the armature voltage is equal to the dc-link voltage V_{dc}, and when the switch is opened, current transfers to the freewheeling diode, essentially setting the armature voltage to zero. Thus the average armature voltage under this condition is equal to

$$V_a = \mathrm{DV}_{\mathrm{dc}}(10.7)$$

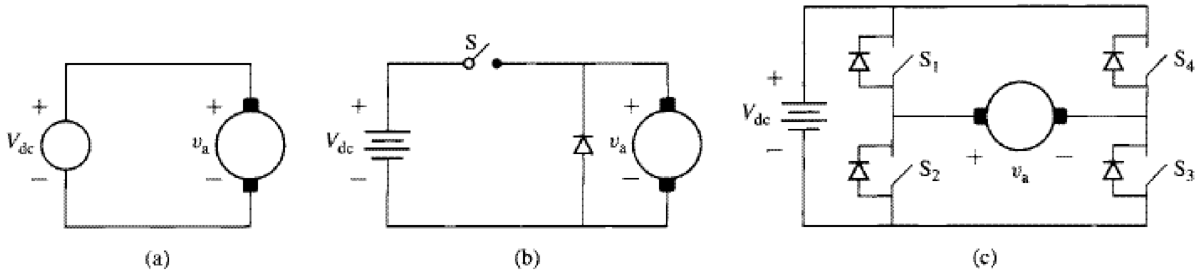


Figure 10.4 Three typical configurations for armature-voltage control.

(a) Variable dc-link voltage (produced by a phase-controlled rectifier)

applied directly to the dc-motor armature terminals.

(b) Constant dc-link voltage with single-polarity pulse-width modulation.

(c)Constant dc-link voltage with a full H-bridge.

where

V_a = average armature voltage (V)

V_{dc} = dc-link voltage (V)

D = PWM duty cycle (fraction of time that switch S is closed)

- Figure 10.4c shows an H-bridge configuration. Note that if switch S3 is held closed while switch S4 remains open, this configuration reduces to that of Fig. 10.4b. However, the H-bridge configuration is more flexible because it can produce both positive- and negative-polarity armature voltage.
- For example, with switches S 1 and S3 closed, the armature voltage is equal to V_{dc} while with switches S2 and S4 closed, the armature voltage is equal to $-V_{\mathrm{dc}}$.
- Using such an H-bridge configuration in combination with an appropriate choice of control signals to the switches allows this PWM system to achieve any desired armature voltage in the range $-V_{\mathrm{dc}} \leq V_a \leq V_{\mathrm{dc}}$.
- Advantages of armature-voltage control: because the voltage drop across the armature resistance is relatively small, a change in the armature terminal voltage of a shunt motor is accompanied in the steady state by a substantially equal change in the speed voltage. Thus, motor speed can be controlled directly by means of the armature terminal voltage.
- Frequently the control of motor voltage is combined with field-current control in order to achieve the widest possible speed range.
- With such dual control, base speed can be defined as the normal-armature-voltage, full-field speed of the motor.
- Speeds above base speed are obtained by reducing the field current; speeds below base speed are obtained by armature-voltage control. The range above base speed is that of a constant-power drive.
- The range below base speed is that of a constant-torque drive because, as in armature resistance control, the flux and the allowable armature current remain approximately

constant.

- The overall output limitations are shown in Fig. 10.6a for approximate allowable torque and in Fig. 10.6b for approximate allowable power.
- The constant-torque characteristic is well suited to many applications in the machinetool industry, where many loads consist largely of

overcoming the friction of moving parts and hence have essentially constant torque requirements.

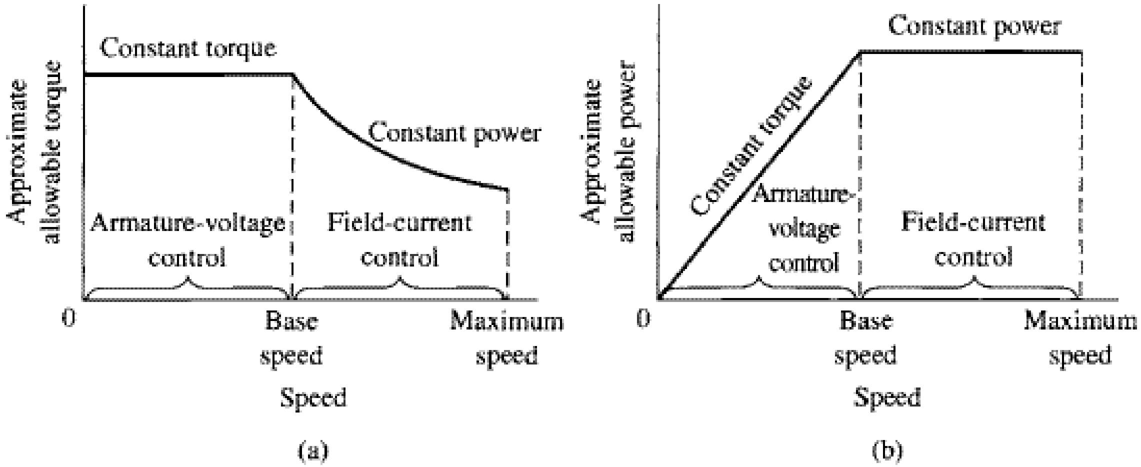


Figure 10.6 (a) Torque and (b) power limitations of combined armature-voltage

and field-current methods of speed control.

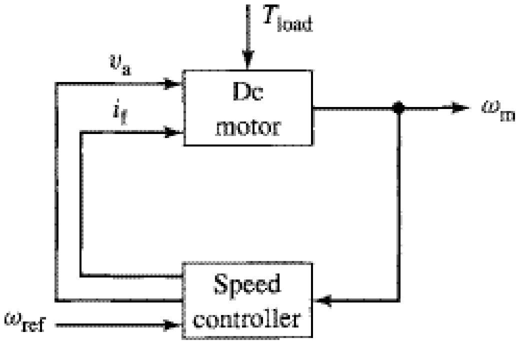


Figure 10.7 Block diagram for a speed-control system

for a separately excited or shunt-connected dc motor.

- Figure 10.7 shows a block diagram of a feedback-control system that can be used to regulate the speed of a separately excited or shunt-

connected dc motor. The inputs to the dc-motor block include the armature voltage and the field current as well as the load torque T_{load}. The resultant motor speed ω_m is fed back to a controller block which represents both the control logic and power electronics and which controls the armature voltage and field current applied to the dc motor, based upon a reference speed signal ω_{ref}.
- Depending upon the design of the controller, with such a scheme it is possible to control the steady-state motor speed to a high degree of accuracy independent of the variations in the load torque.

Torque Control

- The electromagnetic torque in the case of a separately excited or shunt motor

$T_{\text{mech}} = K_f I_f I_a$ (10.8)

and $T_{\text{mech}} = K_m I_a$(10.9)

in the case of a permanent-magnet motor.

- Torque can be controlled directly by controlling the armature current. Fig. 10.8 shows three possible configurations.
- In Fig. 10.8a, a phase-controlled rectifier, in combination with a dc-link filter inductor, can be used to create a variable dc-link current which can be applied directly to the armature terminals of the dc motor.
- In Fig. 10.8b, a constant dc-link current is produced by a diode rectifier. The armature terminal voltage is then varied by a pulse-width modulation scheme in which switch S is alternately opened and closed. When switch S is opened, the current I_{dc} flows into the dc-motor armature while when switch S is closed, the armature is

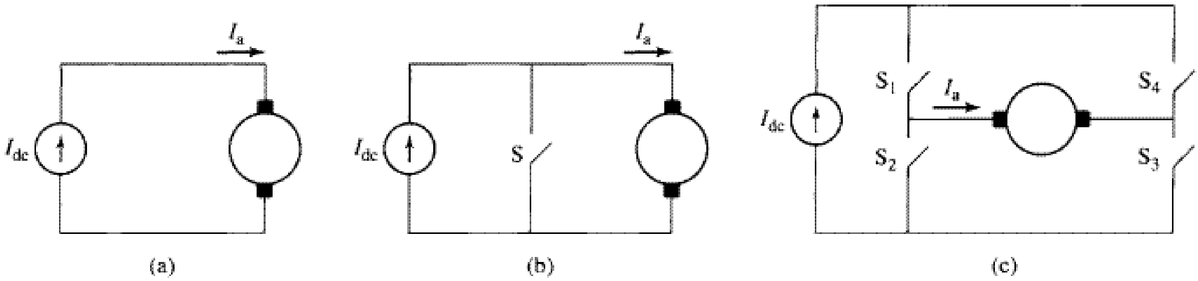


Figure 10.8 Three typical configurations for armature-current control.

(a) Variable dc-link current (produced by a phase-controlled rectifier)

applied directly to the dc-motor armature terminals.

(b) Constant dc-link current with single-polarity pulse-width modulation.

(c) Constant dc-link current with a full H-bridge.

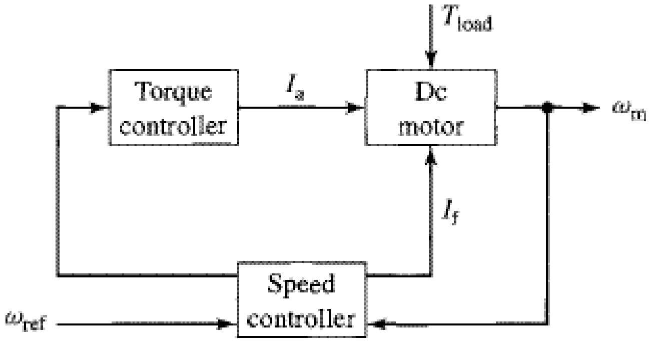


Figure 10.9 Block diagram of a dc-motor

speed-control system using direct-control of motor torque.

shorted and I_a decays. Thus, the duty cycle of switch S will control the average current into the armature.

- Fig 10.8c shows an H-bridge configuration .Appropriate control of the four switches S 1 through S4 allows this PWM system to achieve any desired armature average current in the range $-I_{\text{dc}} \leq I_a \leq I_{\text{dc}}$.

- Note that in each of the PWM configurations of Fig. 10.8b and c, rapid changes in instantaneous current through the dc machine armature can give rise to large voltage spikes, which can damage the machine insulation as well as give rise to flashover and voltage breakdown of the commutator. In order to eliminate these effects, a practical system must include some sort of filter across the armature terminals (such as a large capacitor) to limit the voltage rise and to provide a low-impedance path for the high-frequency components of the drive current.
- Figure 10.9 shows a typical configuration in which the torque control is surrounded by a speed-feedback loop. Instead of controlling the armature voltage, in this case the output of the speed controller is a torque reference signal T_{ref} which in turn serves as the input to the torque controller.

CONTROL OF SYNCHRONOUS MOTORS

Speed Control

- Synchronous motors are essentially constant-speed machines, with their speed being determined by the frequency of the armature currents. The synchronous angular velocity $\omega_s = \left[\frac{2}{\text{poles}}\right]\omega_e$(10.10)

where

$\omega_s =$ synchronous spatial angular velocity of the air-gap mmf wave [rad/sec]

$\omega_e = 2\pi f_e =$ angular frequency of the applied electrical excitation [rad/sec]

$f_e =$ applied electrical frequency [Hz]

- The simplest means of synchronous motor control is speed control via control of the frequency of the applied armature voltage, driving the motor by a polyphase voltage-source inverter shown in Fig. 10.10. This inverter can either be used to supply stepped ac voltage waveforms of amplitude V_{dc} or the switches can be controlled to produce pulse-widthmodulated ac voltage waveforms of variable amplitude. The dc-link voltage V_{dc} can itself be varied, for example, through the use of a phase-controlled rectifier.

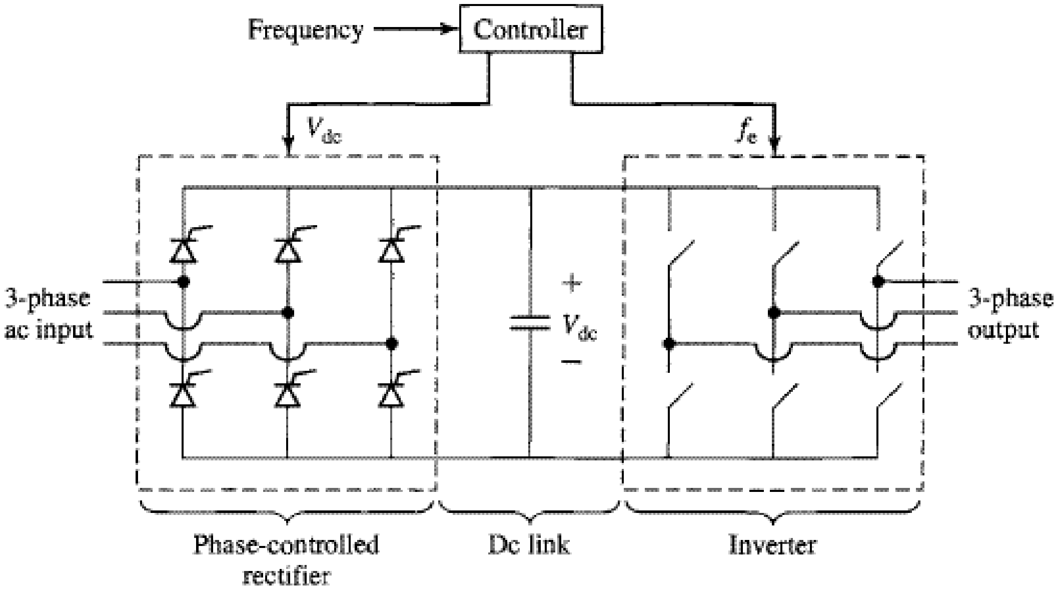


Figure 10.10 Three-phase voltage-source inverter.

- The frequency of the inverter output waveforms can of course be varied by controlling the switching frequency of the inverter switches. For ac-machine applications, coupled with this frequency control must be control of the amplitude of the applied voltage.
- The air-gap component of the armature voltage in an ac machine is proportional to the peak flux density in the machine and the electrical frequency. If we neglect the voltage drop across the armature resistance and leakage reactance,

$$V_a = \left[\frac{f_e}{f_{\text{rated}}}\right]\left[\frac{B_{\text{peak}}}{B_{\text{rated}}}\right]V_{\text{rated}}(10.11)$$

where V_a is the amplitude of the armature voltage, f_e is the operating frequency, and $B_{\text{peak}}$is the peak air-gap flux density. V_{rated}, f_{rated}, and B_{rated} are the corresponding rated-operating-point values.

- Consider a situation in which the frequency of the armature voltage is varied while its amplitude is maintained at its rated value ($V_a = V_{\text{rated}}$). Under these conditions,

$$B_{\text{peak}} = \left[\frac{f_{\text{rated}}}{f_e}\right] B_{\text{rated}} (10.12)$$

For a given armature voltage, the machine flux density is inversely proportional to frequency and thus as the frequency is reduced, the flux density will increase. A significant drop in frequency will increase the flux density to the point of potential machine damage due both to increased core loss and to the increased machine currents required to support the higher flux density.

- As a result, for frequencies less than or equal to rated frequency, it is typical to operate a machine at constant flux density. From Eq. 10.11, with $B_{\text{peak}} = B_{\text{rated}}$

$$V_a = \left[\frac{f_e}{f_{\text{rated}}}\right] V_{\text{rated}} (10.13)$$

$$\frac{V_a}{f_e} = \frac{V_{\text{rated}}}{f_{\text{rated}}} (10.14)$$

- From Eq. 10.14, constant-flux operation can be achieved by maintaining a constant ratio of armature voltage to frequency. This is referred to as constant volts- per-hertz (constant V/Hz) operation. It is typically maintained from rated frequency down to the low frequency at which the armature resistance voltage drop becomes a significant component of the applied voltage.
- If the machine is operated at frequencies in excess of rated frequency with the voltage at its rated value, the air-gap flux density will drop below its rated value. In order to maintain the flux density at its rated value, it would be necessary to increase the terminal voltage for frequencies in excess of rated frequency. In order to avoid insulation damage, it is common to maintain the machine terminal voltage at its rated value for frequencies in excess of rated frequency.
- Figure 10.11 shows a plot of maximum power and maximum torque versus speed for a synchronous motor under variable-frequency

operation. The operating regime below rated frequency and speed is referred to as the constant-torque regime and that above rated speed is referred to as the constant-power regime.

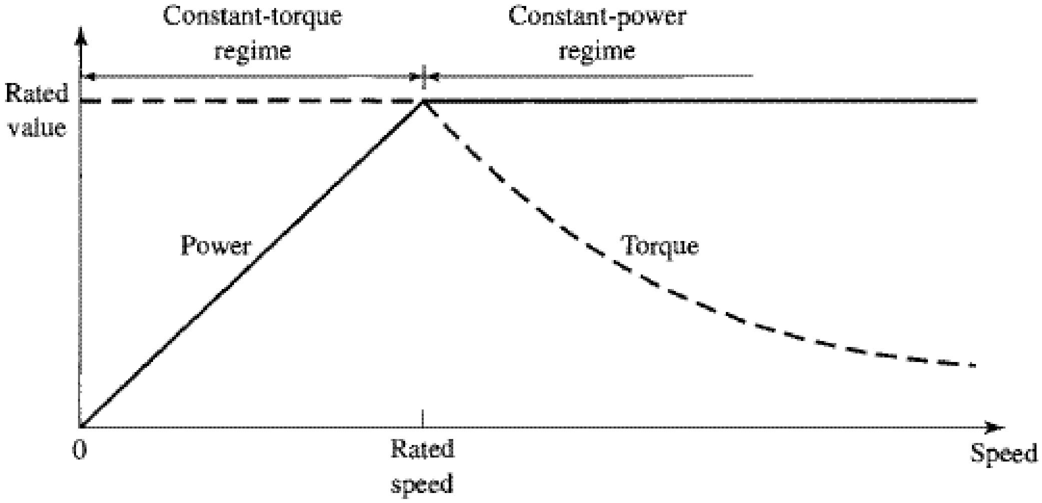


Figure 10.11 Variable-speed operating regimes for a synchronous motor.

- Although during steady-state operation the speed of a synchronous motor is determined by the frequency of the drive, speed control by means of frequency control is of limited use in practice. This is due in most part to the fact that it is difficult for the rotor of a synchronous machine to track arbitrary changes in the frequency of the applied armature voltage.
- Problems with changing speed result from the fact that, in order to develop torque, the rotor of a synchronous motor must remain in synchronism with the stator flux. Control of synchronous motors can be greatly enhanced by control algorithms in which the stator flux and its relationship to the rotor flux are controlled directly.

Torque Control

Direct torque control in an ac machine, which can be implemented in a number of different ways, is commonly referred to as field-oriented control or vector control. To facilitate our discussion of field-oriented control, it is helpful to return to the discussion of Section 5.6.1. Under this viewpoint, which is formalized in Appendix C, stator quantities (flux, current, voltage, etc.) are resolved into components which rotate in synchronism with the rotor. Direct-axis quantities represent those components which are aligned with the field-winding axis, and quadrature-axis components are aligned perpendicular to the field-winding axis.

Section C.2 of Appendix C derives the basic machine relations in dq0 variables for a synchronous machine consisting of a field winding and a three-phase stator winding. The transformed flux-current relationships are found to be

$\lambda = L_d i_d + L_{\mathrm{af}} i_f$(10.15)

$\lambda_q = L_q i_q$(10.16)

$\lambda_f = \frac{3}{2} L_{\mathrm{af}} i_d + L_{\mathrm{ff}} i_f$(10.17)

where the subscripts d, q, and f refer to armature direct-, quadrature-axis, and fieldwinding quantities respectively. Note that throughout this chapter we will assume balanced operating conditions, in which case zero-sequence quantities will be zero and hence will be ignored.

The corresponding transformed voltage equations are

$v_d = R_a i_d + \frac{d\lambda_d}{\mathrm{dt}} - \omega_{\mathrm{me}} \lambda_q$(10.18)

$v_q = R_a i_q + \frac{d\lambda_q}{\mathrm{dt}} + \omega_{\mathrm{me}} \lambda_d$(10.19)

$v_f = R_f i_f + \frac{d\lambda_f}{\mathrm{dt}}$(10.20)

where $\omega_{\mathrm{me}} = (\mathrm{poles}/2)\omega_m$ is the electrical angular velocity of the rotor.

Finally, the electromagnetic torque acting on the rotor of a synchronous motor is shown to be $T_{\mathrm{mech}} = \frac{3}{2}\left[\frac{\mathrm{poles}}{2}\right](\lambda_d i_q - \lambda_q i_d)$(10.21)

Under steady-state, balanced-three-phase operating conditions, $\omega_{\mathrm{me}} = \omega_e$ where ω_e is the electrical frequency of the armature voltage and current in rad/sec. Because the armature-produced mmf and flux waves rotate in synchronism with the rotor and hence with respect to the dq reference frame, under these conditions an observer in the dq reference frame will see constant fluxes, and hence one can set d/dt = 0

Letting the subscripts F, D, and Q indicate the corresponding constant values of field-, direct- and quadrature-axis quantities respectively, the flux-current relationships of Eqs. 10.15 through 10.17 then become

$$\lambda_D = L_d i_D + L_{\mathrm{af}} i_F (10.22)$$

$$\lambda_Q = L_q i_Q (10.23)$$

$$\lambda_F = \frac{3}{2} L_{\mathrm{af}} i_D + L_{\mathrm{ff}} i_F (10.24)$$

Armature resistance is typically quite small, and, if we neglect it, the steady-state voltage equations (Eqs. 10.18 through 10.20) then become

$$v_D = -\omega_e \lambda_Q (10.25)$$

$$v_Q = \omega_e \lambda_D (10.26)$$

$$v_F = R_f i_F (10.27)$$

Finally, we can write Eq. 10.21 as

$$T_{\mathrm{mech}} = \frac{3}{2} \left[\frac{\mathrm{poles}}{2} \right] (\lambda_D i_Q - \lambda_Q i_D) (10.28)$$

From this point on, we will focus our attention on machines in which the effects of saliency can be neglected. In this case, the direct- and quadrature-axis synchronous inductances are equal and we can write

$$L_d = L_q = L_s (10.29)$$

where L_s is the synchronous inductance. Substitution into Eqs. 10.22 and 10.23 and then into Eq. 10.28 gives

$$T_{\text{mech}} = \frac{3}{2}\left[\frac{\text{poles}}{2}\right][(L_s i_D + L_{\text{af}} i_F) i_Q - L_s i_Q i_D]$$

$$= \frac{3}{2}\left[\frac{\text{poles}}{2}\right] L_{\text{af}} i_F i_Q (10.30)$$

Equation 10.30 shows that torque is produced by the interaction of the field flux (proportional to the field current) and the quadrature-axis component of the armature current, in other words the component of armature current that is orthogonal to the field flux. By analogy, we see that the direct-axis component of armature current, which is aligned with the field flux, produces no torque.

This result is fully consistent with the generalized torque expressions which are derived in Chapter 4. Consider for example the equation which expresses the torque in terms of the product of the stator and rotor mmfs (F_s and F_r respectively) and the sine of the angle between them.

$$T = -\left[\frac{\text{poles}}{2}\right]\left[\frac{\mu_0 \pi \text{Dl}}{2\text{g}}\right] F_s F_r \text{sin} \delta_{\text{sr}} (10.31)$$

where δ_r is the electrical space angle between the stator and rotor mmfs. This shows clearly that no torque will be produced by the direct-axis component of the armature mmf which, by definition, is that component of the stator mmf which is aligned with that of the field winding on the rotor.

Equation 10.31 shows the torque in a nonsalient synchronous motor is proportional to the product of the field current and the quadrature-axis component of the armature current. This is directly analogous to torque production in a dc machine for which the equations can be combined to show that the torque is proportional to the product of the field current and the armature current.

The analogy between a nonsalient synchronous machine and dc machine can be further reinforced. Consider the equation, which expresses the rms value of the line-toneutral generated voltage of a synchronous generator as

$$E_{\mathrm{af}} = \frac{\omega_e L_{\mathrm{af}} i_F}{\sqrt{2}} \quad (10.32)$$

Substitution into Eq. 10.30 gives

$$T_{\mathrm{mech}} = \frac{3}{2}\left[\frac{\mathrm{poles}}{\sqrt{2}}\right]\frac{E_{\mathrm{af}} i_Q}{\omega_e} \quad (10.33)$$

This is directly analogous to Eq. $T_{\mathrm{mech}} = E_a I_a / \omega_m$ for a dc machine in which the torque is proportional to the product of the generated voltage and the armature current.

The brushes and commutator of a dc machine force the commutated armature current and armature flux along the quadrature axis such that I_d = 0 and it is the interaction of this quadrature-axis current with the direct-axis field flux that produces the torque. A field-oriented controller which senses the position of the rotor and controls the quadrature-axis component of armature current produces the same effect in a synchronous machine.

Although the direct-axis component of armature current does not play a role in torque production, it does play a role in determining the resultant stator flux and hence the machine terminal voltage, as can be readily shown. Specifically, from the transformation equations of Appendix C,

$$v_a = v_D \cos(\omega_e t) - v_Q \sin(\omega_e t) \quad (10.34)$$

And thus the rms amplitude of the armature voltage is equal to

$$V_a = \sqrt{\frac{v_D^2 + v_Q^2}{2}} = \omega_e \sqrt{\frac{\lambda_D^2 + \lambda_Q^2}{2}}$$

$$= \omega_e \sqrt{\frac{(L_s i_D + L_{\mathrm{af}} i_F)^2 + (L_s i_Q)^2}{2}} \quad (10.35)$$

Dividing V_a by the electrical frequency ω_e, we get an expression for rms armature flux linkages

$$(\lambda_a)_{\mathrm{rms}} = \sqrt{\frac{\lambda_D^2 + \lambda_Q^2}{2}} = \sqrt{\frac{(L_s i_D + L_{\mathrm{af}} i_F)^2 + (L_s i_Q)^2}{2}} \quad (10.36)$$

Similarly, the transformation equations of Appendix C can be used to show that the rms amplitude of the armature current is equal to

$$I_a = \sqrt{\frac{i_D^2+i_Q^2}{2}} \quad (10.37)$$

From Eq.10.33 we see that torque is controlled by the product $i_F i_Q$ of the field current and thee quadrature-axis component of the armature current. Thus, simply specifying a desired torque is not sufficient to uniquely determine either i_F or i_Q.

In fact, under the field-oriented-control viewpoint presented here, there are actually three independent variables, i_F,i_Q and i_D, and , in general, three constraints will be required to uniquely determine them. In addition to specifying the desired torque, a typical controller will implement additional contraints flux-linkages and current using the basic relationships found in Eqs.10.36 and 10.37.

Figure 10.12a shows a typical field-oriented torque-control system in block-diagram form. The torque-controller block has two inputs, T_{ref}, the reference value or set point for torque and $(i_F)_{\text{ref}}$, the reference value or set point for the field current, which is also supplied to the power supply which supplies the current i_F to the motor field winding. $(i_F)_{\text{ref}}$ is determined by an auxiliary controller which also determines

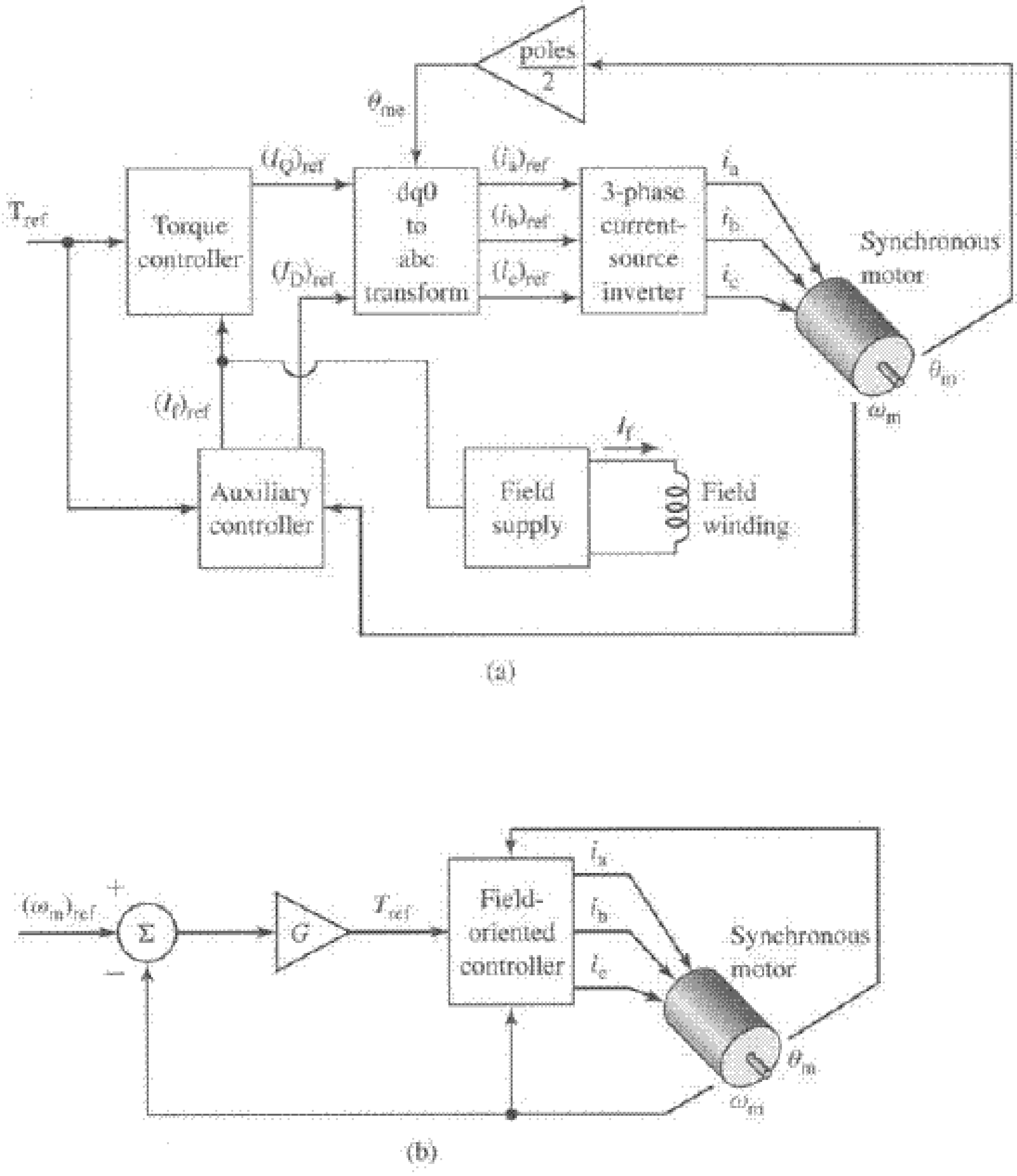


Figure 10.12 (a) Block diagram of a field-oriented torque-control system for a synchronous motor. (b) Block diagram of a synchronous-motor speed-control loop built around a field-oriented torque control system.

the reference value $(i_D)_{\mathrm{ref}}$ of the direct-axis current, based upon desired values for the armature current and voltage. The torque controller calculates the desired quadrature axis current from Eq. 10.30 based upon T_{ref} and $(i_F)_{\text{ref}}$

$$(i_Q)_{\text{ref}} = \frac{2}{3}\left[\frac{2}{\text{poles}}\right]\frac{T_{\text{ref}}}{L_{\text{af}}(i_F)_{\text{ref}}}(10.38)$$

Note that a position sensor is required to determine the angular position of the rotor in order to implement the dq0 to abc transformation.

In a typical application, the ultimate control objective is not to control motor torque but to control speed or position. Figure 10.12a shows how the torque-control system of Fig. 10.12b can be used as a component of a speed-control loop, with speed feedback forming an outer control loop around the inner torque-control loop.

As we have discussed, a practical field-oriented control must determine values for all three currents i_F, i_D, and i_Q. In Example 10.8 two of these values were chosen relatively arbitrarily (i_F = 2.84 and i_D = 0) and the result was a control that achieved the desired torque but with a terminal voltage 30 percent in excess of the motorrated voltage. In a practical system, additional constraints are required to achieve an acceptable control algorithm.

One such algorithm would be to require that the motor operate at rated flux and at unity terminal power factor. Such an algorithm canbe derived with reference to the

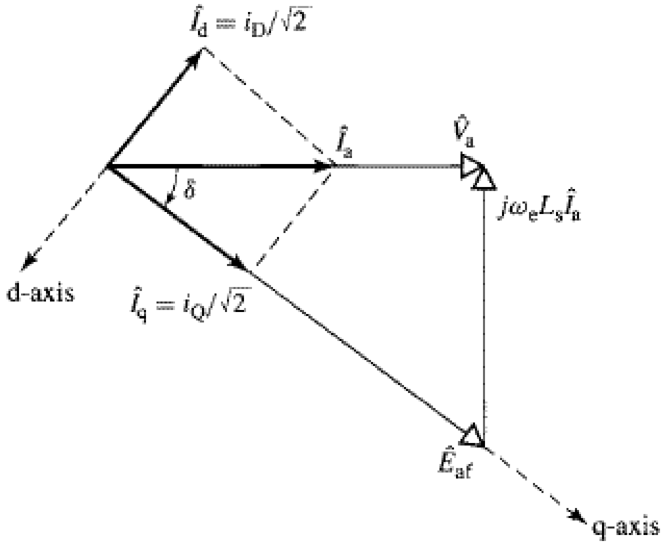

$\hat{I}_d = i_D/\sqrt{2}$
$\hat{I}_a$
$\hat{V}_a$
$j\omega_e L_s \hat{I}_a$
δ
d-axis
$\hat{I}_q = i_Q/\sqrt{2}$
$\hat{E}_{af}$
q-axis

Figure 10.13 Phasor diagram for unity-powerfactor

field-oriented-control algorithm

phasor diagram of Fig 10.13 and can be implemented using the following steps:

Step 1. Calculate the line-to-neutral armature voltage corresponding to rated flux as

$$V_a = (V_a)_{\text{rated}} \left[\frac{\omega_m}{(\omega_m)_{\text{rated}}} \right] (10.39)$$

where ***SORRY, THIS MEDIA TYPE IS NOT SUPPORTED.*** is the rated line-to-neutral armature voltage at rated motor speed, ω_m is the desired motor speed, and $(\omega_m)_{\text{rated}}$ is its rated speed.

Step 2. Calculate the rms armature current from the desired torque T_{ref} as

$$I_a = \frac{P_{\text{ref}}}{3\text{V}_a} = \frac{T_{\text{ref}}\omega_m}{3\text{V}_a} (10.40)$$

where P_{ref} is the mechanical power corresponding to the desired torque.

Step 3. Calculate the angle δ based upon the phasor diagram of Fig 11.14

$$\delta = -\tan^{-1}\left[\frac{\omega_e L_s I_a}{V_a}\right] (10.41)$$

where $\omega_e = \omega_{\text{me}} = (\text{poles}/2)\omega_m$ is the electrical frequency corresponding to the desired motor speed.

Step 4. Calculate $(i_Q)_{\text{ref}}$ and $(i_D)_{\text{ref}}$

$$(i_Q)_{\text{ref}} = \sqrt{2} I_a \cos\delta (10.42)$$

$$(i_D)_{\text{ref}} = \sqrt{2} I_a \sin\delta (10.43)$$

Step 5. Calculate $(i_F)_{\text{ref}}$

$$(i_F)_{\text{ref}} = \frac{2}{3}\left[\frac{2}{\text{poles}}\right]\frac{T_{\text{ref}}}{L_{\text{af}}(i_Q)_{\text{ref}}} (10.44)$$

The discussion of this section has focused upon synchronous machines with field windings and the corresponding capability to control the field excitation. The basic concept, of course, also applies to synchronous machines with permanent magnets on the rotor. However, in the case of permanent-magnet synchronous machines, the effective field excitation is fixed and, as a result, there is one less degree of freedom for the field-oriented control algorithm.

For a permanent-magnet synchronous machine, since the effective equivalent field current is fixed by the permanent magnet, the quadrature-axis current is determined directly by the desired torque. Consider a three-phase permanent-magnet motor whose rated rms, line-to-neutral open-circuit voltage is $(E_{\text{af}})_{\text{rated}}$ at electrical frequency $(\omega_e)_{\text{rated}}$. From Eq. 10.32 we see that the equivalent $L_{\text{af}} I_f$ product for this motor, which we will refer to by the symbol Λ_{PM}, is

$$\Lambda_{\text{PM}} = \frac{\sqrt{2}(E_{\text{af}})_{\text{rated}}}{(\omega_e)_{\text{rated}}} (10.45)$$

Thus, the direct-axis flux-current relationship for this motor, corresponding to Eq. 10.22, becomes

$$\lambda_D = L_d i_D + \mathit{\Lambda}_{\text{PM}} (10.46)$$

and the torque expression becomes

$$T_{\text{mech}} = \frac{3}{2}\left[\frac{\text{poles}}{2}\right]\mathit{\Lambda}_{\text{PM}} i_Q (10.47)$$

From Eq. 10.47 we see that, for a permanent-magnet sychronous machine under field-oriented control, the quadrature-axis current is uniquely determined by the desired torque and Eq. 10.38 becomes

$$(i_Q)_{\text{ref}} = \frac{2}{3}\left[\frac{2}{\text{poles}}\right]\frac{T_{\text{ref}}}{\mathit{\Lambda}_{\text{PM}}} (10.48)$$

Once $(i_Q)_{\text{ref}}$ has been specified, the only remaining control choice remains to determine the desired value for the direct-axis current, $(i_D)_{\text{ref}}$.One possibility is simply to set $(i_D)_{\text{ref}} = 0$.

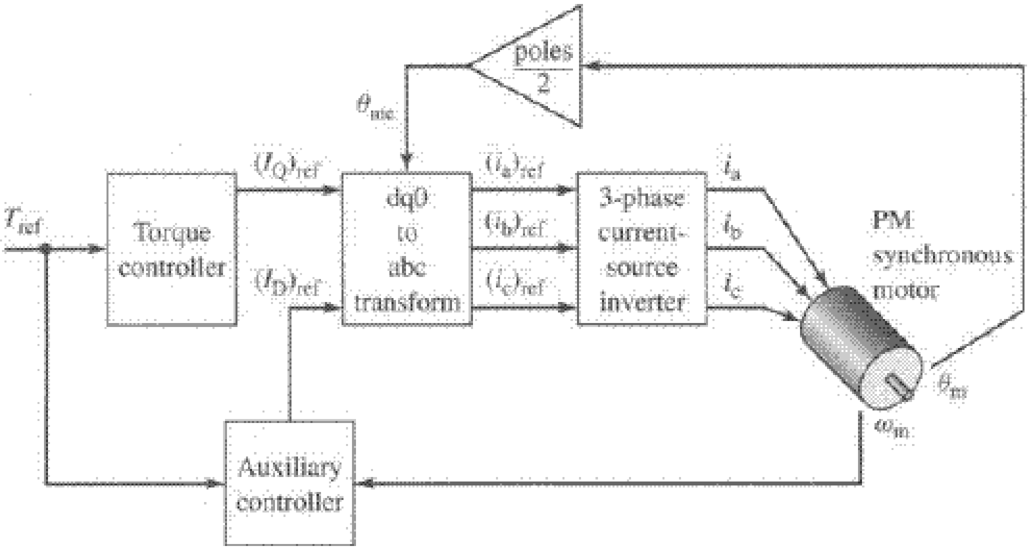


Figure 10.14 Block diagram of a field-oriented torque-control system

for a permanent-magnet synchronous motor.

This will clearly result in the lowest possible armature current for a given torque. However, as we have seen in Example 10.8, this is likely to result in terminal voltages in excess of the rated voltage of the machine. As a result, it is common to supply direct-axis current so as to reduce the direct-axis flux linkage of Eq. 10.22, which will in turn result in reduced terminal voltage. This technique is commonly referred to as flux weakening and comes at the expense of increased armature current. In practice, the chosen operating point is determined by a trade-off between reducing the armature voltage and an increase in armature current. Figure 10.14 shows the block diagram for a field-oriented-control system for use with a permanent-magnet motor.

CONTROL OF INDUCTION MOTORS

Speed Control

Induction motors supplied from a constant-frequency source admirably fulfill the requirements of substantially constant-speed drives. Many motor applications, however, require several speeds or even a continuously adjustable range of speeds. From the earliest days of ac power systems, engineers have been interested in the development of adjustable-speed ac motors.

The synchronous speed of an induction motor can be changed by (a) changing the number of poles or (b) varying the line frequency. The operating slip can be changed by (c) varying the line voltage, (d) varying the rotor resistance, or (e) applying voltages of the appropriate frequency to the rotor circuits. The salient features of speed-control methods based on these five possibilities are discussed in the following five sections.

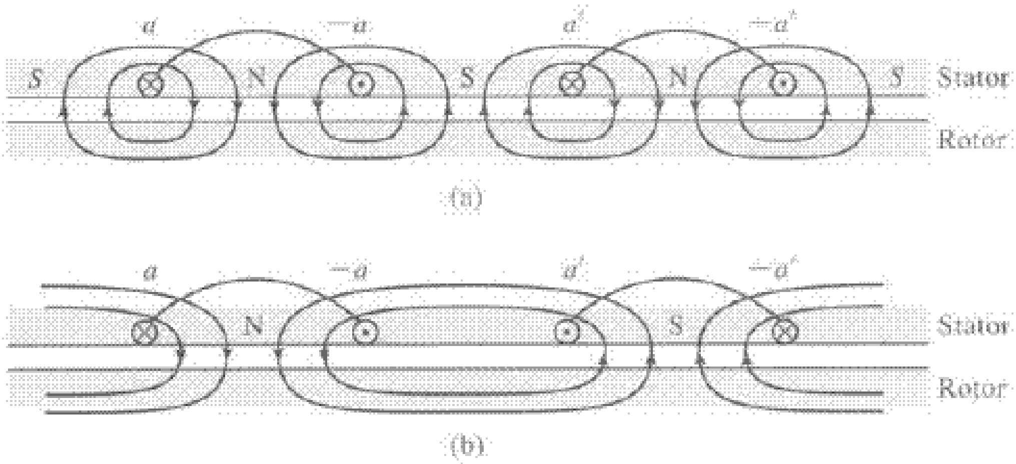


Figure 10.15 Principles of the pole-changing winding.

Pole-Changing Motors In pole-changing motors, the stator winding is designed so that, by simple changes in coil connections, the number of poles can be changed in the ratio 2 to 1. Either of two synchronous speeds can then be selected. The rotor is almost always of the squirrel-cage type, which reacts by producing a rotor field having the same number of poles as the inducing stator field. With two independent sets of stator windings, each arranged for pole changing, as many as four synchronous speeds can be obtained in a squirrel-cage motor, for example, 600, 900, 1200, and 1800 r/min for 60-Hz operation.

The basic principles of the pole-changing winding are shown in Fig. 10.15, in which aa and $a'a'$ are two coils comprising part of the phase-a stator winding. An actual winding would, of course, consist of several coils in each group. The windings for the other stator phases (not shown in the figure) would be similarly arranged. In Fig. 10.15a the coils are connected to produce a four-pole field; in Fig. 10.15b the current in the $a'a'$ coil has been reversed by means of a controller, the result being a two-pole field.

Figure 10.16 shows the four possible arrangements of these two coils: they can be connected in series or in parallel, and with their currents either in the same direction (four-pole operation) or in the opposite direction (two-pole operation). Additionally, the machine phases can be connected either in Y or Δ, resulting in eight possible combinations.

Note that for a given phase voltage, the different connections will result in differing levels of air-gap flux density. For example, a change from a Δ to a Y connection will reduce the coil voltage (and hence the air-gap flux density) for a given coil arrangement by $\sqrt{3}$. Similarly, changing from a connection with two coils in series to two in parallel will double the voltage across each coil and therefore double the magnitude of the air-gap flux density. These changes in flux density can, of course, be compensated for by changes in the applied winding voltage. In any case, they must be considered, along with corresponding changes in motor torque, when the configuration to be used in a specific application is considered.

Armature-Frequency Control: The synchronous speed of an induction motor can be controlled by varying the frequency of the applied armature voltage. This method

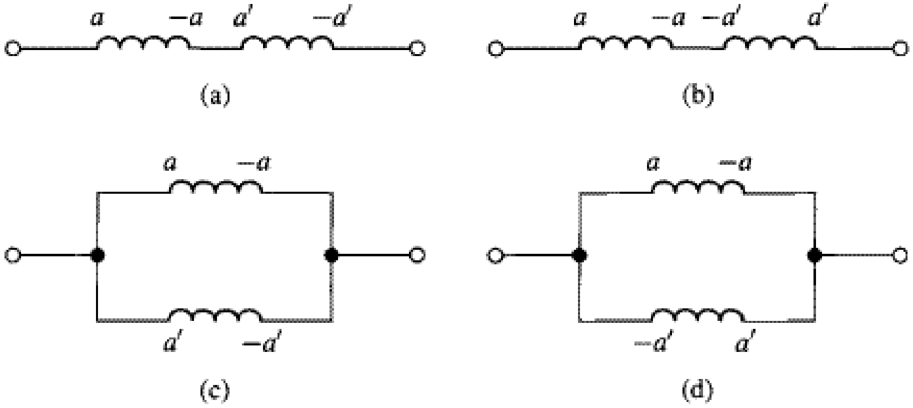


Figure 10.16 Four possible arrangements of phase-a stator coils

in a pole-changing induction motor: (a) series-connected, four-pole;

(b) series-connected, two-pole; (c)parallel-connected, four-pole;

(d) parallel-connected, two-pole.

of speed control is identical to that discussed in Section 10.2.1 for synchronous machines. In fact, the same inverter configurations used for synchronous machines, such as the three-phase voltage-source inverter of

Fig. 10.10, can be used to drive induction motors. As is the case with any ac motor, to maintain approximately constant flux density, the armature voltage should also be varied directly with the frequency (constant-volts-per-hertz).

The torque-speed curve of an induction motor for a given frequency can be calculated by using the methods of Chapter 6 within the accuracy of the motor parameters at that frequency. Consider the torque expression which is repeated here.

$$T_{\mathrm{mech}} = \frac{1}{\omega_s}\left[\frac{n_{\mathrm{ph}} V_{1,\mathrm{eq}}^2 (R_2/s)}{(R_{1,\mathrm{eq}}+(R_2/s))^2+(X_{1,\mathrm{eq}}+X_2)^2}\right](10.49)$$

where $\omega_s = (2/\mathrm{poles})\omega_e$ and ω_e is the electrical excitation frequency of the motor in rad/sec,

$$\hat{V}_{1,\mathrm{eq}} = \hat{V}_1\left[\frac{\mathrm{jX}_m}{R_1+j(X_1+X_m)}\right](10.50)$$

and

$$R_{1,\mathrm{eq}} + \mathrm{jX}_{1,\mathrm{eq}} = \frac{\mathrm{jX}_m(R_1+\mathrm{jX}_1)}{R_1+j(X_1+X_m)}(10.51)$$

To investigate the effect of changing frequency, we will assume that R_1 is negligible. In this case,

$$\hat{V}_{1,\mathrm{eq}} = \hat{V}_1\left[\frac{X_m}{X_1+X_m}\right](10.52)$$

$$R_{1,\mathrm{eq}} = 0(10.53)$$

And

$$X_{1,\mathrm{eq}} = \frac{X_m X_1}{X_1+X_m}(10.54)$$

Let the subscript 0 represent rated-frequency values of each of the induction motor parameters. As the electrical-excitation frequency is varied, we can then write

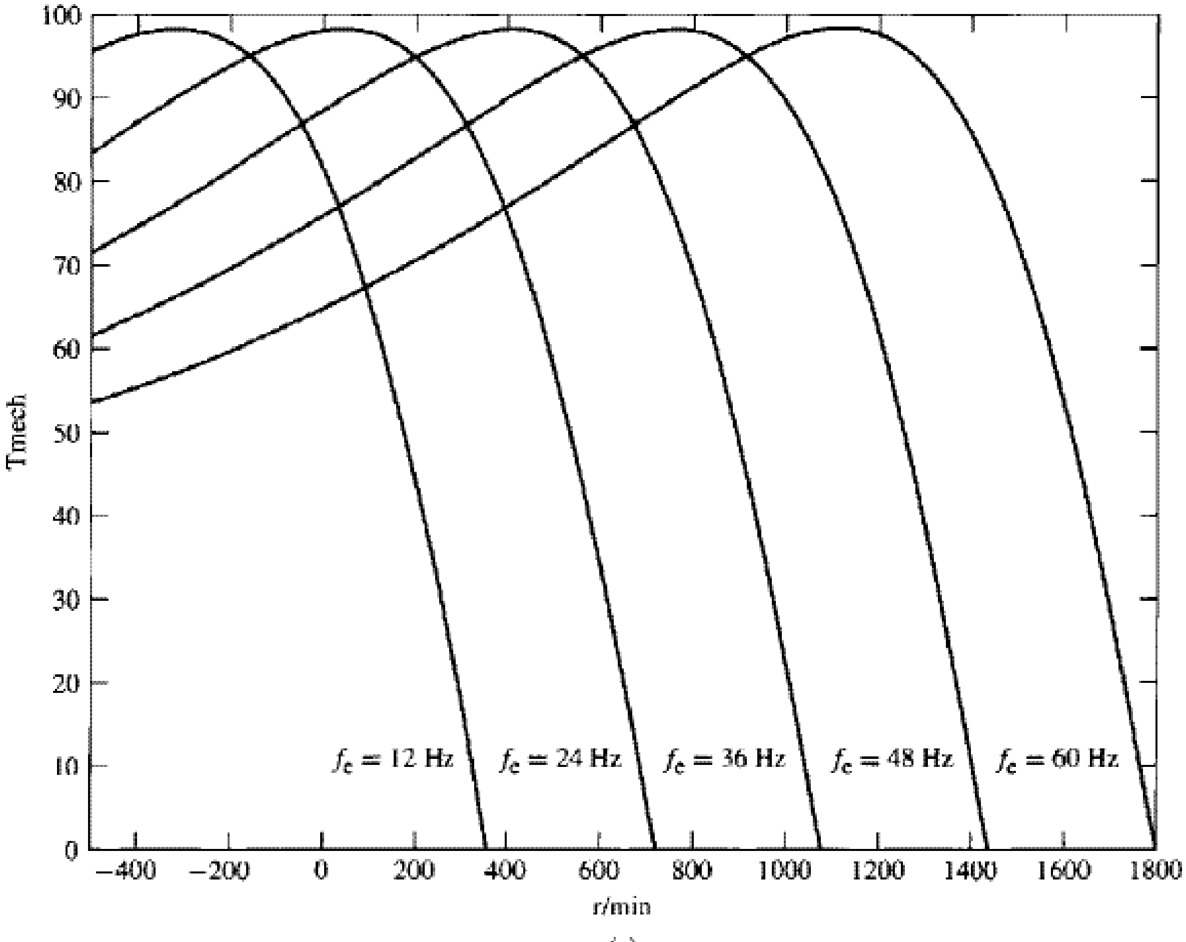
$$(X_{1,\mathrm{eq}} + X_2) = \left[\frac{\omega_e}{\omega_{e0}}\right](X_{1,\mathrm{eq}} + X_2)_0 (10.55)$$

Under constant-volts-per-hertz control, we can also write the equivalent source voltage as

$$\hat{V}_1 = \left[\frac{\omega_e}{\omega_{e0}}\right](\hat{V}_1)_0 (10.56)$$

And hence, since $\hat{V}_{1,\mathrm{eq}}$is equal to $\hat{V}_1$ multiplied by a reactance ratio which stays constant with changing frequency,

$$\hat{V}_{1,\mathrm{eq}} = \left[\frac{\omega_e}{\omega_{e0}}\right](\hat{V}_{1,\mathrm{eq}})_0 (10.57)$$



(a)

Figure 10.17 A family of typical induction-motor speed-torque curves for a four-pole motor for various values of the electrical supply frequency. (a) R_1 sufficiently small so that its effects are negligible.(b) R_1not negligible.

Finally, we can write the motor slip as

$$s = \frac{\omega_s - \omega_m}{\omega_s} = \frac{\text{poles}}{2}\left[\frac{\Delta\omega_m}{\omega_e}\right] \quad (10.58)$$

where $\Delta\omega_m = \omega_s - \omega_m$ is the difference between the synchronous and mechanical angular velocities of the motor.

Substitution of Eqs. 10.55 through 10.58 into Eq. 10.49 gives

$$T_{\text{mech}} = \frac{n_{\text{ph}}[(V_{1,\text{eq}})_0]^2(R_2/\Delta\omega)}{\left[\frac{2\omega_{e0}}{\text{poles}}(R_2/\Delta\omega)\right]^2 + [(X_{1,\text{eq}} + X_2)_0]^2} \quad (10.59)$$

Equation 10.59 shows the general trend in which we see that the frequency dependence of the torque-speed characteristic of an induction motor appears only in the term $R_2/\Delta\omega$. Thus, under the assumption that R_1 is negligible, as the electrical supply frequency to an induction motor is changed, the shape of the speed-torque curve as a function of $\Delta\omega$ (the difference between the synchronous speed and the motor speed) will remain unchanged. As a result, the torque-speed characteristic will simply shift along the speed axis as $\omega_e(f_e)$is varied.

A set of such curves is shown in Fig. 10.17a. Note that as the electrical frequency (and hence the synchronous speed) is decreased, a given value of $\Delta\omega$ corresponds to a larger slip.

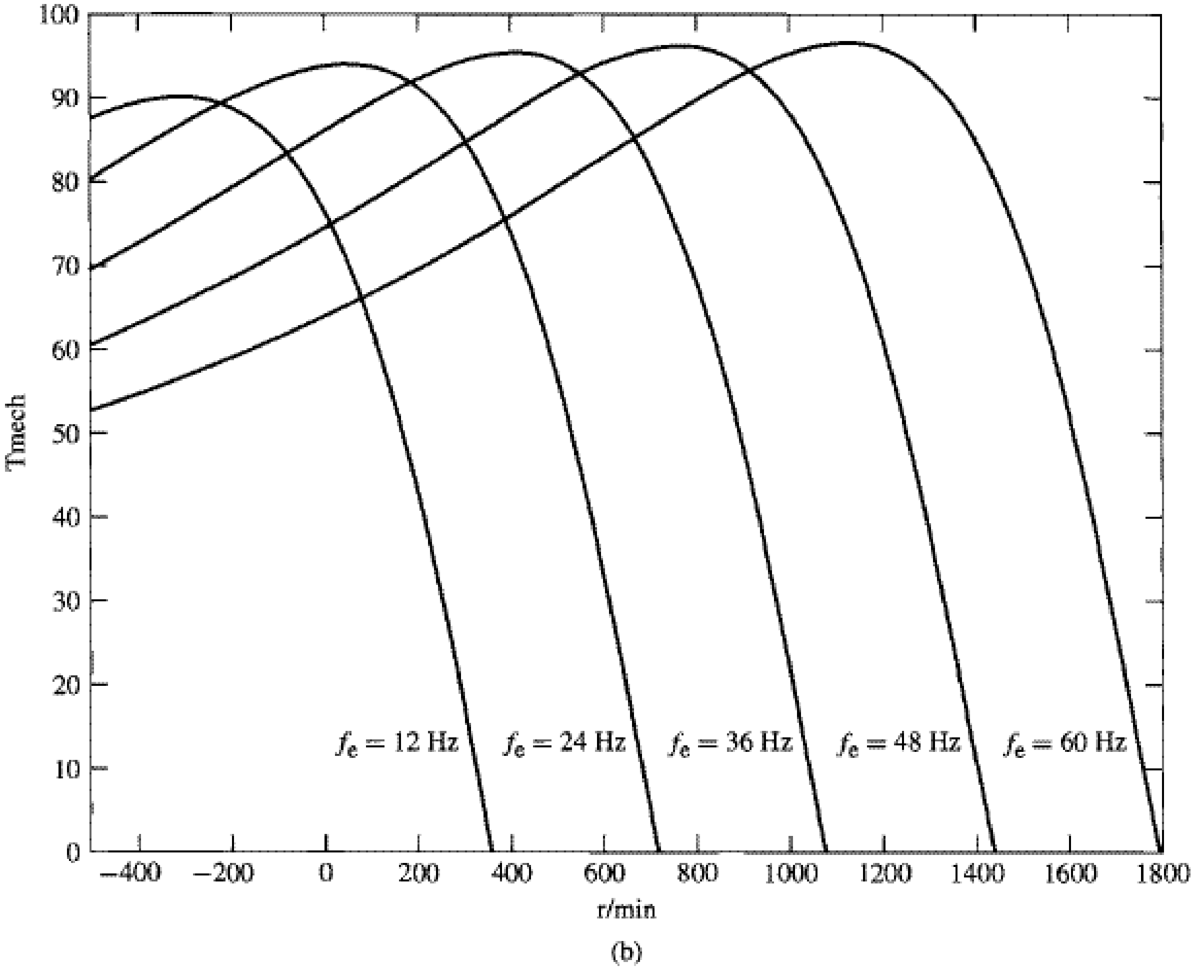


Figure 10.17(continued)

Thus, for example, if the peak torque of a four-pole motor driven at 60 Hz occurs at 1638 r/min, corresponding to a slip of 9 percent, when driven at 30 Hz, the peak torque will occur at 738 r/min, corresponding to a slip of 18 percent.

In practice, the effects of R_1 may not be fully negligible, especially for large values of slip. If this is the case, the shape of the speed-torque curves will vary somewhat with the applied electrical frequency. Figure 10.17b shows a typical family of curves for this case.

Line-Voltage Control The internal torque developed by an induction motor is proportional to the square of the voltage applied to its primary terminals, as shown by the two torque-speed characteristics in Fig. 10.18. If the load has the torque-speed characteristic shown by the dashed line, the speed will be reduced from n_1 to n_2. This method of speed control is commonly used with small squirrel-cage motors driving fans, where cost is an issue and the inefficiency of high-slip operation can be tolerated. It is characterized by a rather limited range of speed control.

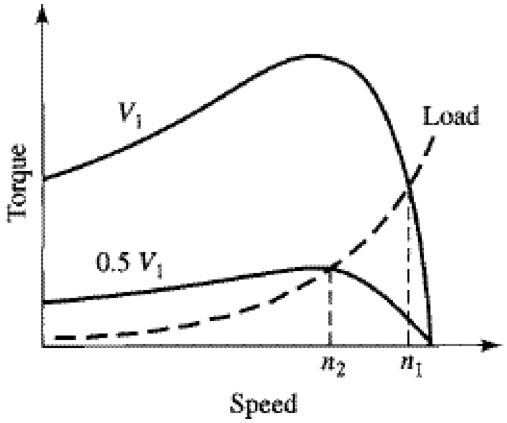


Figure 10.18 Speed control by means of line voltage.

Rotor-Resistance Control The possibility of speed control of a wound-rotor motor by changing its rotor-circuit resistance has already been pointed out in Section 6.7.1.

The torque-speed characteristics for three different values of rotor resistance are shown in Fig. 10.19. If the load has the torque-speed characteristic shown by the dashed line, the speeds corresponding to each of the values of rotor resistance are $n_1, n_2 \text{and} n_3$. This method of speed control has characteristics similar to those of dc shunt-motor speed control by means of resistance in series with the armature.

The principal disadvantages of both line-voltage and rotor-resistance control are low efficiency at reduced speeds and poor speed regulation with respect to change in load. In addition, the cost and maintenance requirements of wound-rotor induction motors are sufficiently high that squirrel-cage motors combined with solid-state drives have become the preferred option in most applications.

Torque Control

In Section 10.2.2 we developed the concept of field-oriented-control for synchronous machines. Under this viewpoint, the armature flux and current are resolved into two components which rotate synchronously with the rotor and with the air-gap flux wave.

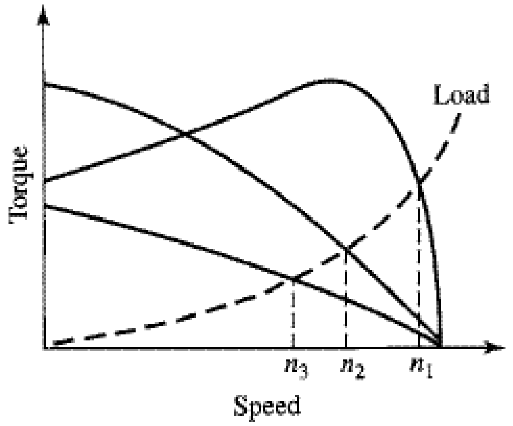


Figure 10.19 Speed control by means of rotor resistance.

The components of armature current and flux which are aligned with the field-winding are referred to as direct-axis components while those which are perpendicular to this axis are referred to as quadrature-axis components.

It turns out that the same viewpoint which we applied to synchronous machines can be applied to induction machines. As is discussed in Section 6.1, in the steadystate the mmf and flux waves produced by both the rotor and stator windings of an induction motor rotate at synchronous speed and in synchronism with each other.

Thus, the torque-producing mechanism in an induction machine is equivalent to that of a synchronous machine. The difference between the two is that, in an induction machine, the rotor currents are not directly supplied but rather are induced as the induction-motor rotor slips with respect to the rotating flux wave produced by the stator currents.

To examine the application of field-oriented control to induction machines, we begin with the dq0 transformation of Section C.3 of Appendix C. This transformation transforms both the stator and rotor quantities into a synchronously rotating reference frame. Under balanced three-phase, steady-state conditions, zero-sequence quantities will be zero and the remaining direct- and quadrature-axis quantites will be constant.

Hence the flux-linkage current relationships of Eqs. C.52 through C.58 become

$\lambda_D = L_S i_D + L_m i_{\mathrm{DR}}$(10.60)

$\lambda_Q = L_S i_Q + L_m i_{\mathrm{QR}}$(10.61)

$\lambda_{\mathrm{DR}} = L_m i_D + L_R i_{\mathrm{DR}}$(10.62)

$\lambda_{\mathrm{QR}} = L_m i_Q + L_R i_{\mathrm{QR}}$(10.63)

In these equations, the subscripts D, Q, DR, and QR represent the constant values of the direct- and quadrature-axis components of the stator and rotor quantities respectively. It is a straight-forward matter to show that the inductance parameters can be determined from the equivalent-circuit parameters as

$L_m = \frac{X_{m0}}{\omega_{e0}}$(10.64)

$L_S = L_m + \frac{X_{10}}{\omega_{e0}}$(10.65)

$L_R = L_m + \frac{X_{20}}{\omega_{e0}}$(10.66)

where the subscript 0 indicates the rated-frequency value.

The transformed voltage equations Eqs. C.63 through C.68 become

$v_D = R_a i_D - \omega_e \lambda_Q$(10.67)

$v_Q = R_a i_Q + \omega_e \lambda_D$(10.68)

$0 = R_{\mathrm{aR}} i_{\mathrm{DR}} - (\omega_e - \omega_{\mathrm{me}}) \lambda_{\mathrm{QR}}$(10.69)

$0 = R_{\mathrm{aR}} i_{\mathrm{QR}} + (\omega_e - \omega_{\mathrm{me}}) \lambda_{\mathrm{DR}}$(10.70)

where one can show that the resistances are related to those of the equivalent circuit as

and

$R_a = R_1$(10.71)

$R_{\mathrm{aR}} = R_2$(10.72)

For the purposes of developing a field-oriented-control scheme, we will begin with the torque expression of Eq. C.70

$T_{\mathrm{mech}} = \frac{3}{2} \left[\frac{\mathrm{poles}}{2} \right] \left[\frac{L_m}{L_R} \right] (\lambda_{\mathrm{DR}} i_q - \lambda_{\mathrm{QR}} i_d)$(11.73)

For the derivation of the dq0 transformation in Section C.3, the angular velocity of the reference frame was chosen to the synchronous speed as determined by the stator electrical frequency ω_e. It was not necessary for the purposes of the derivation to specify the absolute angular location of the reference frame. It is convenient at this point to choose the direct axis of the reference frame aligned with the rotor flux.

If this is done

$\lambda_{\mathrm{QR}} = 0$(10.74)

and the torque expression of Eq. 11.75 becomes

$T_{\mathrm{mech}} = \frac{3}{2} \left[\frac{\mathrm{poles}}{2} \right] \left[\frac{L_m}{L_R} \right] \lambda_{\mathrm{DR}} i_Q$(10.75)

From Eq. 10.69 we see that

$i_{\mathrm{DR}} = 0$(10.76)

and thus

$\lambda_{\mathrm{DR}} = L_m i_D$(10.77)

and

$\lambda_D = L_S i_D$(10.78)

From Eqs. 11.77 and 11.78 we see that by choosing set $\lambda_{\mathrm{QR}} = 0$ and thus aligning the synchronously rotating reference frame with the axis of the rotor flux, the directaxis rotor flux (which is, indeed, the total rotor flux) as well as the direct-axis flux are determined by the direct-axis component of the armature current. Notice the direct analogy with a dc motor. In a dc motor, the field- and direct-axis armature fluxes are determined by the field current and in this field-oriented control scheme, the rotor and direct-axis armature fluxes are determined by the direct-axis armature current.

In other words, in this field-oriented control scheme, the direct-axis component of armature current serves the same function as the field current in a dc machine.

The torque equation, Eq. 10.75, completes the analogy with the dc motor. We see that once the rotor direct-axis flux λ_{DR} is set by the direct-axis armature current, the torque is then determined by the quadrature-axis armature current just as the torque is determined by the armature current in a dc motor.

In a practical implementation of the technique which we have derived, the directand quadrature-axis currents $i_D \mathrm{and} i_Q$ must be transformed into the three motor phase currents $i_a(t),i_b(t),\mathrm{and} i_c(t)$. This can be done using the inverse dq0 transformation of Eq. C.48 which requires knowledge of θ_S, the electrical angle between the axis of phase a, and the direct-axis of the synchronously rotating reference frame.

Since it is not possible to measure the axis of the rotor flux directly, it is necessary to calculate θ_S, where $\theta_S = \omega_e t + \theta_0$ as given by Eq. C.46. Solving Eq. 10.70 for ω_e gives $\omega_e = \omega_{\mathrm{me}} - R_{\mathrm{aR}} \left[\frac{i_{\mathrm{QR}}}{\lambda_{\mathrm{DR}}} \right]$(10.79)

From Eq. 10.63 with λ_{QR} = 0 we see that

$$i_{\mathrm{QR}} = -\left[\frac{L_m}{L_R}\right] i_Q \text{ (10.80)}$$

Eq. 10.80 in combination with Eq. 10.77 then gives

$$\omega_e = \omega_{\mathrm{me}} + \frac{R_{\mathrm{aR}}}{L_R}\left[\frac{i_Q}{i_D}\right] = \omega_{\mathrm{me}} + \frac{1}{\tau_R}\left[\frac{i_Q}{i_D}\right] \text{(10.81)}$$

where $\tau_R = L_R / R_{\mathrm{aR}}$ is the rotor time constant. We can now integrate Eq. 10.81 to find

$$\hat{\theta}_S = \left[\omega_{\mathrm{me}} + \frac{1}{\tau_R}\left[\frac{i_Q}{i_D}\right]\right] t + \theta_0 \text{(11.82)}$$

where $\hat{\theta}_S$ indicates the calculated value of θ_S (often referred to as the estimated value of θ_S). In the more general dynamic sense

$$\hat{\theta}_S = \int_o^t \left[\omega_{\mathrm{me}} + \frac{1}{\tau_R}\left[\frac{i_Q}{i_D}\right]\right] dt' + \theta_0 \text{(10.83)}$$

Note that both Eqs. 10.82 and 10.83 require knowledge of θ_0, the value of θ_S at t = 0. Although we will not prove it here, it turns out that in a practical implementation, the effects of an error in this initial angle decay to zero with time, and hence it can be set to zero without any loss of generality.

Figure 10.20 a shows a block diagram of a field-oriented torque-control system for an induction machine. The block labeled "Estimator" represents the calculation of Eq. 10.83 which calculates the estimate of θ_S required by the transformation from dq0 to abc variables.

Note that a speed sensor is required to provide the rotor speed measurement required by the estimator. Also notice that the estimator requires knowledge of the rotor time constant $\tau_R = L_R / R_{\mathrm{aR}}$. In general, this will not be known exactly, both due to uncertainty in the machine parameters as well as due to the fact that the rotor

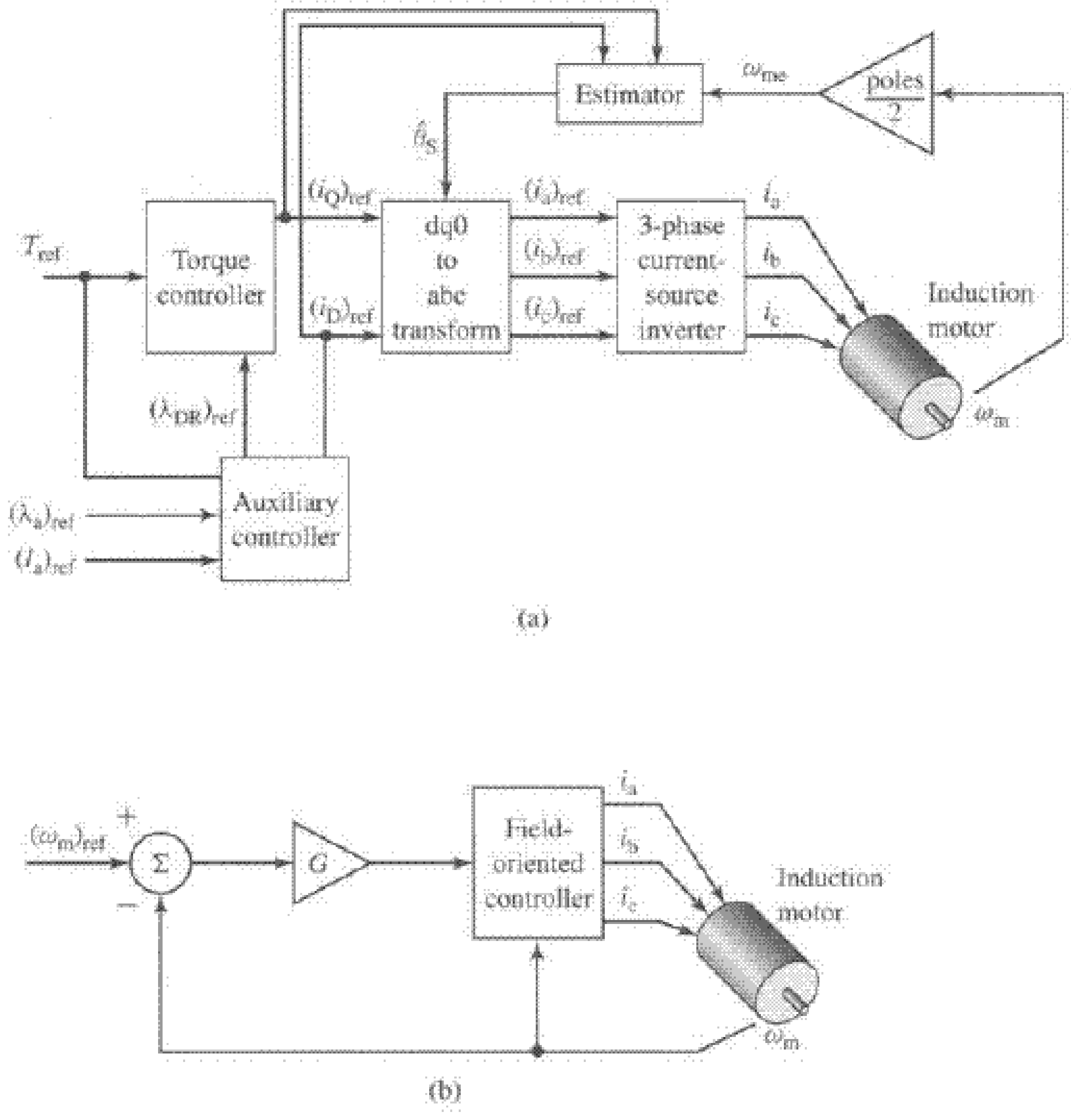


Figure 10.20 (a) Block diagram of a field-oriented torque-control system for an induction motor. (b) Block diagram of an induction-motor speed-control loop built around a field-oriented torque control system.

resistance R_{aR} will undoubtedly change with temperature as the motor is operated. It can be shown that errors in τ_R result in an offset in the estimate of θ_S, which in turn will result in an error in the estimate for the position of the rotor flux with the result that the applied armature currents will not be exactly aligned with the direct- and quadrature-axes. The torque controller will still work basically as expected, although there will be corresponding errors in the torque and rotor flux.

As with the synchronous motor, the rms armature flux-linkages can be found from Eq. 10.36 as

$$(\lambda_a)_{\text{rms}} = \sqrt{\frac{\lambda_D^2+\lambda_Q^2}{2}} \tag{10.84}$$

Combining Eqs. 10.61 and 10.80 gives

$$\lambda_Q = L_S i_Q + L_m i_{\text{QR}} = \left[L_S - \frac{L_m^2}{L_R}\right] i_Q \tag{10.85}$$

Substituting Eqs. 10.78 and 10.85 into Eq. 11.84 gives

$$(\lambda_a)_{\text{rms}} = \sqrt{\frac{L_S^2 i_D^2 + \left[L_s - \frac{L_m^2}{L_R}\right]^2 i_Q^2}{2}} \tag{10.86}$$

Finally, as discussed in the footnote to Eq. 10.35, the rms line-to-neutral armature voltage can be found as

$$V_a = \sqrt{\frac{v_D^2+v_Q^2}{2}} = \sqrt{\frac{(R_a i_D - \omega_e \lambda_Q)^2 + (R_a i_Q + \omega_e \lambda_D)^2}{2}}$$

$$= \sqrt{\frac{\left[R_a i_D - \omega_e \left[L_S - \frac{L_m^2}{L_R}\right] i_Q\right]^2 + (R_a i_Q + \omega_e L_S i_D)^2}{2}} \tag{10.87}$$

These equations show that the armature flux linkages and terminal voltage are determined by both the direct- and quadrature-axis components of the armature current.

Thus, the block marked "Auxiliary Controller" in Fig. 10.20 a, which calculates the reference values for the direct- and quadrature-axis currents, must calculate the reference currents $(i_D)_{\text{ref}}$ and $(i_Q)_{\text{ref}}$ which achieve the desired torque subject to constraints on armature flux linkages (to avoid saturation in the motor), armature current, $(I_a)_{\text{rms}} = \sqrt{(i_D^2 + i_Q^2)/2}$ (to avoid excessive armature heating) and armature voltage (to avoid potential insulation damage).

Note that, as we discussed with regard to synchronous machines in Section 10.2.2, the torque-control system of Fig. 10.20 a is typically imbedded within a larger control loop. One such example is the speed-control loop of Fig. 10.20 b.

The ability to independently control the rotor flux and the torque has important control implications. Consider, for example, the response of the direct-axis rotor flux to a change in direct-axis current. Equation C.66, with $\lambda_{\mathrm{qR}} = 0$, becomes

$$0 = R_{\mathrm{aR}} i_{\mathrm{dR}} + \frac{d\lambda_{\mathrm{dR}}}{\mathrm{dt}} (10.88)$$

Substituting for i_{dR} in terms of λ_{dR}

$$i_{\mathrm{dR}} = \frac{\lambda_{\mathrm{dR}} - L_m i_d}{L_R} (10.89)$$

gives a differential equation for the rotor flux linkages λ_{DR}

$$\frac{d\lambda_{\mathrm{dR}}}{\mathrm{dt}} + \left[\frac{R_{\mathrm{aR}}}{L_R}\right]\lambda_{\mathrm{dR}} = \left[\frac{L_m}{L_R}\right] i_d (10.90)$$

From Eq. 10.90 we see that the response of the rotor flux to a step change in direct axis current id is relatively slow; λ_{dR} will change exponentially with the rotor time constant of $\tau_R = L_R / R_{\mathrm{aR}}$. Since the torque is proportional to the product $\lambda_{\mathrm{dR}} i_q$ we see that fast torque response will be obtained from changes in i_q. Thus, for example, to implement a step change in torque, a practical control algorithm might start with a step change in $(i_Q)_{\mathrm{ref}}$ to achieve the desired torque change, followed by an adjustment in $(i_D)_{\mathrm{ref}}$ (and hence λ_{dR}) to readjust the armature current or terminal voltage as desired. This adjustment in $(i_D)_{\mathrm{ref}}$ would be coupled with a compensating adjustment in $(i_Q)_{\mathrm{ref}}$ to maintain the torque at its desired value.

CONTROL OF VARIABLE-RELUCTANCE

MOTORS

Unlike dc and ac (synchronous or induction) machines, VRMs cannot be simply "plugged in" to a dc or ac source and then be expected to run. As is dicussed in Chapter 8, the phases must be excited with (typically unipolar) currents, and the timing of these currents must be carefully correlated with the position of the rotor poles in order to produce a useful, time-averaged torque. The result is that although the VRM itself is perhaps the simplest of rotating machines, a practical VRM drive system is relatively complex.

VRM drive systems are competitive only because this complexity can be realized easily and inexpensively through power and microelectronic circuitry. These drive systems require a fairly sophisticated level of controllability for even the simplest modes of VRM operation. Once the capability to implement this control is available, fairly sophisticated control features can be added (typically in the form of additional software) at little additional cost, further increasing the competitive position of VRM drives.

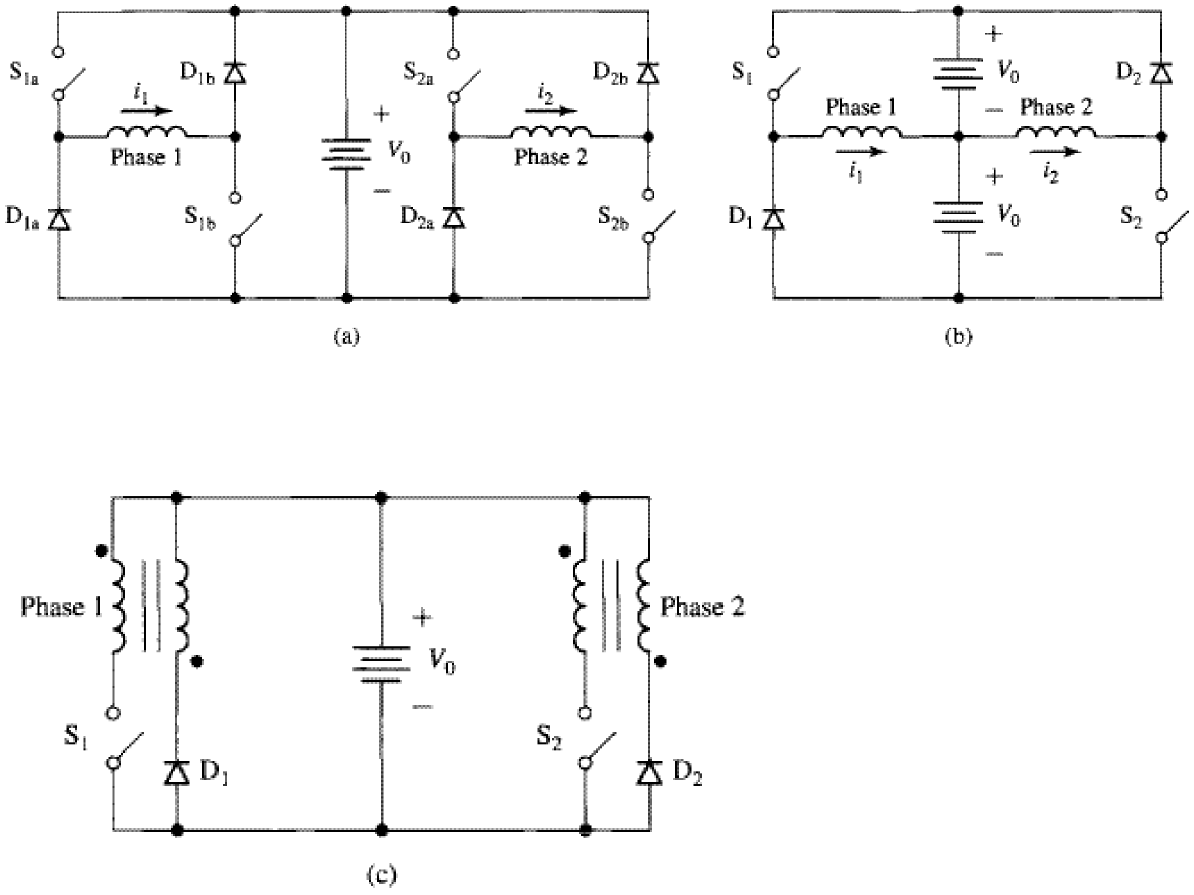
In addition to the VRM itself, the basic VRM drive system consists of the following components: a rotor-position sensor, a controller, and an inverter. The function of the rotor-position sensor is to provide an indication of shaft position which can be used to control the timing and waveform of the phase excitation. This is directly analogous to the timing signal used to control the firing of the cylinders in an automobile engine.

The controller is typically implemented in software in microelectronic (microprocessor) circuitry. Its function is to determine the sequence and waveforms of the phase excitation required to achieve the desired motor speed-torque characteristics. In addition to set points of desired speed and/or torque and shaft position (from the shaftposition sensor), sophisticated controllers often employ additional inputs including shaft-speed and phase-current magnitude. Along with the basic control function of determining the desired torque for a given speed, the more sophisticated controllers attempt to provide excitations which are in some sense optimized (for maximum efficiency, stable transient behavior, etc.).

The control circuitry consists typically of low-level electronics which cannot be used to directly supply the currents required to excite the motor phases. Rather its output consists of signals which control an inverter which in turn supplies the phase currents. Control of the VRM is achieved by the application of an appropriate set of currents to the VRM phase windings.

Figures 10.21a to c show three common configurations found in inverter systems for driving VRMs. Note that these are simply H-bridge inverters of the type discussed in Section 10.3. Each inverter is shown in a two-phase configuration. As is clear from the figures, extension of each configuration to drive additional phases can be readily accomplished.

The configuration of Fig. 10.21a is perhaps the simplest. Closing switches $S_{1\mathrm{a}}$ and $S_{1\mathrm{b}}$ connects the phase-1 winding across the supply ($v_1 = V_0$) and causes the winding current to increase. Opening just one of the switches forces a short across



(a)

(b)

(c)

Figure 11.23 Inverter configurations.

(a) Two-phase inverter which uses two switches per phase.

(b) Two-phase inverter which uses a split supply and one switch per phase.

(c) Two-phase inverter with bifilar phase windings and one switch per phase.

the winding and the current will decay, while opening both switches connects the winding across the supply with negative polarity through the diodes ($v_1 = -V_0$) and the winding current will decay more rapidly. Note that this configuration is capable of regeneration (returning energy to the supply), but not of supplying negative current to the phase winding. However, since the torque in a VRM is proportional to the square of the phase current, there is no need for negative winding current. As discussed in Section 10.3.2, the process of pulse-width modulation, under which a series of switch configurations alternately charge and discharge a phase winding, can be used to control the average winding current. Using this technique, an inverter such as that of Fig. 10.21a can readily be made to supply the range of waveforms required to drive a VRM.

The inverter configuration of Fig. 10.21a is perhaps the simplest of H-bridge configurations which provide regeneration capability. Its main disadvantage is that it requires two switches per phase. In many applications, the cost of the switches (and their associated drive circuitry) dominates the cost of the inverter, and the result is that this configuration is less attractive in terms of cost when compared to other configurations which require one switch per phase.

Figure 10.21b shows one such configuration. This configuration requires a split supply (i.e., two supplies of voltage V_0) but only a single switch and diode per phase.

Closing switch S 1 connects the phase-1 winding to the upper dc source. Opening the switch causes the phase current to transfer to diode D 1, connecting the winding to the bottom dc source. Phase 1 is thus supplied by the upper dc source and regenerates through the bottom source. Note that to maintain symmetry and to balance the energy supplied from each source equally, phase 2 is connected oppositely so that it is supplied from the bottom source and regenerates into the top source.

The major disadvantages of the configuration of Fig. 10.21b are that it requires a split supply and that when the switch is opened, the switch must withstand a voltage of 2 V_0. This can be readily seen by recognizing that when diode D 1 is forward-biased, the switch is connected across the two supplies. Such switches are likely to be more expensive than the switches required by the configuration of Fig. 10.21a. Both of these issues will tend to offset some of the economic advantage which can be gained by the elimination of one switch and one diode as compared with the inverter circuit of Fig. 10.21a.

A third inverter configuration is shown in Fig. 10.21c. This configuration requires only a single dc source and uses only a single switch and diode per phase. This configuration achieves regeneration through the use of bifilar phase windings. In a bifilar winding, each phase is wound with two separate windings which are closely coupled magnetically (this can be achieved by winding the two windings at the same time) and can be thought of as the primary and secondary windings of a transformer.

When switch S 1 is closed, the primary winding of phase 1 is energized, exciting the phase winding. Opening the switch induces a voltage in the secondary winding (note the polarity indicated by the dots in Fig. 11.23c) in such a direction as to forwardbias D 1. The result is that current is transferred from the primary to the secondary winding with a polarity such that the current in the phase decays to zero and energy is returned to the source.

Although this configuration requires only a single dc source, it requires a switch which must withstand a voltage in excess of 2 V_0 (the degree of excess being determined by the voltage developed across the primary leakage reactance as current is switched from the primary to the secondary windings) and requires the more complex bifilar winding in the machine. In addition, the switches in this configuration must include snubbing circuitry (typically consisting of a resistor-capacitor combination) to protect them from transient overvoltages. These overvoltages result from the fact that although the two windings of the bifilar winding are wound such that they are as closely coupled as possible, perfect coupling cannot be achieved. As a result, there will be energy stored in the leakage fields of the primary winding which must be dissipated when the switch is opened.

As is discussed in Section 10.3, VRM operation requires control of the current applied to each phase. For example, one control strategy for constant torque production is to apply constant current to each phase during the time that $\mathrm{dL}/d\theta_m$ for that phase is constant. This results in constant torque proportional to the square of the phasecurrent magnitude. The magnitude of the torque can be controlled by changing the magnitude of the phase current.

The control required to drive the phase windings of a VRM is made more complex because the phase-winding inductances change both with rotor position and with current levels due to saturation effects in the magnetic material. As a result, it is not possible in general to implement an open-loop PWM scheme based on a precalculated algorithm. Rather, pulse-width-modulation is typically accomplished through the use of current feedback. The instantaneous phase current can be measured and a switching scheme can be devised such that the switch can be turned off when the current has been found to reach a desired maximum value and turned on when the current decays to a desired minimum value. In this manner the average phase current is controlled to a predetermined function of the rotor position and desired torque.

This section has provided a brief introduction to the topic of drive systems for variable-reluctance machines. In most cases, many additional issues must be considered before a practical drive system can be implemented. For example, accurate rotor-position sensing is required for proper control of the phase excitation, and the control loop must be properly compensated to ensure its stability. In addition, the finite rise and fall times of current buildup in the motor phase windings will ultimately limit the maximum achievable rotor torque and speed.

The performance of a complete VRM drive system is intricately tied to the performance of all its components, including the VRM, its controller, and its inverter.

In this sense, the VRM is quite different from the induction, synchronous, and dc machines discussed earlier in this chapter. As a result, it is useful to design the complete drive system as an integrated package and not to design the individual components (VRM, inverter, controller, etc.) separately. The inverter configurations of Fig. 11.21 are representative of a number of possible inverter configurations which can be used in VRM drive systems. The choice of an inverter for a specific application must be made based on engineering and economic considerations as part of an integrated VRM drive system design.

SUMMARY

This chapter introduces various techniques for the control of electric machines. The broad topic of electric machine control requires a much more extensive discussion than is possible here so our objectives have been somewhat limited. Most noticeably, the discussion of this chapter focuses almost exclusively on steady-state behavior, and the issues of transient and dynamic behavior are not considered.

Much of the control flexibility that is now commonly associated with electric machinery comes from the capability of the power electronics that is used to drive these machines. This chapter builds therefore on the discussion of power electronics in Chapter 10.

The starting point is a discussion of dc motors for which it is convenient to subdivide the control techniques into two categories: speed and torque control. The algorithm for speed control in a dc motor is relatively straight forward. With the exception of a correction for voltage drop across the armature resistance, the steadystate speed is determined by the condition that the generated voltage must be equal to the applied armature voltage. Since the generated voltage is proportional to the field flux and motor speed, we see that the steady-state motor speed is proportional to the armature voltage and inversely proportional to the field flux.

An alternative viewpoint is that of torque control. Because the commutator/brush system maintains a constant angular relationship between the field and armature flux, the torque in a dc motor is simply proportional to the product of the armature current and the field flux. As a result, dc motor torque can be controlled directly by controlling the armature current as well as the field flux.

Because synchronous motors develop torque only at synchronous speed, the speed of a synchronous motor is simply determined by the electrical frequency of the applied armature excitation. Thus, steady-state speed control is simply a matter of armature frequency control. Torque control is also possible. By transforming the stator quantities into a reference frame rotating synchronously with the rotor (using the dq0 transformation of Appendix C), we found that torque is proportional to the field flux and the component of armature current in space quadrature with the field flux. This is directly analogous to the torque production in a dc motor. Control schemes which adopt this viewpoint are referred to as vector or field-oriented control.

Induction machines operate asynchronously; rotor currents are induced by the relative motion of the rotor with respect to the synchronously rotating stator-produced flux wave. When supplied by a constant-frequency source applied to the armature winding, the motor will operate at a speed somewhat lower than synchronous speed, with the motor speed decreasing as the load torque is increased. As a result, precise speed regulation is not a simple matter, although in most cases the speed will not vary from synchronous speed by an excessive amount.

Analogous to the situation in a synchronous motor, in spite of the fact that the rotor of an induction motor rotates at less than synchronous speed, the interaction between the rotor and stator flux waves is indeed synchronous. As a result, a transformation into a synchronously rotating reference frame results in rotor and stator flux waves which are constant. The torque can then be expressed in terms of the product of the rotor flux linkages and the component of armature current in quadrature with the rotor flux linkages (referred to as the quadrature-axis component of the armature current) in a fashion directly analogous to the field-oriented viewpoint of a synchronous motor. Furthermore, it can be shown that the rotor flux linkages are proportional to the direct-axis component of the armature current, and thus the direct-axis component of armature current behaves much like the field current in a synchronous motor. This field-oriented viewpoint of induction machine control, in combination with the power-electronic and control systems required to implement it, has led to the widespread applicability of induction machines to a wide range of variable-speed applications.

Finally, this chapter ends with a brief discussion of the control of variablereluctance machines. To produce useful torque, these machines typically require relatively complex, nonsinusoidal current waveforms whose shape must be controlled as a function of rotor position. Typically, these waveforms are produced by pulse-width modulation combined with current feedback using an H-bridge inverter of the type discussed in Chapter 10. The details of these waveforms depend heavily upon the geometry and magnetic properties of the VRM and can vary significantly from motor to motor.